# Stability Analysis of a SEIQRS Model with Graded Infection Rates for Internet Worms

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Abstract—Internet worms have drawn a significant attention owing to their enormous threats to the Internet infrastructure and services. In order to effectively defend against them, this paper proposes a novel epidemic *SEIQRS* model with graded infection rates for Internet worms. Using this *SEIQRS* model, we obtain the basic reproduction number for determining whether the worm dies out completely. The global stabilities of worm-free equilibrium and endemic equilibrium are proved, and determined by the basic reproduction number. The impact of different parameters of this model is studied. Simulation results show that the number of susceptible and infected hosts are consistent with theoretical analysis. The model provides a theoretical foundation for control and forecasting for Internet worms.

*Index Terms*—network security, Internet worm, stability analysis, endemic equilibrium, basic reproduction number

## I. INTRODUCTION

Internet worms are malicious codes or programs which can replicate themselves and spread via Internet. Numerous worms have appeared on the Internet over the last decade whose goal is to compromise the confidentiality, integrity, and availability of infected computing systems. With the ever increasing number of Internet applications and the emergence of new technologies, Internet worms have become a great threat to our work and daily life, caused tremendous economic losses. Especially, the advent of the Internet of things would make the threat increasingly serious. How to combat Internet worms effectively is an urgent issue confronted with defenders. Therefore, it is necessary to comprehend the long-term behavior of worms and to propose effective strategies to defend against Internet worms. The similarity between the spread of biological viruses and that of Internet worms encourages researchers to adopt appropriately modifying epidemic models to describing the propagation of worms across the Internet.

Based on the infectivity between a worm and a biological virus, some epidemic models representing worm propagations were presented to depict the propagation of worms, e.g., SIR model [1], SIRS model [2], [3], SIQ model [4], SEIR model [5], SEIRS model [6], [7], SEIQV model [8], SEIQRS model [9], which assume that infected hosts in which the worm resides are in an exposed state can not infect other hosts. Actually, an infected host which is in latency can infect other hosts by means of some methods, e.g., vulnerability seeking. All the previous models do not take this passive infectivity into consideration. Recently, Yang et al. proposed some models [10], [11], [12], by taking into account the fact that a host immediately possesses infectivity once it is infected. These models, however, all make an assumption that exposed hosts and infected hosts have the same infectivity. This is not consistent with the reality. Although an exposed host also sends scanning packets to find susceptive hosts with certain vulnerabilities, the scanning packets sent by an exposed host are less than an infected one. Usually, the infection rate of exposed hosts is less than that of infected ones. Therefore, they should have a different infection rate.

Due to the frequent occurrence of worms over the Internet in the last decade, users usually install some antivirus softwares or firewalls to protect their hosts avoiding. Once a user feels that the performance of his host is degraded or there exists some useless data in disks (e.g., Witty can do it), he will clean worms by the means of antivirus softwares. In order to protect his important files, the user spontaneously cleans worms even if he is not sure the existence of worms in his host. Additionally, an infected host often represents more obvious characteristics than an exposed host, the user could take some more effective measurements, e.g., patching, dynamic instruction sequences [13]. Therefore, the cured rate of exposed hosts would be lower than that of infected ones. The feature should be considered when modeling Internet worms.

In this paper, we propose a new worm attack model, referred to as *SEIQRS* (susceptible - exposed - infected - quarantined - recovered - susceptible) model, which incorporates the two features mentioned above. Using the basic reproduction number, we derive global stabilities of a worm-free equilibrium and a unique endemic equilibrium by a Lyapunov function and a geometric approach. Based on these results and further analysis, some effective methods for controlling worms are recommended.

The rest of this paper is organized as follows. Section II formulates the new model and obtains its basic reproduction number. Section III proves the local and global stabilities of the worm-free equilibrium. Section IV examines the local and stabilities of the endemic equilibrium. Section V covers the numerical analysis and the simulations. Section VI summarizes the paper with some future directions.

## **II. MATHEMATICAL MODEL FOMULATION**

The total host population N is partitioned into five groups and any host can potential be in any of these groups at any time t: the susceptible, exposed, infectious, quarantined, recovered, with sizes denoted by S, E, I, Q, R, respectively. The total number of population N at time t is given by N(t) = S(t) + E(t) + I(t) + Q(t) + R(t). The dynamical transfer is depicted in the following figure.



Figure 1. The states and state transitions in SEIQRS model.

Our model is based on the following assumptions: (1) Initially, all hosts are vulnerable to attack. These quarantined hosts, without considering the quarantine time, will move to the recovered state or susceptible state after installing the required security patches or updates. (2) All hosts in the recovered state only have a temporary immunity, because after the run of anti-virus software as

soon as we surf Internet or use secondary devices, they again become susceptible to malicious objects attack. (3) Exposed hosts have a lower infection rate than infected ones.

Fig. 1 shows the five states and state transition in *SEIQRS*. Based on the compartment model presented Fig. 1, the *SEIQRS* model having infectious force in the exposed, infected period is described by the following system of differential equations:

$$\begin{cases} S'(t) = \Pi + \gamma R + \theta Q - \beta_1 S E - \beta_2 S I - \mu S, \\ E'(t) = \beta_1 S E + \beta_2 S I - (\mu + \alpha_1 + \delta_2 + \omega) E, \\ I'(t) = \omega E - (\mu + \alpha_2 + (1 - p)\delta_1 + p) I, \\ Q'(t) = p I - (\mu + \eta + \theta) Q, \\ R'(t) = \delta_2 E + (1 - p)\delta_1 I + \eta Q - (\mu + \gamma) R, \end{cases}$$
(1)

where  $\Pi$  is a constant recruitment of susceptible hosts.  $\beta_1$ ,  $\beta_2$  are the rates of the efficient contact in the latent, infected period, respectively. The positive parameter  $\mu$ is the rate of natural death,  $\alpha_1$ ,  $\alpha_2$  are non-negative constant and denote the rates of worm-caused death, and  $\alpha_1 < \alpha_2$ . The parameter  $\gamma$  is the transfer rate between the recovered and the susceptible.  $\theta$  denotes the transfer rate between the quarantined and the susceptible.  $\delta_2$ ,  $\omega$ are the transfer rates between the exposed and the recovered, between the exposed and the infectious, respectively. The parameter p denotes the quarantined rate.  $\delta_1^{-1}$  is the average cured time. The parameter  $\eta$ denotes the transfer rate between the quarantined and the recovered.

Summing the equations of the system (1), we obtain  

$$N'(t) = \Pi - \mu N - \alpha_1 E - \alpha_2 I.$$
 (2)

Therefore, the total population *N* may vary with time *t*. In the absence of disease, the total population size N(t) converges to the equilibrium  $\Pi / \mu$ . It follows from Equation (2) that  $\lim_{t\to\infty} \inf N(t) \le \Pi / \mu$ . We thus study our system (1) in the following feasible region:

 $\Omega = \{ (S, E, I, Q, V) \in \mathbb{R}^5_+ : S + E + I + Q + V \le \Pi / \mu \},\$ 

Which is a positively invariant set of the model (1). We next consider the dynamic behavior on  $\Omega$ .

Firstly, we obtain the basic reproduction number by the method of next generation matrix [15]. It is easy to see that the model (1) always has a worm-free equilibrium,  $P_0 = (\Pi / \mu, 0, 0, 0, 0)$ . Let  $x = (E, I, R, Q, S)^T$ , then the model (1) can be written as dx/dt = F(x) - V(x), where

$$F(x) = \begin{pmatrix} \beta_1 SE + \beta_2 SI \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$
  
$$V(x) = \begin{pmatrix} (\mu + \alpha_1 + \delta_2 + \omega)E \\ (\mu + \alpha_2 + (1 - p)\delta_1 + p)I - \omega E \\ (\mu + \gamma)R - \delta_2 E - (1 - p)\delta_1I - \eta Q \\ (\mu + \eta + \theta)Q - pI \\ \beta_1 SE + \beta_2 SI + \mu S - \Pi - \gamma R - \theta Q \end{pmatrix}.$$

Differentiating F(x) and V(x) with respect to E, I, R, Q, S, and evaluating at the worm-free equilibrium  $P_0 = (\Pi / \mu, 0, 0, 0, 0)$ , respectively, we have

where  $n = \mu + \alpha_1 + \delta_2 + \omega$ ,  $m = \mu + (1 - p)\delta_1 + p + \alpha_2$ . Thus, the spectral radius of the next generation matrix

$$FV^{-1} \text{ can be found as,}$$

$$\rho(FV^{-1}) = \frac{\Pi(\beta_1 m + \beta_2 \omega)}{\mu m n}.$$
(3)

 $R_0 = \Pi(\beta_1 m + \beta_2 \omega) / (\mu m n).$ 

#### III. THE WORM-FREE EQUILIBRIUM AND ITS STABILITY

It is easily obtained that the model has a worm-free equilibrium given by  $P_0 = (\Pi / \mu, 0, 0, 0, 0)$ .

*Lemma 1*: When  $R_0 < 1$ , the worm-free equilibrium  $P_0$  is locally asymptotically stable in  $\Omega$ . When  $R_0 > 1$ , the worm-free equilibrium  $P_0$  is an unstable saddle point.

**Proof**: The Jacobian matrices of the model (1) at  $P_0$  is

$$J(P_0) = \begin{pmatrix} -\mu & -\beta_1 \Pi / \mu & -\beta_2 \Pi / \mu & \theta & \gamma \\ 0 & \beta_1 \Pi / \mu - n & \beta_2 \Pi / \mu & 0 & 0 \\ 0 & \omega & -m & 0 & 0 \\ 0 & 0 & p & -(\eta + \mu + \theta) & 0 \\ 0 & \delta_2 & (1 - p)\delta_1 & \eta & -(\mu + \gamma) \end{pmatrix}.$$

Obviously,  $J(P_0)$  has three negative eigenvalues  $\lambda_1 = -\mu$ ,  $\lambda_2 = -(\eta + \mu + \theta)$ , and  $\lambda_3 = -(\mu + \gamma)$ , the other eigenvalues of  $J(P_0)$  are determined by the following equation

$$\lambda^2 + (m+n-\beta_1\Pi/\mu)\lambda + mn - (m\beta_1 + \omega\beta_2)\Pi/\mu = 0.$$
 (4)

When  $R_0 < 1$ , then  $mn > (m\beta_1 + \omega\beta_2)\Pi/\mu$ . For  $mn > (m\beta_1 + \omega\beta_2)\Pi/\mu$ , we can easily obtain  $m + n > m + \beta_1\Pi/\mu + \Pi\beta_2\omega/(\mu m)$ , therefore  $m + n - \beta_1\Pi/\mu > m + \beta_2\Pi/\mu > 0$ , which means the Eq. (4) has two negative roots. Therefore, the worm-free equilibrium  $P_0$  is locally asymptotically stable.

When  $R_0 > 1$ , then, which means the Eq. (4) has a positive root and a negative root. Therefore, the worm-free equilibrium  $P_0$  is unstable saddle point.

**Lemma 2**: When  $R_0 \le 1$ , the worm-free equilibrium  $P_0$  is globally asymptotically stable in  $\Omega$ . When  $R_0 > 1$ , all solutions starting in  $\Omega$  and sufficiently close to  $P_0$  move away from  $P_0$ .

**Proof:** Consider the following Lyapunov function:

$$L = \frac{\beta_1 m + \beta_2 \omega}{mn} E + \frac{\beta_2}{m} I.$$

Its derivative along the solutions to the model (1) is

$$L' = \frac{\beta_1 m + \beta_2 \omega}{mn} (\beta_1 SE + \beta_2 SI - nE) + \frac{\beta_2}{m} (\omega E - mI)$$
  
$$= \frac{\beta_1 m + \beta_2 \omega}{mn} (\beta_1 SE + \beta_2 SI) - (\beta_1 E + \beta_2 I)$$
  
$$= (\beta_1 E + \beta_2 I) (\frac{\beta_1 m + \beta_2 \omega}{mn} S - 1)$$
  
$$\leq (\beta_1 E + \beta_2 I) (\frac{\Pi(\beta_1 m + \beta_2 \omega)}{mn\mu} - 1)$$
  
$$= (\beta_1 E + \beta_2 I) (R_0 - 1) \leq 0.$$

Furthermore, L'=0 if and only if E = I = 0 or  $R_0 = 1$ . Thus, the largest compact invariant set in  $\{(S, E, I, Q, R) | L'=0\}$  is the singleton  $\{P_0\}$ . When  $R_0 \le 1$ , the global stability of  $P_0$  follows from LaSalle's invariance principle [15]. LaSalle's invariance principle [15] implies that  $P_0$  is globally asymptotically stable. When  $R_0 > 1$ , it follows from the fact L'>0 if E > 0 and I > 0.

# IV. THE ENDEMIC EQUILIBRIUM AND ITS STABILITY

The endemic equilibrium  $P^*(S^*, E^*, I^*, Q^*, R^*)$  of the model (1) is determined by equations

$$\begin{aligned} \Pi + \gamma R + \theta Q - \beta_1 S E - \beta_2 S I - \mu S &= 0, \\ \beta_1 S E + \beta_2 S I - n E &= 0, \\ \omega E - m I &= 0, \\ p I - l Q &= 0, \\ \delta_2 E + (1 - p) \delta_1 I + \eta Q - (\mu + \gamma) R &= 0, \\ \Pi - \mu N - \alpha_1 E - \alpha_2 I &= 0, \end{aligned}$$

$$(5)$$

where  $l = \mu + \eta + \theta$ .

By a simple computation, we obtain

$$S = \frac{\Pi[\delta_2 ml + (1-p)\delta_1 \omega l + \eta p \,\omega]\gamma + \eta p \,\omega(\mu + \gamma)}{[(\beta_1 m + \beta_2 \omega)(\Pi - \mu N) + \mu(\alpha_2 m + \alpha_1 \omega)]l(\mu + \gamma)},$$

$$E = \frac{m(\Pi - \mu N)}{(\alpha_2 m + \alpha_1 \omega)},$$

$$I = \frac{\omega(\Pi - \mu N)}{(\alpha_2 m + \alpha_1 \omega)l},$$

$$Q = \frac{p \,\omega(\Pi - \mu N)}{(\alpha_2 m + \alpha_1 \omega)l},$$

$$R = S = \frac{(\Pi - \mu N)[\delta_2 ml + (1-p)\delta_1 \omega l + \eta p \omega]}{(\alpha_2 m + \alpha_1 \omega)l(\mu + \gamma)}.$$
(6)

Substituting Eq. (6) into the second equation of the system (5), N satisfies the following equation:

 $F(N)(\Pi - \mu N) = 0,$ where,

 $F(N) = [\mu mnN - A_1 - \mu mn(\alpha_2 m + \alpha_1 \omega)]A_2\gamma,$ where  $A_1 = \Pi(mn - \alpha_2 m - \alpha_1 \omega)(\beta_1 m + \beta_2 \omega)$ ,  $A_2 = \delta_2 m l + (1 - p) \delta_1 \omega + \eta p \omega.$ 

For  $R_0 > 1$ ,  $F(0) = -(A_1 + \mu mn\omega\alpha)A_2\gamma < 0$  and  $F(\Pi/\mu) = -\mu(\alpha_2m + \alpha_1\omega)mnA_2\gamma(R_0 - 1)$ , thus F(N) is monotone increasing and  $F(\Pi/\mu) > 0$ . Within the interval  $(0, \Pi/\mu)$ , F(N) has only a positive root. That is, the model (1) has a unique endemic equilibrium  $P^*(S^*, E^*, I^*, Q^*, V^*)$ , where  $S^*, E^*, I^*, Q^*, R^*$  are determined by (6).

**Lemma 3:** When  $R_0 > 1$ , the endemic equilibrium  $P^*$  is locally asymptotically stable in  $\Omega$ .

**Proof:** Replacing S with N - E - I - Q - R in the model (1), we obtain

$$\begin{cases} E'(t) = (\beta_{1}E + \beta_{2}I)(N - E - I - Q - R) - nE, \\ I'(t) = \omega E - mI, \\ Q'(t) = pI - lQ, \\ V'(t) = \delta_{2}E + (1 - p)\delta_{1}I + \eta Q - (\mu + \gamma)R, \\ N'(t) = \Pi - \mu N - \alpha_{1}E - \alpha_{2}I = 0. \end{cases}$$
(7)

The Jacobian matrices of the model (7) at  $P^*(E^*, I^*, Q^*, V^*, N^*)$  is

$$J(P^*) = \begin{pmatrix} \beta_1 B - b - n & \beta_2 B - b & -b & -b & b \\ \omega & -m & 0 & 0 & 0 \\ 0 & p & -l & 0 & 0 \\ \delta_2 & (1 - p)\delta_1 & \eta & -(\mu + \gamma) & 0 \\ -\alpha_1 & -\alpha_2 & 0 & 0 & -\mu \end{pmatrix}, \quad (8)$$

where  $B = mn / (\beta_1 m + \beta_2 \omega)$ ,  $b = \beta_1 E + \beta_2 I$ .

Its characteristic equation is  $\det(\lambda I - J(P^*)) = 0$ , where *I* is the unit matrix. Therefore,  $\det(\lambda I - J(P^*)) = (\lambda_1 + \mu + \gamma)(\lambda_2 + l)(\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3) = 0$ , where

$$C_1 = b + m + \mu + n - \frac{mn\beta_1}{\beta_1 m + \beta_2 \omega} = b + m + \mu + \frac{n\omega\beta_2}{\beta_1 m + \beta_2 \omega} > 0$$
  
$$C_2 = b(m + \mu + \omega + \alpha_1) + m\mu + mn + n\mu$$

 $-\left(\frac{\beta_1 m^2 n + \beta_1 m n \mu + \beta_2 m n \omega}{\beta_1 m + \beta_2 \omega}\right)$  $= b(m + \mu + \omega + \alpha_1) + m\mu + \frac{\beta_2 \omega n \mu}{\beta_1 m + \beta_2 \omega} > 0,$  $C_3 = b(m\mu + \omega\mu + m\alpha_1 + \omega\alpha_2) + mn\mu - \frac{\beta_1 m^2 n \mu + \beta_2 m n \mu \omega}{\beta_1 m + \beta_2 \omega}$ 

 $= b(m\mu + \omega\mu + m\alpha_1 + \omega\alpha_2) > 0.$ 

By a direct calculation, we obtain that  $C_1C_2 - C_3 > 0$ . According to the theorem of Routh-Hurwitz, the endemic equilibrium  $P^*$  is locally asymptotically stable.

For the model (7), we consider global stability of the endemic equilibrium  $P^*$  when  $\alpha_1 = \alpha_2 = 0$ . Since  $\lim_{t\to\infty} \inf N(t) \le \Pi/\mu$ , the model (9) is a four-dimensional asymptotically autonomous differential system with the limit system

$$\begin{cases} E'(t) = (\beta_{1}E + \beta_{2}I)(\Pi / \mu - E - I - Q - R) - nE, \\ I'(t) = \omega E - mI, \\ Q'(t) = pI - lQ, \\ V'(t) = \delta_{2}E + (1 - p)\delta_{1}I + \eta Q - (\mu + \gamma)R. \end{cases}$$
(9)

Next, we apply the geometrical approach [16] to investigate the global stability of the endemic equilibrium  $P^*$  in the region  $\Omega$ .

**Theorem 1:** Consider the following systems [16]:  $x' = f(x), x \in \Omega$ .

If the following conditions are satisfied: (1) The system (\*) exists a compact absorbing set  $K \subset \Omega$  and has a unique equilibrium  $P^*$  in  $\Omega$ ; (2)  $P^*$  is locally asymptotically stable; (3) The system (\*) satisfies a Poincaré-Bendixson criterion; (4) A periodic orbit of the system (\*) is asymptotically orbitally stable, then the only equilibrium  $P^*$  is the globally asymptotically stable in  $\Omega$ .

**Lemma 4:** If  $R_0 > 1$ , the unique positive equilibrium  $P^*$  of model (9) is globally asymptotically stable in  $\Omega$ .

**Proof:** We only need to prove that all assumptions of Theorem 1 hold.

If  $R_0 > 1$ , then  $P_0$  is unstable according to Lemma 1. Moreover, the behaviour of the local dynamics near the region  $P_0$  described in Lemma 1 implies that model (9) is uniformly persistent in the region  $\Omega$ . That is, there exists a constant c > 0, such that any solution (E(t), I(t), Q(t), R(t)) of model (9) with initial value (E(0), I(0), Q(0), R(0)) in  $\Omega$  satisfies min  $\lim_{t \to \infty} \inf E(t), \liminf_{t \to \infty} I(t), \liminf_{t \to \infty} Q(t), \lim_{t \to \infty} \inf V(t) \ge c$ .

This can be proved by applying a uniform persistent result in [17] and by the use of a similar argument as in the proof [18]. The uniform persistence of system (9) in the bounded set  $\Omega$  is equivalent to the existence of a compact  $K \subset \Omega$  that is absorbing for system (9). In Section III, during the process of obtaining the endemic equilibrium  $P^*$ , we can know that  $P^*$  is the unique equilibrium in the interval  $(0, \Pi/\mu)$ . Assumption (1) holds.

According to Lemma3, we know that the endemic equilibrium  $P^*$  is locally asymptotically stable in the region  $\Omega$ . Assumption (2) holds.

The Jacobian matrix of model (9) is denoted by

$$J(P^{*}) = \begin{pmatrix} \beta_{1}S - b - n & \beta_{2}S - b & -b & -b \\ \omega & -m & 0 & 0 \\ 0 & p & -l & 0 \\ \delta_{21} & (1 - p)\delta_{1} & \eta & -\mu - \gamma \end{pmatrix}.$$
 (10)

Choosing the matrix *H* as  $H = diag\{1,-1,1,1\}$ , it is easy to prove that *HJH* has non-positive off-diagonal elements, thus we can obtain that system (9) is competitive. This verifies the assumption (3).

The second compound matrix  $J^{[2]}(P^*)$  of  $J(P^*)$  [19] can be calculated as follows:

$$J^{[2]}(P^*) = \begin{pmatrix} D_1 & 0 & 0 & b & b & 0 \\ p & D_2 & 0 & \beta_2 S - b & 0 & b \\ (1-p)\delta_1 & \eta & D_3 & 0 & \beta_2 S - b - b \\ 0 & \omega & 0 & D_4 & 0 & 0 \\ -\delta_2 & 0 & \omega & \eta & D_5 & 0 \\ 0 & -\delta_2 & 0 & -(1-p)\delta_1 & p & D_6 \end{pmatrix},$$

where,

$$\begin{split} D_1 &= -(b+n+m-\beta_1 S) \,, \\ D_2 &= -(b+n+l-\beta_1 S) \,, \\ D_3 &= -(b+n-\beta_1 S) \,, \\ D_4 &= -(m+l) \,, \\ D_5 &= -(m+\mu+\gamma) \,, \\ D_6 &= -(l+\mu+\gamma) \,. \end{split}$$

The second compound system of the model (9) in a periodic solution can be represented by the following differential equations:

$$\begin{aligned} X'(t) &= -(b+n+m-\beta_1 S)X + bL + bM, \\ Y'(t) &= pX - (b+n+l-\beta_1 S)Y - (b-\beta_2 S)L + bU, \\ Z'(t) &= (1-p)\delta_1 X + \eta Y - D_3 Z - (b-\beta_2 S)M - bU, \\ L'(t) &= \omega Y - (m+l)L, \\ M'(t) &= -\delta_2 X + \omega Z + \eta L - (m+\mu+\gamma)M, \\ U'(t) &= -\delta_2 Y - (1-p)\delta_1 L + pM - (l+\mu+\gamma)U. \end{aligned}$$
(11)

Next, we will prove that the system (11) is asymptotically stable.

We consider the following Lyapunov function:

V(X,Y,Z,L,M,U;E,I,Q,R)

$$= \sup\{|X| + |Y| + |Z|, \frac{E}{I}(|L| + |M| + |U|)\}.$$

By the use of the uniform persistence, we obtain that the orbit of P(t) = (E(t), I(t), Q(t), V(t)) remains a positive distance from the boundary of  $\Omega$ , thus we know that there exists a constant *c* satisfying

V(X,Y,Z,L,M,U;E,I,Q,R)

 $\geq c \sup\{|X|, |Y|, |Z|, |L|, |M|, |U|\},\$ 

for all  $(X, Y, Z, L, M, U; E, I, Q, R) \in \mathbb{R}^6$  and and  $(E, I, Q, R) \in \mathbb{P}(t)$ .

For the differential equations in Eq. (11), we can obtain the following differential inequalities by direct calculations:

$$\begin{split} D_+(|X|+|Y|+|Z|) &\leq -x(|X|+|Y|+|Z|) \\ &+ \frac{E}{I}(\beta_1 S + \beta_2 S \frac{I}{E})(|L|+|M|+|U|), \\ D_+(|L|+|M|+|U|) &\leq \omega(|X|+|Y|+|Z|) \\ &- y(|L|+|M|+|U|), \\ \text{where, } x &= (2\mu + p + (1-p)\delta_1 + \alpha_1 + \delta_2 + \omega), \\ y &= 2\mu + p + (1-p)\delta_1. \\ \text{Then,} \\ D_+ \frac{E}{I}(|L|+|M|+|U|) &\leq \omega \frac{E}{I}(|X|+|Y|+|Z|) \\ &+ (\frac{E'}{E} - \frac{I'}{I} - y)\frac{E}{I}(|L|+|M|+|U|). \end{split}$$

From the previous formula, we can obtain  $D_+ |V(t)| \le \max\{g_1(t), g_2(t)\}V(t),$ 

where,

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$$\begin{split} g_{1}(t) &= -x + (\beta_{1}S + \beta_{2}SI / E), \ g_{2}(t) = \omega \frac{E}{I} + \frac{E}{E} - \frac{T}{I} - y \ . \\ \text{From the model (1), we can obtain} \\ E' / E &= \beta_{1}S + \beta_{2}SI / E - (\mu + \alpha_{1} + \delta_{2} + \omega), \\ I' / I &= \omega E / I - (\mu + \alpha_{2} + (1 - p)\delta_{1} + p) \ . \\ \text{Therefore,} \\ g_{1}(t) &= E' / E - \mu - p - (1 - p)\delta_{1}, \ g_{2}(t) = E' / E - \mu \ . \\ \text{We find } \sup\{g_{1}(t), g_{2}(t)\} \leq E' / E - \mu \ , \\ \text{and thus} \end{split}$$

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$$\int_{0}^{\xi} \sup\{g_{1}(t), g_{2}(t)\} dt \le \ln E(t) \mid_{0}^{\xi} -\mu\xi = -\mu\xi < 0$$

which implies that  $(X(t), Y(t), Z(t), L(t), M(t), U(t)) \rightarrow 0$ , as  $t \rightarrow \infty$ . Thus, the second compound system (11) is asymptotically stable. This verifies the assumption (4).

We verify all the assumptions of Theorem 1. Therefore,  $P^*$  is globally asymptotically stable in  $\Omega$ .

# V. NUMERICAL SIMULATIONS

In order to simulate the real behavior of the spread of a worm, the parameters in the experiments are practical values for when worms break out in our real life. We choose the Slammer as basic behavior of a worm in this experiment. Slammer worm is chosen because, despite its simplicity, it still holds the world record of fastest-spread worm [20]. To obtain the spread of worms in a largescale network, 750,000 hosts are selected as the population size. According to the real conditions of the Slammer worm, its average scan rate is s = 4,000 per second. Slammer worm's infection rate can then be computed as  $\beta_2 = s/2^{32} = 9.3 \times 10^{-7}$ ,  $\beta_2 = 9 \times 10^{-7}$ . At the beginning, the number of susceptible, exposed, infected, quarantined and recovered hosts are S(0) = 50,000, E(0) = 0, I(0) = 700,000, Q(0) = 0, R(0) = 0, respectively.

Other parameters in these simulations are given as follows:  $\Pi = 75$  ,  $\mu = 0.0001$  ,  $\gamma = 0.3$  ,  $\theta = 0.3$  ,  $\alpha_1 = 0.0001, \ \alpha_2 = 0.0002, \ \delta_1 = 0.8, \ \delta_2 = 0.7, \ \omega = 0.1,$ p = 0.4,  $\eta = 0.5$ , where  $R_0 = 0.97 < 1$ . We change some parameters about the reproduction number  $R_0$  to obtain different  $R_0$ , e.g.,  $\omega$ . When  $\omega = 0.6$ ,  $R_0 = 0.90 < 1$ . The worm will gradually disappear according to Lemma 1 and 2. Fig. 2 illustrates the number of susceptible and infected hosts when  $R_0$  is 0.97 and 0.90, respectively. From Fig. 2, we can clearly see that the tendency of the worm propagation is depressive, which is consistent with Lemma 1 and 2. Finally, all infected hosts vanish and the population, in the long term, is in a susceptible state. In order to effectively defend against such worms, we must adopt some feasible methods to decrease the infection rate [21], [22], improve the accuracy of intrusion detection systems [23], or increase the following parameters (e.g., the transfer rates between the exposed and the recovered, between the exposed and the infectious) to guarantee the basic reproduction number  $R_0 < 1$ .



Figure 2. Globally asymptotically stable worm-free equilibrium.

In the second experiment, the number of susceptible, exposed, infected, guarantined and recovered hosts are

S(0) = 749,990, E(0) = 0, I(0) = 10, Q(0) = 0, R(0) = 0, respectively. When  $\delta_2 = 0.2$ ,  $\omega = 0.1$ , we can obtain  $R_0 = 2.59 > 1$ . For  $\delta_2 = 0.2$ ,  $\omega = 0.9$ ,  $R_0 = 1.28 > 1$ . Other parameters do not vary. We can see the results in Fig. 3. As can be seen from Fig. 3, the number of susceptible and infected hosts eventually become positive values between 0 and  $\Pi / \mu$ . The worm does not disappear. This is fully consistent with the conclusions of Lemma 3 and 4. In this experiment, the basic reproduction number  $R_0$  has not an obvious effect on the number of infected hosts, however, significantly affects the number of susceptible hosts. The larger the basic reproduction number is, the more the number of susceptible hosts become.



Figure 3. Globally asymptotically stable endemic equilibrium.

With other parameters remaining the same, the quarantined rate p is set to different value each time in order to state that the number of infected hosts is affected by every different value of the quarantined rate. Fig. 4 shows the effect of changing the quarantined rate (which vary between 0.1 and 0.9) on worm propagations. As expected, a larger quarantined rate results in diminishing the worm propagation speed, prolonging the time at which infected population reaches its peak, more importantly, lowering the total number of infected hosts. Quarantined rate p relies mainly on the accuracy and detection speed of intrusion detection algorithms. Some methods have been proposed on to reach the goal, e.g., a pulse quarantine strategy [4], an orchestration approach [24].



Figure 4. Effect of the quarantined rate on infected hosts.

# VI. CONCLUSION

This paper presented a mathematical model describing the dynamical behavior of a SEIQRS epidemic model with graded infection rates for Internet worms. Firstly, by the method of next generation matrix, we give the basic reproduction number determining whether the worm extinguishes. Secondly, the global asymptotic stabilities of our model have proved by the use of the Lyapunov function and a geometric approach. When the basic reproduction number is less than or equal to 1, the proposed model has only a worm-free equilibrium which is globally stable, it implies the worm dies out eventually; when the basic reproduction number is larger than 1, our model has a unique endemic equilibrium which is globally stable, it implies that the worm persists in the whole host population and tends to a steady state. Finally, some numerical examples are given to verify our conclusions. Our future work will expand this model which can characterize more features of Internet worms, e.g., taking delay or impulse into consideration.

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