

# Access Control Capability Assessment Method Based on Security Entropy

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**<sup>1</sup>Abstract**—In this paper, we propose analysis methods based on security entropy to overcome the problem of quantitative analysis, after going through the study of access control capability assessment for computer information system. At First, we computed the uncertainty how system determine irregular access behavior using the security entropy theory. Next, we defined the security theorem of classificatory information system, and proposed the standard of access control capability. Finally, we analyzed the typical access control models using the methods, and compared security and applicability of them. It proved that the method is appropriate for security quantitative analysis of access control model and for the evaluation of access control capability in information system.

**Index Terms**—Information entropy; Security entropy; Classificatory access control model; Direct unauthorized access; Right about access; Indirectly unauthorized access

## I. INTRODUCTION

The key to prove the access control model security is to find a recognized and self-evident security axiom, which can be used to deduce or prove security assumptions proposed in the model, so as to make it more reliable. However, even the formally proved BLP model<sup>[1,2]</sup> can't prove the rationality, completeness and safety of "simple security axiom" and " \* - property axiom", which is proposed by BLP model. Therefore, some scholars point out that the security axioms of BLP<sup>[3]</sup> cannot completely prove the security of BLP. A complete access

control model must clearly tell us which security requirements to be met, *what* access violation to be prevented, and how to reduce the uncertainty of the access violation *that* the system allows.

Entropy is the tool for measuring uncertainty, which is originally used in thermodynamics. An American mathematician, Claude Shannon, introduced it to the information theory, and put forward the concept of information entropy for disordered degree of information<sup>[4]</sup>. Since the information entropy theory is proposed, *it* has been applied to many fields such as Engineering Science and Social Science. Some scholars have successfully introduced it into the quantification analysis of information security risk and event uncertainty<sup>[5-7]</sup>.

In this paper, we put forward the concept of security entropy and measures the uncertainty of system's response to access violations, based on the thought and method where information entropy measures the uncertainty of things, so as to provide a scientific method for the safety analysis of classificatory access control model.

## II. SECURITY ENTROPY

### A. Definition of Security Entropy

In the information system, when a user sends out an access request, the system will respond in two ways: to allow or to deny. The access request is also divided into two types: legal request or illegal request. If we take the system as a black box, the system will give four kinds of responses to a user's every access request: allow legal access, refuse legal access, allow access violation and refuse access violation. Obviously, the response can be considered as the basis for judging whether a system is

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good. The more denial responses legal access gets, the poorer availability the system has.

In order to comprehensively measure the uncertainty of a system's response to various access requests, the security entropy is defined as follows:

Definition 1 (security entropy): If a group of access request like  $B = b_1, b_2, \dots, b_q$  is seen as the input, and the system's request responses to each access result is taken as the object of study and the variable  $X$  as the response results, then there will be four values for  $X$ : allow legal access, deny legal access, allow illegal access, and deny illegal access, which are recorded as  $a_1, a_2, a_3, a_4$  respectively. If we use Symbol  $p(a_i)$  to represent statistical probability of  $a_i$ , then the probability space  $[X, p(X=a_i)]$  of  $X$  will be

$$\begin{bmatrix} X \\ P(x) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ p(a_1) & p(a_2) & p(a_3) & p(a_4) \end{bmatrix}$$

$$p(a_i) \geq 0 (i=1,2,3,4), \sum_{i=1}^4 p(a_i) = 1.$$

We assign a weight  $w_i$ , the impact factor of the system security, for each response result. The greater  $w_i$  is, the higher  $a_i$ 's influence on system safety will be, and vice versa. If the distribution of  $w_i$  is

$$\begin{bmatrix} X \\ w \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ w_1 & w_2 & w_3 & w_4 \end{bmatrix}, \quad 0 \leq w_i \leq 1,$$

$$\sum_{i=1}^4 w_i = 1,$$

The security entropy of  $X$  will be:

$$H(X) = -\sum_{i=1}^4 w_i p(a_i) \log p(a_i) \quad (1)$$

### B. The Meaning of Safety Entropy

Based on common sense of information security, the response  $a_2$  gives negative effects on the usability of the system, and the response  $a_3$  gives negative effects on the confidentiality of the system, meanwhile the response  $a_1$  and  $a_4$  has less influence on system security. Therefore, if we let  $w_2, w_3 \square w_1, w_4$ , the meaning of safety entropy in formula (1) will be the average uncertainty of harmful responses occurring. The bigger value security entropy has, the higher uncertainty a harmful response will occur. On the other hand, the smaller value security entropy has, the lower uncertainty a harmful response will occur. As for the same set of access request, the smaller the security entropy of different access control model is, the less possibility model will make harmful response.

If we make  $w_2 > 0, w_3 > 0, w_1 = w_4 = 0$ , and at the same time  $w_2 + w_3 = 1$ , then security entropy can be used to judge whether the system satisfies usability and confidentiality. If  $w_2 = 1, w_3 = w_1 = w_4 = 0$ , security entropy of formula (1) can be used to judge whether the system satisfies usability. If  $w_3 = 1, w_1 = w_2 = w_4 = 0$ , security entropy of formula (1) can be used to judge whether the system satisfies confidentiality.

The number of the four responses is related to the number of input samples. If all input samples are legal access, then  $a_3$  and  $a_4$  will be 0. If all input samples are illegal access, the  $a_1$  and  $a_2$  will be 0. In order to make the safety entropy accurately reflect the system security, the input samples must be complete. In addition, the responses are related to the number of input samples. If the input number of the access request is much way more than others, the response will be distorted.

Therefore, when security entropy is calculated, the input samples (access requests) must be complete and its probability distribution must be uniform.

When the security entropy is smaller, there will be less uncertainty of harmful response to the system, hence the security of the model becomes better. When the security entropy approaches 0, the model will achieve the theoretical security.

### C. Security Entropy of Different Types of Illegal Accesses

Whether an access is illegal or not is related to security requirements. According to the access control requirements from National Grade of Protection Standard GB17859-1999<sup>[8]</sup>, illegal access can be classified into three types: direct legal access, right about access, and indirect legal access. The "direct legal access" refers to explicitly violating the authorized strategy such as the access control matrix and so on.

The "right about access" refers to the one which leads to violating information flow direction that the system stipulates, or in other words, the one which leads information flow from higher class to lower class. The "indirect legal access" refers to the one that violates the authorized strategy through information indirect transmission.

For instance, there are two users ( $s_1, s_2$ ) and two resources ( $o_1, o_2$ ) in the information system, and the relationship between each security level is  $f(s_1) \triangleright f(s_2) \triangleright f(o_1) = f(o_2)$ . The authorized system strategy is that "s<sub>1</sub> reads o<sub>2</sub>", "s<sub>2</sub> reads o<sub>1</sub>", "s<sub>2</sub> writes o<sub>2</sub>".

Let's see the following four events: b<sub>1</sub>: s<sub>2</sub> reads o<sub>1</sub>, b<sub>2</sub>: s<sub>2</sub> writes o<sub>2</sub>, b<sub>3</sub>: s<sub>1</sub> reads o<sub>2</sub>, b<sub>4</sub>: s<sub>1</sub> reads o<sub>1</sub>. Since b<sub>4</sub> explicitly violates the authorized strategy, b<sub>4</sub> is therefore "direct legal access"; the sequence of access b<sub>1</sub>b<sub>2</sub>b<sub>3</sub> causes the information to flow from s<sub>1</sub> to o<sub>1</sub>, which equals ~~that~~ to s<sub>1</sub> reading o<sub>1</sub> indirectly. Therefore b<sub>1</sub>b<sub>2</sub>b<sub>3</sub> is "indirect legal access". b<sub>1</sub> and b<sub>3</sub> causes the information flowing to the violation in the direction stipulated by the system, so b<sub>1</sub> and b<sub>3</sub> is "right about access".

For different types of legally access, the meaning of (1) is different. If the legal access is defined as "direct legal access", then the security entropy of (1) is called "direct security entropy" being recorded as  $H_D(X)$ .

Again, if the illegal access is defined as "right about access", then the security entropy of (1) is called "mandatory security entropy" recorded as  $H_M(X)$ . If the legal access is defined as "indirect legal access", then the security entropy of (1) is called "indirect security entropy".

recorded as  $H_I(X)$ .

### III. SAFETY CONDITIONS OF CLASSIFICATORY ACCESS CONTROL MODEL

#### A. Security Attributes Based On Security Entropy of Safety

$$H_D(X) = -\sum_{i=1}^4 w_i p(a_i) \log p(a_i) \equiv 0,$$

$$w_2 > 0, w_3 > 0, w_2 + w_3 = 1.$$

Theorem 1 (direct safety of access control model): Access control model has direct safety, if and only if

$$H_D(X) = -\sum_{i=1}^4 w_i p(a_i) \log p(a_i) \equiv 0, \text{ in which}$$

$$w_2 > 0, w_3 > 0, w_2 + w_3 = 1.$$

Prove:

Here we need to prove that when  $H_D(X) = 0$ , the event “refuse legal access” and “allow access violation” will never happen, that is,  $p(a_2) = p(a_3) = 0$ .

We mark the total number of  $a_i$  as  $n_i (i=1,2,3,4)$ ,  $n_1 + n_2 = s$ ,  $n_3 + n_4 = t$ , since  $a_1, a_2, a_3$  and  $a_4$  are different responses to the same access,  $p(a_1) + p(a_2) = s/q$ ,  $p(a_3) + p(a_4) = t/q$ .

Based on common sense, access requests can't be all legal or all illegal, so  $s, t > 0$ . Since  $\sum_{i=1}^4 p(a_i) = 1$ ,  $p(a_2) \neq 1, p(a_3) \neq 1$ .

Since  $w_2 > 0, w_3 > 0, w_2 + w_3 = 1$ , so  $w_1 = w_4 = 0$ .

End.

Similarly, we can get theorems as follows:

Theorem 2 (mandatory safety of access control model): the access control model has mandatory safety, if and only if

$$H_M(X) = -\sum_{i=1}^4 w_i p(a_i) \log p(a_i) \equiv 0, \text{ in which}$$

$$w_2 > 0, w_3 > 0, w_2 + w_3 = 1.$$

Theorem 3 (indirect safety of access control model): the access control model has indirect safety, if and only if

$$H_I(X) = -\sum_{i=1}^4 w_i p(a_i) \log p(a_i) \equiv 0, \text{ in which}$$

$$w_2 > 0, w_3 > 0, w_2 + w_3 = 1.$$

#### B. Safety Theorem of Classificatory Access Control Model

Symbol  $\Theta_2$  represents class 2 access control model,  $\Theta_3$  represents class 3 access control model, and  $\Theta_4$  represents class 4 access control model.

Now, according to the safety needs of class 2, 3 and 4 information system, we put forward the safety theorem of classificatory access control model based on the above security attributes.

Theorem 4 (safety of classificatory access control

model): Class 2 access control model  $\Theta_2$  satisfies safety needs, if and only if  $H_D(X)|\Theta_2 \equiv 0$ , in which  $w_2 > 0, w_3 > 0, w_2 + w_3 = 1$ ; Class 3 access control model  $\Theta_3$  satisfies safety needs, if and only if  $H_D(X)|\Theta_3 \equiv 0$  and  $H_M(X)|\Theta_3 \equiv 0$ , in which  $w_2 > 0, w_3 > 0, w_2 + w_3 = 1$ ; Class 4 access control model  $\Theta_4$  satisfies safety needs, if and only if  $H_D(X)|\Theta_4 \equiv 0$ ,  $H_M(X)|\Theta_4 \equiv 0$  and  $H_I(X)|\Theta_4 \equiv 0$ , in which  $w_2 > 0, w_3 > 0, w_2 + w_3 = 1$ ;

### IV. ANALYSIS OF TYPICAL ACCESS CONTROL MODEL BASED ON SECURITY ENTROPY

Now we apply the theory to analyze the security of typical access control model, in order to verify the practicability of this method, and to point out the disadvantages of each access control model.

#### A. Security Analysis to HRU MODEL

##### (1) Direct safety

Suppose there are  $m$  users in the system:  $u_1, u_2, \dots, u_m$ , and  $n$  resources:  $o_1, o_2, \dots, o_n$ , then access requests can be divided into reading and writing atomic requests; and there will be  $2mn$  access request requests, which can be expressed respectively with symbol  $b_1, b_2, \dots, b_q (q=2mn)$ . Results of the access can be divided into two types: legal access  $B^+ = b_1^+, b_2^+, \dots, b_s^+$ , and direct legal access  $B^- = b_1^-, b_2^-, \dots, b_t^- (s+t=q)$ .

Based on the access control matrix, HRU<sup>[9]</sup> controls access behaviors. As long as access behaviors disobeys the policy, it will be refused. So the responds to any  $b_j^- \in B^-$  is  $a_4$ ; As long as access behaviors obeys the policy, it would be allowed. Thus the responds to any  $b_i^+ \in B^+$  is  $a_1$ , and so  $p(a_2) = 0$  and  $p(a_3) = 0$ .

The statistical probability distribution of responses is

$$\begin{bmatrix} X \\ P(x) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ \frac{s}{q} & 0 & 0 & \frac{t}{q} \end{bmatrix}$$

Since  $H_D(X)|HRU \equiv 0$ , the model HRU can satisfy direct safety.

##### (2) Mandatory safety

Suppose we divide all requests  $B = b_1, b_2, \dots, b_q (q=2mn)$  into three kinds: the request  $B^\uparrow = \{b_1^\uparrow, b_2^\uparrow, \dots, b_{q/3}^\uparrow\}$  that causes information to flow from low level to high level, the request  $B^\downarrow = \{b_1^\downarrow, b_2^\downarrow, \dots, b_{q/3}^\downarrow\}$  that causes information to flow from high level to low level, and the request  $B^{\leftrightarrow} = \{b_1^{\leftrightarrow}, b_2^{\leftrightarrow}, \dots, b_{q/3}^{\leftrightarrow}\}$  that causes information to flow between the same levels. Obviously, request  $B^\downarrow$  in the second kind is a “right about access”.

Since the model HRU judges the legality of the access request by visiting the access control matrix,

the access request  $b_i^\uparrow$  and  $b_i^\leftrightarrow$  does not necessarily satisfy the access control matrix, and it may be refused or be allowed, so  $p(a_2) \equiv 0$  can not be always deduced.

(3) Indirect safety

“Indirect illegal access” is composed of several direct non-illegal accesses, so it can be denoted as  $f_i^- = b_{i_1}^+ b_{i_2}^+ \dots b_{i_q}^+ (b_{i_1}^+, b_{i_2}^+, \dots, b_{i_q}^+ \in B^+)$ . For  $H_D(X) | HUR \equiv 0$ , the system will allow every direct non-illegal access in  $f_i^-$ . Consequently,  $f_i^-$  will be allowed, therefore  $p(a_3) > 0$  is deduced.

With  $H_I(X) | HRU > 0$ , it shows that HRU model doesn't satisfy indirect safety.

Through analysis we can conclude that the model HRU satisfies direct safety, but it doesn't satisfy mandatory safety and indirect safety.

B. Security Analysis to BLP

(1) Direct safety and indirect safety

The model BLP uses two methods: DAC and MAC. DAC uses the HRU model, so the direct safety and the indirect safety of the BLP model coincide with that of the HRU, which means BLP satisfies direct safety but doesn't satisfy indirect safety.

(2) Mandatory safety

The BLP model forbids high level subjects writing low level objects and vice versa, and prevents the information flowing from high level into low security level. Therefore any “right about access”  $b_i^\downarrow \in B^\downarrow$  will be refused by BLP, and any non “right about access”  $b_i^\rightarrow \in B^\leftrightarrow$  and  $b_i^\uparrow \in B^\uparrow$  will be allowed. Consequently, the probability distribution of BLP's response X is

$$\begin{bmatrix} X \\ P(x) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ \frac{2q}{3} & 0 & 0 & \frac{q}{3} \end{bmatrix}$$

So  $H_M(X) | BLP \equiv 0$ , and it shows that BLP satisfies mandatory safety.

C. Security Analysis to RBAC

The model RBAC<sup>[9,10]</sup> assigns a role for the user, and then grants authorization based on these roles. The RBAC's rights management and access control manner are similar to HRU's. Thus its safety is similar to that of HRU, which satisfies direct safety but not mandatory safety and indirect safety.

D. Security Analysis to FGBAC

The FGBAC<sup>[12]</sup> is the improved BLP, which introduces the information flow graph as a judgment auxiliary tool. In FGBAC, any “direct illegal access”, “right about access” or “indirect illegal access” will be refused. So

$$\begin{aligned} &H_D(X) | FGBAC \\ &= H_M(X) | FGBAC \\ &= H_I(X) | FGBAC \\ &\equiv 0 \end{aligned}$$

It shows that the model satisfies direct safety, mandatory safety and indirect safety.

According to the above safety analysis of the typical access control model, and in the light of the safety needs of 2, 3 and 4 class systems, we conclude the typical model's security and applicability as below:

TABLE I  
THE SECURITY OF TYPICAL ACCESS CONTROL MODEL

typical access control model	direct safety	mandatory safety	indirect safety
HUR	satisfy	not satisfy	not satisfy
RBAC	satisfy	not satisfy	not satisfy
BLP	satisfy	satisfy	not satisfy
FGBAC	satisfy	satisfy	satisfy

TABLE II  
THE APPLICABILITY OF TYPICAL ACCESS CONTROL MODEL

typical access control model	Scope of application
HUR	Class 2
RBAC	Class 2
BLP	Class 3
FGBAC	Class 4

V. CONCLUSION

This paper puts forward the concept of “security entropy” for measuring uncertainty of system's response to access request, and proposes its security theorems based on security entropy. The theory can be widely applied to security analysis of access control mode and system.

Based on the theory, this paper analyses the typical access control model, verifies the practicability of the method, and concludes the security and application scope of the available models.

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