# Restricted Nussbaum Gain Control Method and Its Application in One Order System

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***Abstract*—The unknown control direction is one of the most open difficult question in nonlinear control theory. A new kind of restricted Nussbaum method is proposed to solve the unknown control direction problem without using high gain feedback. Although High gain has some advantages for improving the dynamic performance of a control system such as it can improve the quickness of the system and it can make the steady state error as small as possible. But now experienced engineers also realized that it causes many problems such as make the system unstable for big signal or high frequency noise. A kind of restricted Nussbaum gain method is firstly proposed to solve the problem. And simulation results show that it has better control effect compared with the traditional Nussbaum gain method. What is worthy pointing out is that the robustness of the Nussbaum gain strategy is improved obviously.**

***Index Terms*—Nussbaum gain; Unknown control direction; Stability; Uncertainty; Adaptive**

## I. INTRODUCTION

Since there are typical results for linear system control, more and more researchers are interesting in control of nonlinear system [1-3]. The main difficulties for nonlinear control lie to the uncertainties and nonlinearity. Also there are some special method is proposed to solve special nonlinear systems; such as special controller [4, 5] designed for a kind of strict feedback nonlinear systems. Or the controller is designed based on differential geometric approach theory[6], Neural-network method [7], Robust control theory[8,9,10], linear feedback[11], sliding mode method[12,13,14,15]. But the unknown control direction is one of the most open difficult questions in nonlinear control theory.

Since Nussbaum first proposed the Nussbaum gain method [16, 17], the unknown control direction problem has arouse many researchers' interests. And Nussbaum gain method was applied in first order system and achieved obvious effect. Adaptive methods used by DING Z[18] to solve unknown control direction problem, and this problem can also be called without a priori knowledge of control direction[19,20] or with unknown virtual control coefficients. Xuedong Ye [21] considered the over parameterization problem during coping unknown control directions. NN method is applied to solve this problem and simultaneously the input constraint is considered by Weisheng Cheng [22]. And T.P.Zhang [23] also use adaptive neural method to solve this problem with dead-zones.

Also unknown control direction with discrete time, time varying , time-delay and output feedback problems are studied by Yang C[24] and Ge, S. S[25,26] and Liu Yunguang [27]. There are some similar problems such as unknown high frequency gains or unmodeled dynamics are research by KE Hai-sen [28] and M. Krstic[29]. Since there is no clear line among those problems so methods for unknown high frequency and unmodeled dynamics can also be used in solve unknown control direction problem.

The Nussbaum gain method can deal with the unknown control direction problem, which is discussed in many papers recently. But it is also has some problems when this method is applied in real engineering control object. The first problem is how to design a proper gain for the system.

It is very common for real physical systems that the gain is not allowed being too big. Also the experienced engineers will be very care to choose a proper gain for every loop. In fact, the big gain can make the system have better performance, such as high response speed, small steady state error and so on. But the problem is that high gain can also cause a lot of problems. First one is that it can make the system too sensitive to noise and also make the system unstable in some special situation. The second disadvantage is that it is too expense to realize a high gain device or it is impossible to realize the high gain system in some situation.


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So it is always important to choose a proper gain for every system with any control method. Nussbaum gain control is a kind of complex nonlinear adaptive method. It should also obey the above principle. In fact, some Nussbaum gain algorithms can be unstable in some situation and the main reason is that the Nussbaum gain is not proper, or in most situations it is too big. In this paper, a new kind of restricted Nussbaum method is proposed to solve the above problem.

## II. Problem Description

Considering a typical one order system with unknown control direction

$$\dot{x} = f(x) + bu \quad . \tag{1}$$

Where $x$ is the state of system and $u$ is the controller or input, $f(x)$ is the known nonlinear function and $b$ is the unknown control direction or it can be called control coefficient.

The control objective of restricted Nussbaum gain method is to design a control $u = q(x,k)$ and $|k| \le k_{sat}$, where $k_{sat}$ is the restriction of the system gain condition, such that system state $x$ can track desired value $x^d$ even if the sign of control $b$ is changed from positive to negative or from negative positive.

The solution of restricted gain problem is based on the traditional Nussbaum gain method since it is effective for the unknown control direction problem. But to make the gain is bounded; a bounded gain function is introduced. With a design of double bounded gain layer, the restriction of the gain is realized; also the whole system signals can be guaranteed to be bounded.

## III. Assumption

For above systems, five assumptions are proposed to make the following design easier.

Assumption 1: The given gain $k_{sat}$ is big enough and it satisfies the energy need of the physical system.

Assumption 2: Without loss of generality, assume the desired value to be constant so $\dot{x}^d = 0$.

Assumption 3: The control direction $b$ is bounded and its value is in an unknown field $I = [l_1^-, l_1^+]$ where $0 \notin I$, so the sign of $b$ is unknown.

Assumption 4: The bound of $b$ is known, so there exists a known constant $b_{\max}$ such that $|b| \le b_{\max}$.

Assumption 5: The number of the sign change of $b$ is not infinite.

## IV. Design of Restricted Nussbaum Gain

Design a new error variable as $z = x - x^d$, the error of the system can be written as

$$\dot{z} = f(x) + bu \, . \tag{2}$$

Use the poles placement method to design the virtual control as

$$u^d = -f(x) - k_1 z - k_2 \int z dt \quad . \tag{3}$$

Considering the unknown control direction problem, design the Nussbaum gain control as:

$$u = -N(k) u^d \quad . \tag{4}$$

Then

$$\dot{z} = -k_1 z - k_2 \int z dt - bN(k)u^d - u^d \quad . \tag{5}$$

Design the turning law of Nussbaum gain as

$$\dot{l} = k_l z u^d \quad . \tag{6}$$

Design a bounded gain function as

$$k = f_s(l) \quad . \tag{7}$$

Where $f_s(l)$ can be chosen as triangular functions.

The error function can be written as:

$$z\dot{z} = -k_1 z^2 - k_2 z \int z dt - \frac{1}{k_l}(bN(k)+1)dl \quad . \tag{8}$$

Construct a type of integral Lyapunov function as follows:

$$V = \frac{1}{2}z^2 + \frac{k_1}{2}\int z^2 dt + \frac{k_2}{2}(\int z dt)^2 \quad . \tag{9}$$

Solve the derivative of Lyapunov function as

$$k_l \dot{V} = -(bN(k)+1)dl \quad . \tag{10}$$

With integration in both sides of the equation, it holds:

$$\begin{aligned} k_l V(t) - k_l V(0) &= \int -(bN(k)+1)dl \\ &= \int_{l(0)}^{l(t)} -bN(k)dl + l(0) - l(t) \end{aligned} \quad . \tag{11}$$

To make it simple, choose the restricted Nussbaum gain as $k = 10\sin(l)$, Nussbaum, and choose the Nussbaum gain function as $N(k) = k^2 \sin k$.

It is obvious that the system gain satisfies below restrictions with the above design:

$$N(k) = k^2 \cos k < 100 \quad . \tag{12}$$

Also different restriction can be chosen to meet the different demand of real systems.

With the proof below, the system can be proved to be stable and the control law can adapted to the disturbance of control direction change.

First, assume the gain $l$ is unbounded. Without loss of generality, consider the situation $l \to +\infty$, divide by $l$ on both side of the equation it holds:

$$\frac{k_l V(t) - k_l V(0) + l(0)}{l} = \frac{1}{l}\int_{l(0)}^{l(t)} -bN(k)dl - 1 \quad . \tag{13}$$

Since $-bN(k)$ is bounded, there exist positive constants $\varepsilon_1$ and $\varepsilon_2$ such that

$$-\varepsilon_1 \le \frac{1}{l}\int_{l(0)}^{l(t)} -bN(k)dl - 1 \le \varepsilon_2 \quad . \tag{14}$$

It is obvious that if $k_l > 0$, it holds:

$$-\frac{l}{k_l}\varepsilon_1 \le V(t) \le \frac{l}{k_l}\varepsilon_2 \quad . \tag{15}$$

If $k_l < 0$, it holds:

$$\frac{l}{k_l}\varepsilon_2 \le V(t) \le -\frac{l}{k_l}\varepsilon_1 \ . \quad (16)$$

Assume that the limit of above integration exists:

$$\lim_{l\to\infty}\frac{1}{l}\int_{l(0)}^{l(t)} -bN(k)dl = \varepsilon + 1 \ . \quad (17)$$

So it always has

$$V(t) = \varepsilon k_l l \ . \quad (18)$$

It is obvious that contradiction will appear if $k_l > 0$ or $k_l < 0$ because it will make $V(t) \le 0$.

Also if the integration does not exist, so there exists a $t$ such that

$$\lim_{l\to\infty}\frac{1}{l}\int_{l(0)}^{l(t)} -bN(k)dl = \varepsilon_3 \ . \quad (19)$$

where

$$-\varepsilon_1 \le \varepsilon_3 \le \varepsilon_2 \ . \quad (20)$$

So whether $\varepsilon_3$ is positive or negative, there exists $k_l > 0$ or $k_l < 0$ such that $V(t) \le 0$, then contradiction appears.

Now, it is easy to prove that $l(t)$ is bounded. Also $V(t)$ is bounded and $z(t)$ is bounded. Finally, $u^d$ is proved to be bounded. Because of the construction of integral type Lyapunov function, it is easy to prove that $z(t) \to 0$.

## V. Numerical Simulation

Take a simple one order system with unknown control direction as an example, the model can be written as

$$\dot{x} = 3x\sin x + bu \ . \quad (21)$$

Where the unknown control direction $b$ is designed as:

$$b = \begin{cases} 1 & 0 < t < 3 \\ -2 & 3 < t < 6 \\ 2 & t > 6 \end{cases} \ . \quad (22)$$

Choose the initial value of Nussbaum gain as $l(0) = 1$, the initial condition as $x(0) = 0$, set the desired value of state as $x^d = 2$, set the simulation step as 0.001s and do the simulation with Eular method , the simulation result can be show as below figures.

Figure 1 shows the state of x1 and at time 3s and 6s, the state x is disturbed by the change of control direction. But it can be stabled by Nussbaum gain law successfully.

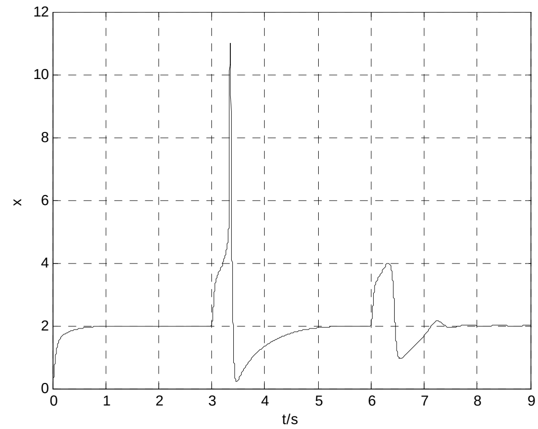


Figure 1. Curve of state x.

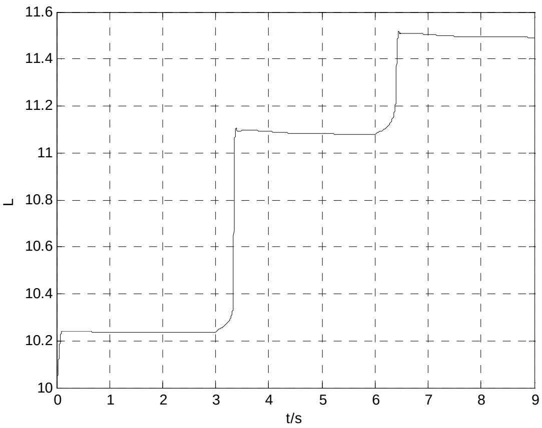


Figure 2. Curve of Gain L

Figure 2 shows the restricted Nussbaum Gain L and it is clear that the Nussbaum gain can adapt to a proper gain after the disturbance of control direction.

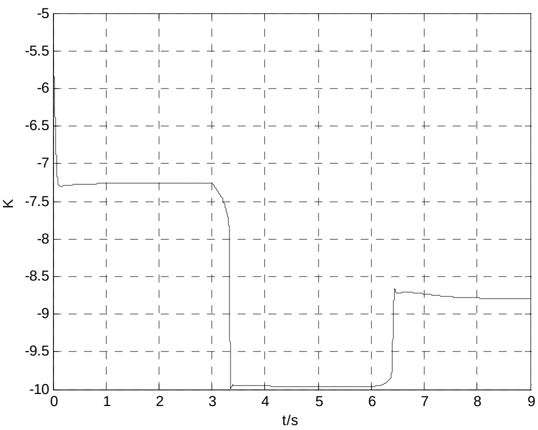


Figure 3. Curve of Gain K.

Figure 3 shows the real Nussbaum Gain K and it is obvious that the Nussbaum gain can adapt to the unknown change of control direction.

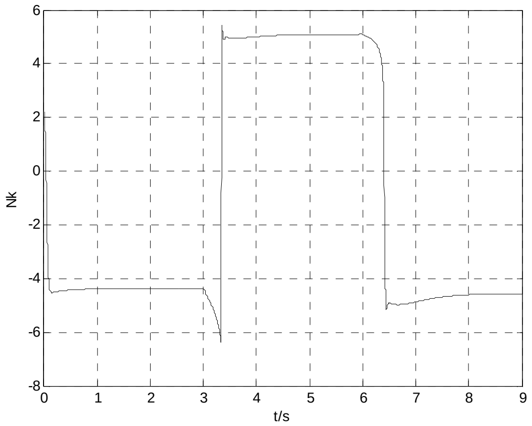


Figure4. Curve of Gain Nk

Figure 4 shows the real gain of control and it can follow the change of control direction very quickly.

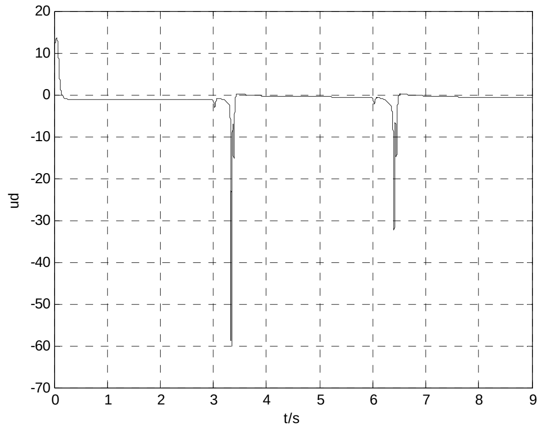


Figure 5. Ideal control Ud.

Figure 5 shows the ideal control and the overshoot is obvious at the time when control direction is changed.

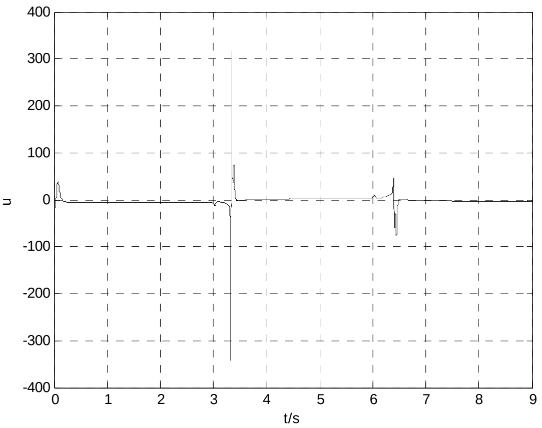


Figure6. Curve of Control U

Figure 6 shows the real control and it shows that at the time the control direction changes the control law need enough energy to make the system stable.

In order to test the robustness of this limit Nussbaum gain method under different initial gain value, we done a simulation assume that the initial gain is $l(0)=10$, and simulation result is show as below.

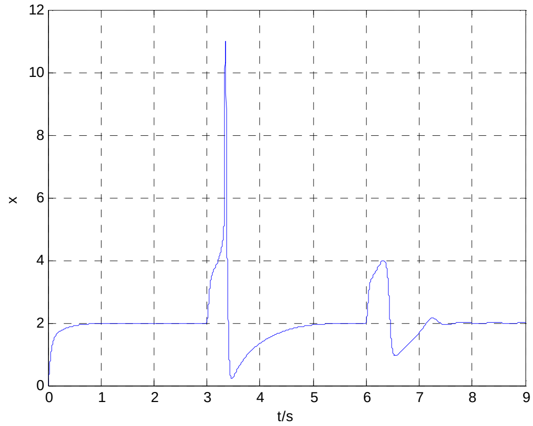


Figure 7. Curve of state x.

Figure 7 shows the state of x1 and at time 3s and 6s, the state x is disturbed by the change of control direction.

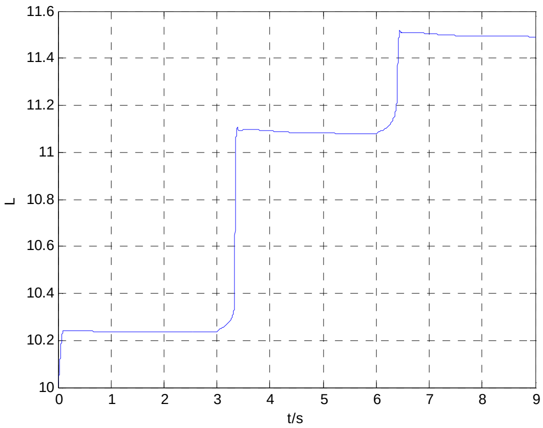


Figure 8. Curve of Gain L

Figure 8 shows the restricted Nussbaum Gain L and the initial value is set to 10.

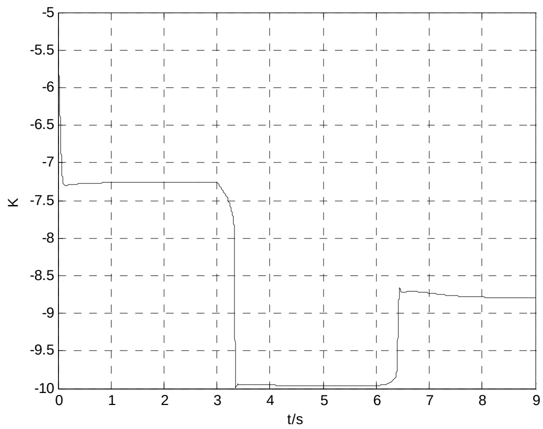


Figure 9. Curve of Gain K

Figure 10 shows the real Nussbaum Gain K and it is begin from -5.5 and adapt to the unknown change of control direction and finally stop at about -9.5.

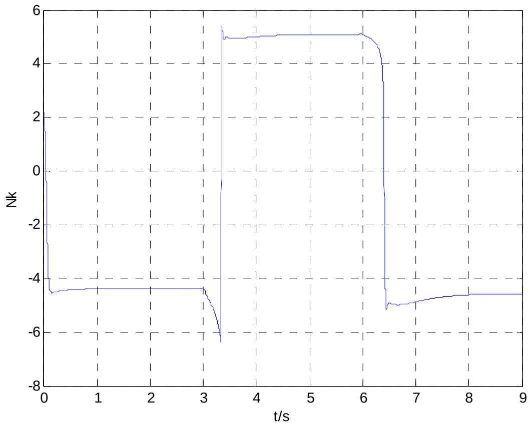


Figure 10. Curve of Gain Nk

Figure 11 shows the ideal gain of control and it can follow the change of control direction very quickly.

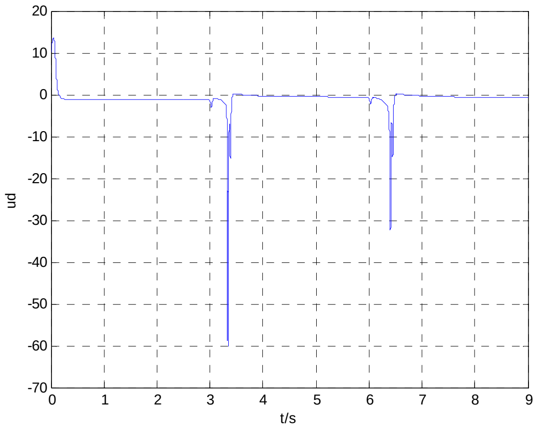


Figure 11. Ideal control Ud

Figure 12 shows the ideal control and the overshoot. We can make a conclusion from above simulation results that the restricted Nussbaum gain method is effective and robust under different initial gain value.

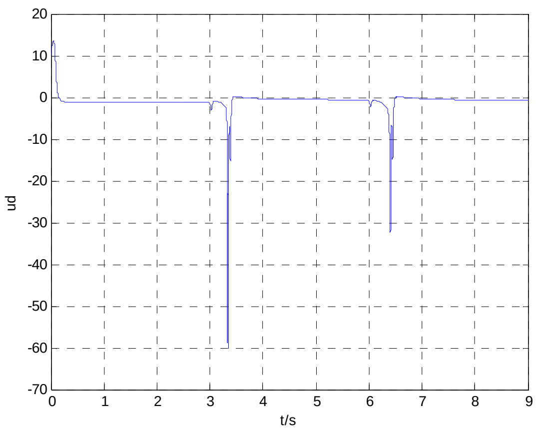


Figure 12. Curve of real control U

To test the robustness of the restricted Nussbaum method under different initial value of system state, we done a simulation and assume that the initial value the first order system is $x(0)=5$. And the simulation result is shown as below.

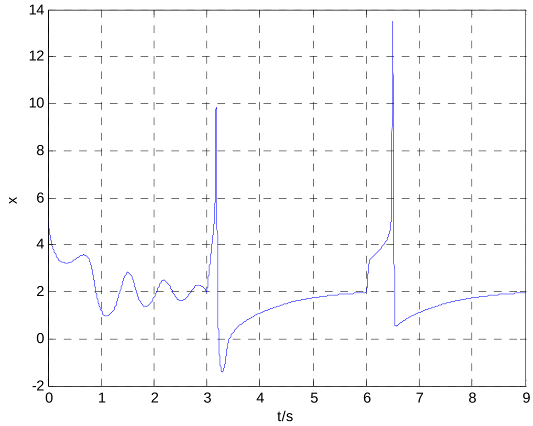


Figure 13. Curve of state x.

Figure 13 shows the state of x1 and at time 3s and 6s, the state x is disturbed by the change of control direction, and the initial state is 5.

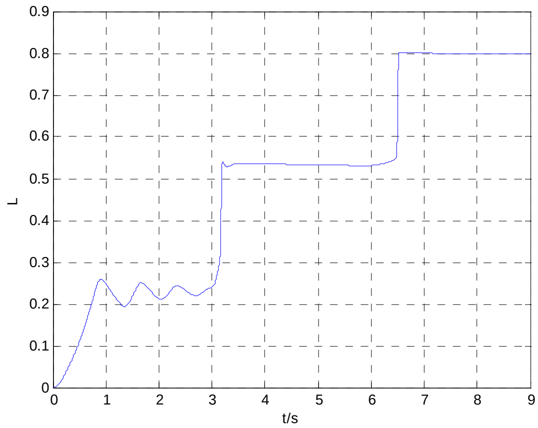


Figure 14. Curve of Gain L

Figure 14 shows the restricted Nussbaum Gain L and the initial value is set to 10.

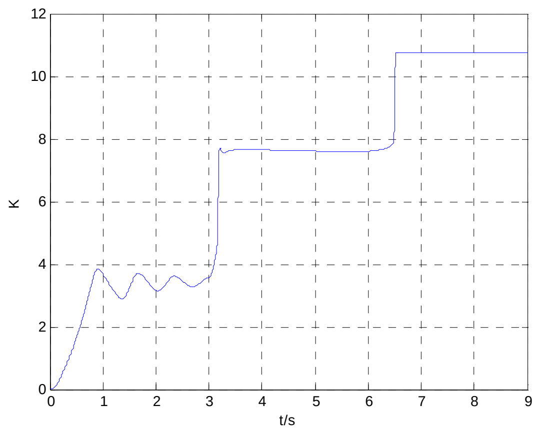


Figure 15. Curve of Gain K

Figure 15 shows the real Nussbaum Gain K and it is begin from 0 and adapt to the unknown change of control direction and finally stop at about 10.8.

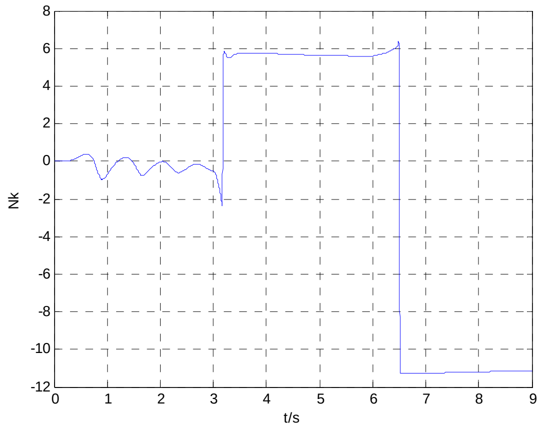


Figure 16. Curve of Gain Nk

Figure 16 shows the ideal gain of control and it can follow the change of control direction very quickly.

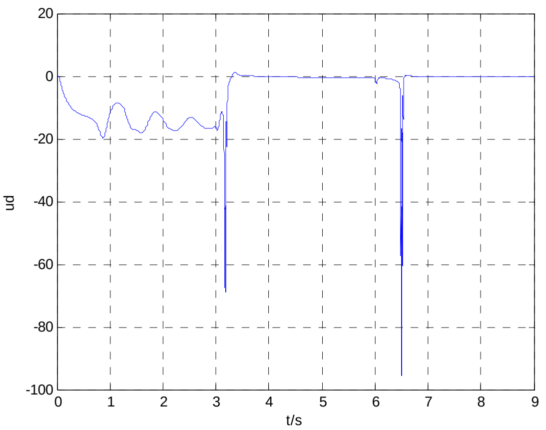


Figure 17. Ideal control Ud

Figure 17 shows the ideal control and the overshoot and Figure 18 shows the real control of the Nussbaum gain method. We can make a conclusion from above simulation results that the restricted Nussbaum gain method is effective and robust under different initial system state.

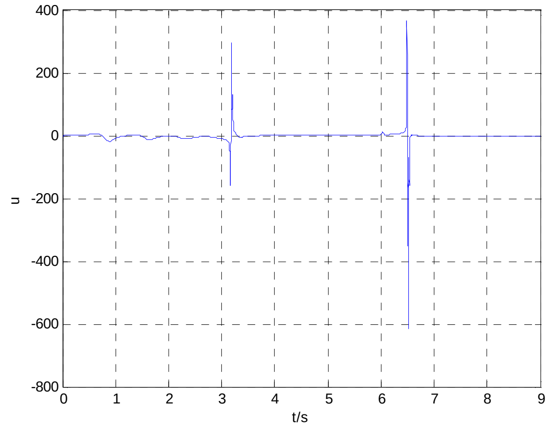


Figure 18. Curve of real control U

Also choose another initial value as $x(0) = 6.6$, it is the biggest initial value the proposed restricted method can cope. And compared with common Nussbaum gain method, this value is improved. And the simulation result is shown as below.

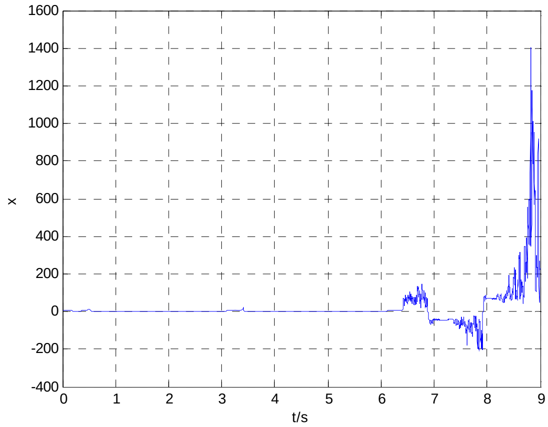


Figure 19. Curve of state x

Figure 19 shows the state of x1 and at time 3s and 6s, the state x is disturbed by the change of control direction. It is obvious that the system is unstable since the initial value of the system state is too big.

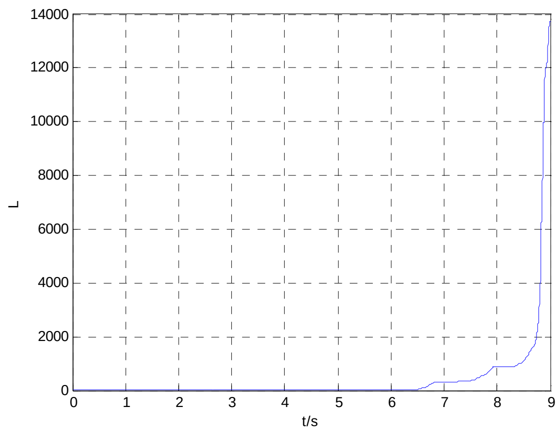


Figure 20. Curve of Gain L

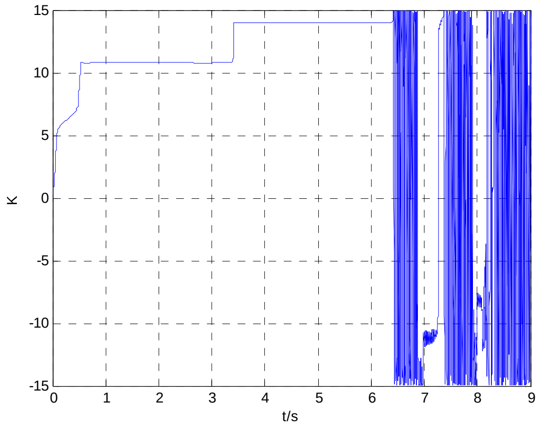


Figure 21. Curve of Gain K

Figure 20 shows the restricted Nussbaum Gain L and the initial value is set to 0, and figure 21 shows the Nussbaum gain K and it is restricted from -15 to 15. Both gain K and L is unstable as time increase.

So we can make a conclusion that the restricted Nussbaum can improve the system stability compared with common Nussbaum gain method. But it is still can not guarantee the system is always stable if the initial value is too big. That is mainly because the Nussbaum gain is a nonlinear control law and the stability is not global. And also because the simulation step is not small enough on some point the gain of the system is too big.

## VI. Conclusions

A kind of restricted Nussbaum gain method is proposed in this paper. It can be used to solve the complex unknown control direction problem. The advantage of this method is that the gain of controller is limited. It can also improve the traditional Nussbaum gain design that it makes the Nussbaum gain strategy more stable since the gain is restricted. Also the stability is proved in this paper that it is not affected by the limit gain.

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