Connectivity-based Performance Evaluation for Mobile Cognitive Radio Network

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Abstract—In this paper, we propose a model of mobile cognitive radio network after fully considering the analysis method of network connectivity based on random way-point (RWP) mobility scheme. The closed-form solution of connectivity probability has been derived to evaluate the network performance in this model. And it shows that the network connectivity is related to the size of mobile cognitive radio network, the number of secondary users, the transmission range of secondary users and primary users, the interference range of secondary users and primary users, the number of primary users and the activity factor of primary users. Simulation results are good agreement with numerical results, which verifies our theoretical analysis is correct and reasonable.

Index Terms—mobile cognitive radio network, random way-point mobility model, network connectivity, connectivity probability

I. INTRODUCTION

Cognitive radio has emerged as a promising technology to enhance spectrum utilization by sensing the spectrum and opportunistically using the spectrum of primary users [1-4]. In cognitive radio network, there are two kinds of users: primary users (PUs) and secondary users (SUs). PUs are licensed users with high priority in the utilization of the spectrum, which constitute the primary network. SUs are unlicensed users with low priority in the utilization of the spectrum, which constitute the secondary network. It is important to understand the connectivity of the secondary network so that proper network operations can be performed. In traditional wireless ad hoc network, the communication among nodes is realized by using the same frequency. The distance between the transmitter and the receiver and the transmission power of the transmitter are the only parameters affecting the network connectivity. But in cognitive radio network, two SUs can connect if they are in radio visibility and have at least one available common channel. As a consequence, the node position, the transmission power and their free spectrum bands all affect cognitive radio network connectivity. In other words, the connectivity is the foundation of reliable data transmission in cognitive radio network. A communication link exists between two SUs if the two SUs meet the following two conditions: (1) they are within each other’s transmission range; (2) they have a common spectrum band determined by the transmitting and receiving activities of nearby PUs.

Several recent works have analyzed and proposed the methods for the network connectivity. Network connectivity of one-dimensional ad hoc network was studied in [5-7]. The movement of nodes with impact on the connectivity of ad hoc network was elaborated in [5]. In [8-9], the research method of ad hoc network connectivity has been applied to not only one-dimensional network but also two-dimensional network, the relationship between network parameters (e.g., transmitting range and node degree) and network connectivity was analyzed in detail. But the authors in [5-9] only studied connectivity of traditional ad hoc network without studying the cognitive radio network connectivity (CRNC). In [10-13], the CRNC based on the connectivity of graph was studied, and the second smallest eigenvalue of Laplacian matrix was used to denote the CRNC, the analysis method provides a methodology to evaluate the CRNC. However, the analysis method is only based on the static network topology with fully available topological information on SUs and PUs without considering the movement of SUs. In [14-18], percolation theory was used to analyze the CRNC, but it is only applicable to large scale cognitive radio network. Two basic properties of cognitive radio network [19], capacity and connectivity, were studied to characterize the relationship and tradeoff among key system parameters involved in these properties. Connectivity opportunity selection algorithm [20] was proposed to select the best connectivity alternative for the user in a heterogeneous wireless multi-hop network. In [21], connectivity-aware minimum-delay routing was discussed in vehicular ad-hoc networks. We can see from [19-21] the CRNC is very useful to the projecting and routing of network. As far as we know, the mobile CRNC has not been adequately analyzed so far.
In this paper, we propose the model of mobile cognitive radio network (MCRN) after the model of one-dimensional mobile ad hoc network in [5] gives us inspiration. Our model is very useful for certain applications such as BusNet in [22] where buses are mobile and bus stops are stationary. In this case, BusNet may be a secondary network which is composed of SUs. The PUs are some network devices. In other words, the developed model can be used to realize the communication among vehicle to vehicle along roads by using the unlicensed spectrum bands, which provides many applications (e.g., accident warning and traffic indication). It is noted that we only investigate CRNC and don’t consider what protocol is used to realize communication in the paper.

With the proposed model, we firstly derive the closed formula of connectivity probability in MCRN, and then analyze the impact of network parameters on connectivity probability. Lastly we further verify that the theoretical analysis is reasonable through simulation.

The rest of the paper is organized as follows: In Section 2, we present the system model for evaluating the MCRN connectivity. Section 3 is dedicated to the analysis of connectivity based on the system model. Simulation results are reported in Section 4. Finally, we conclude the paper in Section 5.

II. SYSTEM MODEL

Consider a channel model with path loss and small scale fading governed by the propagation-related power attenuation model, \( P_r = P_t \left( \frac{h}{d} \right)^{\alpha} \), where \( P_r \) is the received power, \( P_t \) is the transmission power, \( d \) is the distance between the transmitter and receiver under consideration, \( \alpha \geq 2 \) is the path loss exponent and \( h \) represents the small scale fading component. Obviously, the distance between the transmitter and receiver is given by

\[
d = \left( \frac{P_t}{P_r} \right)^{\frac{1}{\alpha}}
\]  

(1)

![Figure 1. Model of mobile cognitive radio network.](image)

The developed network model is illustrated in Fig. 1, we assume the MCRN is composed of \( N_p \) motionless PUs. We consider a road as a line in the model, and mobile SUs are uniformly placed along the line between A and B at the beginning. Then they move along the line between A and B based on RWP mobility model. The network size is represented with the length \( L \) and the width \( D \) which means the distance between A and B is \( L \). PUs are uniformly distributed within the cognitive radio network region.

We assume PUs and SUs are equipped with omni-direction antenna, denoting the primary transmission range with \( R_p \) and the secondary transmission range with \( R_s \). The circle with radius \( R_p \) denotes the coverage area of the PU, and the circle with radius \( R_s \) denotes the coverage area of the SU. Now we investigate connectivity problem and only consider bidirectional links in the secondary network when we define connectivity. Therefore, when we determine whether there exists a communication link between two SUs, we need to check the existence of spectrum opportunities in both directions and the distance between these two nodes. The secondary network consists of all the SUs and all the bidirectional secondary links. As for primary network, we also consider that any primary link between two PUs is bidirectional, and a potential primary link exists between any two PUs if and only if the distance between these two nodes is less than the primary transmission range \( R_p \), the primary links are illustrated in Fig. 1.

We assume that each PU\(_j\) \((j = 1, \ldots, N_p)\) has a licensed spectrum band with carrier frequency \( f_j \), denoted as channel \( ch_j \), and all the PUs transmit with a given transmission power \( P \), and each PU\(_j\) is characterized by an on-off transmission with activity factor \( \beta_j \).

\[
\beta_j = \frac{t_{on}}{t_{on} + t_{off}}
\]

(2)

where \( t_{on} \) and \( t_{off} \) are the average duration of the activity period of the PU and the average duration of silence period of the PU, respectively. In addition, let \( E_j \) represent the event that " PU\(_j\) does not transmit at an arbitrary time t " . Then the probability that this event happens is as follows.

\[
P(E_j) = 1 - \beta_j
\]

(3)

Each secondary user opportunistically exploits locally unused licensed spectrum without interfering transmissions of PUs. In the same time, let \( A_j \) represent the event that " PU\(_j\) transmits at an arbitrary time t " with probability of \( P(A_j) = \beta_j \). Assuming that the events \( A_1, A_2, \ldots, A_{N_p} \) are independent and unrelated. The probability that all the PUs are active (i.e., the probability that all the PUs transmit at the same time) is as follows.

\[
P_{active} = P(\bigcap_{j=1}^{N_p} A_j) = \prod_{j=1}^{N_p} P(A_j)
\]

(4)
Here we represent the interference region of a primary user as a dish of radius \( R_p \) and the interference region of a secondary user as a dish of radius \( R_s \) (see Fig. 2). So the behavior of a secondary link at any given time slot is dependent on two conditions: firstly, all active PUs are out of the two interference regions of the two associated SUs; secondly, these two SUs are out of the interference regions of all currently active PUs. Assuming that an active SU or PU may refer to either a transmitter or a receiver at a given time slot. Then the minimum distance between an active PU and an active SU is equal to \( R_i = \max \{ R_p, R_s \} \), which corresponds to the maximum interference range, as illustrated in Fig. 2 where \( R_p > R_s \), \( R_p > R_s \). To reflect the characteristic of the real wireless communication environment, we use the Log-Normal Shadowing Model [23] and assume \( R_p = (6 / 5) R_p \) and \( R_s = (6 / 5) R_s \). Notice that we define the interference region centered at some PU or SU with radius \( R_i \) as the maximum interference region (MIR). Therefore, a SU \( i \) can transmit by using channel \( ch_i \) in two different cases:
1) when the distance between PU \( j \) and SU \( i \) is longer than or equal to \( R_i \), SU \( i \) always identifies the channel \( ch_i \) as unused;
2) when the distance between PU \( j \) and SU \( i \) is shorter than \( R_i \) and PU \( j \) is inactive.

If one of these two cases occurs, the channel \( ch_i \) is available for the SU \( i \).

According to the proposed network model, there are \( N_s + 1 \) segments to form the secondary network. For the convenience of connectivity analysis, additional terms are defined as follows:

- \( X \) : A continuous random variable describing the location of a mobile cognitive node at any instant time, which represents the distribution of secondary nodes. The PDF and CDF of \( X \) are \( f(x) \) and \( F(x) \).
- \( K_i \) : A random variable denoting the length of a segment with the left edge locating at \( x \). The PDF and CDF of \( K_i \) are \( q_i(k) \) and \( Q_i(k) \), respectively.
- \( L_i \) : A random variable denoting the location of the left edge of the i-th segment. Let \( r_i(l) \) represent the PDF of \( L_i \).
- \( p_{i,s} \) : The probability that the i-th segment has a length shorter than or equal to \( k \) when all the PUs are inactive. In practical application, \( k \) is a constant representing the distance between the transmitter and the receiver. Assuming that \( R_i \) is equal to \( k \) throughout the paper.
- \( p_{i,s}^r \) : The probability that the i-th segment has a length shorter than or equal to \( k \) when PUs located in the MIR of SU \( i-1 \) or SU \( i \) are active and there is at least one common channel between SU \( i-1 \) and SU \( i \).

\( p_c \) : The connectivity probability of MCRN when PUs located in the MIR of any secondary user are inactive.

\( p_c^r \) : The connectivity probability of MCRN when PUs located in the MIR of any secondary user are active.

### III. Connectivity Analysis

In this section, we analyze connectivity of MCRN. If all the PUs located in the MIR of any secondary user are inactive, SUs always have available common channel. The network composed by SUs can be considered as a traditional one dimensional mobile ad hoc network. The link between two nodes is only influenced by the distance between two nodes. Now we review an existing conventional one dimensional mobile ad hoc network connectivity analysis [5].

\[
q_i(k) = N_s[(f(x+k))[1-(F(x+k)-F(x))]^{N-1} \tag{5}
\]

where \( 0 \leq x \leq L \) and \( 0 \leq k \leq L-x \). Obviously the CDF of \( K_i \) is as follows:

\[
Q_i(k) = 1-[1-(F(x+k)-F(x))]^N \tag{6}
\]

The left edge of the i-th segment will be located at \( l \) if one node is located at \( l \), and the remaining \( i-2 \) and \( N_s-i+1 \) nodes are located on the left and right sides of the node located at \( l \), respectively. Hence, the following equation can be obtained

\[
r_i(l) = N_s[f(l)][C_i^{N-2}][F(l)]^{l-2}[1-F(l)]^{N-i+1} \tag{7}
\]

where \( 2 \leq l \leq N_s+1 \) and \( 0 \leq l \leq L \). From (5),(6), and (7), the probability that the i-th segment with its left edge located at \( x \) has a length shorter than or equal to \( k \) can be described as \( Q_i(k)x_r(x) \). Then \( p_{i,s} \) is defined as

\[
p_{i,s} = \int_0^{L-k} Q_i(k)x_r(x)dx + \int_{L-k}^L Q_i(k)x_r(x)dx \tag{8}
\]

In (8), due to the border effect, when the left edge of a segment falls between \( L-k \) and \( L \), the segment is always shorter than \( k \), indicating a connection, i.e. \( Q_i(k)=1 \). Hence the connectivity probability in conventional one dimensional mobile ad hoc network when the above analysis is applied becomes the following:

\[
p_c = \begin{cases} 
Q_i(k) & N_s \geq \frac{L}{k} \\
0 & N_s < \frac{L}{k} 
\end{cases} \tag{9}
\]

In practice, since some PUs are active, the topology of MCRN will be affected by active PUs, and SUs may interfere with communication of PUs when SUs apply the same frequency bands as the active PUs. So the connectivity analysis method of conventional mobile ad hoc network is inappropriate. For instance, when the distance of the i-th link between SU \( i-1 \) and SU \( i \) is shorter
than or equal to $k$, the i-th segment is considered to be connected which means the i-th link is valid in conventional mobile ad hoc network. But the link may be invalid if there is no available channel between them in cognitive radio network. Hence, the analysis method described above must be revised.

In Fig. 2, one can see transmission range ($R_x, R_y$) and interference range ($R_{xI}, R_{yI}$) for SU and PU, respectively. To realize communication between SU$_{i-1}$ and SU$_i$, we must consider behavior of PUs and choose channels which are not used by PUs. This implies that the SU$_{i-1}$ (SU$_i$) can transmit data to SU$_i$ (SU$_{i-1}$) if the transmission from SU$_{i-1}$ (SU$_i$) does not interfere with nearby primary nodes. To solve the interference problem, we consider the minimum distance between an active primary user and an active secondary user is equal to $R_y = \max\{R_{p}, R_y\}$ when we analyze the network connectivity. We can see from Fig. 2 the channel of primary link 1 is always available to the link between SU$_{i-1}$ and SU$_i$, the primary link 2 and the primary link 3 are not the case because of the relative location of PUs and SUs. There are two possible cases for the i-th secondary link which depend on the location of SU$_{i-1}$ and SU$_i$. Let $R_y < x < L - R_y$ represent the first case of the MIR of SU$_{i-1}$ or SU$_i$ being completely within cognitive radio network region and $0 \leq x < R_y$ or $L - R_y \leq x \leq L$ corresponds to the other case which is the MIR of SU$_{i-1}$ or SU$_i$ being partly within cognitive radio network region.

For the case of $R_y < x < L - R_y$: Fig. 2 shows SU$_{i-1}$ is located at $(x, 0)$, and the MIR centered at SU$_{i-1}$ is indicated with $C_{i-1} = \pi R_y^2$. We denote PDF of PUs with $g_{SU} (w, v) = \frac{1}{DL}$. The probability that a primary user is located within the scope of $C_{i-1}$ is indicated with $p_{i-1}(x)$.

\[
p_{i-1}(x) = \int_{C_{i-1}} g_{SU} (w, v) dA = \frac{\pi R_y^2}{DL} (10)
\]

Similarly, SU$_i$ is located at $(x + k, 0)$, and the MIR centered at SU$_i$ is indicated with $C_i = \pi R_y^2$. The probability that a primary user is located within $C_i$ is denoted with $p_i(x + k)$.

\[
p_i(x + k) = \int_{C_i} g_{SU} (w, v) dA = \frac{\pi R_y^2}{DL} (11)
\]

Overlapping area of the MIR which is covered by SU$_{i-1}$ and SU$_i$ is represented with $C_{con}$.

\[
C_{con} = \frac{R_y^2}{2} \arccos \left( \frac{k^2 - 2R_y^2}{2R_y^2} - \sqrt{R_y^2 - k^2} \right) (12)
\]

The probability that all the PUs are located in the MIR of SU$_{i-1}$ or SU$_i$ is denoted with $p_{all-pu}$ and given by

\[
p_{all-pu} = \sum_{j=0}^{N_p} C_{i-1}^{j} [p_{i-1}(x)]^{j} [wp_i(x + k)]^{N_p - j}, N_p \geq 1 \quad 0, N_p = 0 \quad (13)
\]

where, $w = (C_i - C_{con}) / C_i$. If all the PUs are located in the MIR of SU$_{i-1}$ or SU$_i$, and all the PUs are active, the probability that there is no available common channel between SU$_{i-1}$ and SU$_i$ is given by

\[
p_{ac} = p_{all-pu} p_{active} \quad (14)
\]

For the case of $0 \leq x < R_y$ or $L - R_y \leq x \leq L$: This happens when not all the MIRs of SU$_{i-1}$ or SU$_i$ are located within cognitive radio network region. Notice that the probability that all the PUs are located in the MIR of SU$_{i-1}$ or SU$_i$ within cognitive radio network region is different from (13). Due to the symmetry of the model, the analysis method of $0 \leq x < R_y$ is the same as it of $L - R_y \leq x \leq L$, so we only analyze the case of $0 \leq x < R_y$ in the paper. The MIR which is covered by SU$_{i-1}$ or SU$_i$ within the scope of MCRN is denoted by $C_{b-total}$ and is given by

\[
C_{b-total} = \frac{3}{2} \pi R_y^2 + k \sqrt{R_y^2 - k^2}^2 - 2R_y^2 \arcsin \left( \frac{R_y^2 - k^2}{4R_y} \right) (15)
\]

The probability that all the PUs are located in the MIR of SU$_{i-1}$ or SU$_i$ within cognitive radio network region is $p_{all-pu}^b$ and given by

\[
p_{all-pu}^b = \begin{cases} \left( \frac{C_{b-total}}{DL} \right)^{N_p}, & N_p \geq 1 \\ 0, & N_p = 0 \end{cases} (16)
\]

The probability that there is no common channel between SU$_i$ and SU$_{i-1}$ is $p_{b-ac}$ and given by
\begin{equation}
P_{b_{nc}} = p^{b}_{n-c}p_{active}
\end{equation}

In MCRN, the link between two nearest neighboring secondary users relies on the distance and available common channel between them. If the two nodes are within the transmission range of each other and they have at least one available common channel, the link is valid, otherwise the link is noneffective. Therefore, the connectivity probability of the i-th segment between SU_{i-1} and SU_i is given by

\begin{equation}
p'_{c,i,k} = \beta_{b_{nc}} \int_{R_i}^{L_i} Q_{s}(k) r_{i}(x)dx + \beta_{c_{nc}} \int_{L_i}^{R_i} Q_{s}(k) r_{i}(x)dx
+ \beta_{b_{nc}} \int_{L_i}^{L_i} Q_{s}(k) r_{i}(x)dx + \beta_{b_{nc}} \int_{L_i}^{L_i} Q_{s}(k) r_{i}(x)dx
\end{equation}

where \( \beta_{b_{nc}} = (1-p_{nc}) \) or \( \beta_{c_{nc}} = (1-p_{nc}) \) is the probability that there is at least one available common channel between SU_{i-1} and SU_i for \( 0 \leq x < R_i \) and \( L - R_i \leq x \leq L \) or \( R_i < x < L - R_i \), respectively. The connectivity probability of MCRN is

\begin{equation}
p'_{c} = \begin{cases} 
Q_{s}(k) \prod_{k=1}^{N_s} p'_{c,i,k}, & N_s \geq \lfloor \frac{L}{k} \rfloor - 1 \\
0, & N_s < \lfloor \frac{L}{k} \rfloor - 1
\end{cases}
\end{equation}

The steady-state mobile cognitive users distribution of the RWP mobility model is given in [24], and the PDF and CDF is shown as follows.

\begin{equation}
\begin{cases}
f(x) = -\frac{6}{E} x^2 + \frac{6}{E} x, & 0 \leq x \leq L \\
F(x) = -\frac{2}{E} x^3 + \frac{3}{E} x^2, & 0 \leq x \leq L
\end{cases}
\end{equation}

Using above equations, the connectivity probability of mobile cognitive radio network can be calculated via numerical method.

IV. NUMERICAL RESULTS

Let us now verify these analytical results by computer simulation. In our simulation environment, SU's move according to the RWP model in a line, whose length and width have been set as \( L = 1000m, D = 780m \), respectively. Each secondary user picks a random spot in a line and moves there with a constant speed which is randomly chosen in \([0, 5m/s]\). Upon reaching this point, the node randomly picks a new destination and repeats the process. The transmission range and interference range of the PUs, whose positions are assumed static, have been set to \( R_p = 180m \) and \( R_{pi} = 216m \), respectively. The interference range of SU's is related to transmission range \( R_s \). We assume any mobile cognitive node will move towards the contrary direction at the same speed if it reaches the border (i.e., A or B) during the sampling interval. Our simulation results are repeated 10000 times, 200 random network topologies are generated during simulation. Such the experiment is repeated 50 times for every network topology, and finally averaged over all 200 random network topologies (i.e., 10000 experiments).

Given \( N_p = 7 \) and the activity factor of primary users \( \beta_j = 1 (j=1,...,N_p) \), the resulting curves are shown in Fig. 3 and Fig. 4. It may be observed that this simulation yields the same qualitative behavior as the analytical plots.

In Fig. 3, it shows simulation results are in good agreement with the theoretical results for \( R_s = 0.15km, 0.18km, 0.2km \), respectively. As expected, in all the three cases, the value of \( p'_{c} \) increases with increase of \( N_s \) for given \( R_s \) or with increase of transmission range \( R_s \) for given \( N_s \). One can see from Fig. 3 that \( p'_{c} \) is close to 1 when \( N_s \) is larger than 55 for fixed \( R_s = 200m \). To reach some connectivity target \( p'_{c} \), the smaller the transmission range of SU's is, the more number of SU's in the secondary network is.

Fig. 4 gives the curve obtained with our formula and that produced by the simulations, we obtain the acceptable results between analytical and simulation results. Fig. 4 demonstrates that \( p'_{c} \) increases with increase of transmission range \( R_s \), and the number of SU's. For fixed \( N_p \), the relationship among \( p'_{c} \), \( N_s \) and \( R_s \) is similar to Fig. 3. Notice that an increase of the transmission power of some SU leads to an increase of interference with PUs and other SU's, and the energy consumption increases accordingly. So we should take the full consideration of the tradeoff between energy consumption and network connectivity.

![Figure 3. Analytical and simulation results for different Rs](image)
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