# Buckling Capacity Optimization of Stiffened Rectangular Plate under Uniform Normal Compression

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***Abstract*—The stiffened rectangular plate was usually adopted in the blast airtight doors. In order to improve the buckling capacity of stiffened rectangular plate under uniform normal compression, the optimization model of stiffened rectangular plate was set up based on APDL and ANSYS commands, and the sequential linear programming method was executed to optimize the thickness of plate and the sizes of stiffeners. Moreover, we compared the mechanical property of the optimized stiffened rectangular plate with the theoretical value of no-stiffener plate with equal volume, and obtained the reasonable stiffener distribution based on the optimization results of five different longitudinal and transverse stiffener patterns. The results showed that the buckling capacity of stiffened rectangular plate under uniform normal compression could be improved by approximately 50% on the condition of reasonable stiffeners distribution.**

***Index Terms*—buckling; normal compression; stiffened rectangular plate; optimization**

## I. INTRODUCTION

The stiffened rectangular plate was usually adopted in the blast airtight doors of coal mine refuge chambers. Usually, the stiffened plates have bigger buckling capacity than the no-stiffener plates with equal quality, which can reach the stress level at 40% to 50% of the material's yield stress. Meanwhile, it has become a prominent problem that how to distribute the stiffeners to improve the buckling capacity of plate structures under normal compression according to the plate structures and load characteristics.

The theoretical and experimental research on buckling of thin plates and shells was studied relatively early. And the rectangular plate limit condition only suffering the film stress has been applied to solve the ultimate load problems [1]. The overall deformation process of elastic thin rectangular plate under symmetrical normal compression has been studied. And the buckling stress empirical formula for rectangular plate suffering normal compression at the axes has been suggested [2-5]. Recently, Some researchers have studied the plastic buckling and post-buckling of thin plate and shell under normal compression. Moreover, the elastoplastic numerical solution of thin plate's buckling and post-buckling has been obtained [6]. Furthermore, based on the research of the buckling deformation pattern of this structure on combined load, the most important influence factor in stiffened plate and plate has been obtained [7,8].

Recently, lots of researchers have adopted different methods in the plate structure optimization, such as energy principle method, flexible tolerance polyhedron method, and full-stress standard method [9-14].

In view of the above, we have a lot of work to do to improve the buckling capacity of stiffened rectangular plate in the field of plate structure optimization [15,16]. The author has studied the buckling capacity optimization of stiffened cylindrical shell under uniform axial compression [17]. In this paper, we set up the optimization model of stiffened rectangular plate based on APDL and ANSYS commands, and executed the serial linear programming optimization procedure to optimize the plate structure.

## II. MATHEMATICAL MODEL

With the research on the structure optimization deepening, structure optimization softwares have developed greatly. Moreover, the ANSYS software has been the most outstanding finite element analysis software of structure optimization [18], the whole parameterized options of which can be chosen as the optimize parameters. And APDL (ANSYS Parametric Design Language) is an indispensable important technology of ANSYS software, which can realize the parameterized finite element analysis, batched analysis, secondary development and optimization design.

The process of optimization design based on APDL is as follows. And the parameter analysis file is built for optimal circulation based on APDL and ANSYS commands, then the analysis processing is executed in OPT and the serials of optimal design is checked up.

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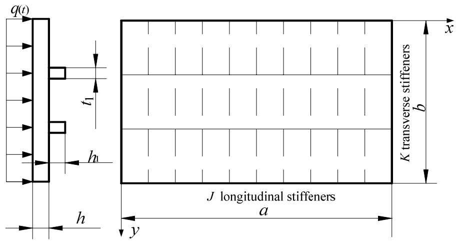


Fig.1 Stiffened plate under uniform normal compression

In this paper, finite element analysis software ANSYS was employed and secondarily the optimization model was set up with APDL. Fig.1 shown that the rectangular plate with *J* longitudinal stiffeners and *K* transverse stiffeners under uniform normal compression *q*(*t*), where *a* is the length of plate, *b* is the width of plate, *h* is the thickness of plate, $t_1$ is the width of stiffeners and $h_1$ is the height of stiffeners. In addition, the low alloy steel (Q345R) was adopted in the rectangular plate structure model, which could be assumed as perfect elastic material. In the optimization analysis, it was assumed that the rectangular plate was supported simply on four sides. And the length *a* was chosen as 1200 mm and the width was chosen as 600 mm. According to engineering experiment, the thickness was chosen as 10 mm. The sections of longitudinal stiffeners and transverse stiffeners were both rectangle and the same to each other, whose width and height were 5 mm and 20 mm respectively.

Taking the thickness of rectangular plate *h*, the width of stiffeners $t_1$ and the height of stiffeners $h_1$ as optimization design variables, the optimization model was built as following forms:

$$\begin{cases} \max q_{cr} = q_0 + C_1(x_1 - x_{1,0}) + \\ \qquad C_2(x_2 - x_{2,0}) + C_3(x_3 - x_{3,0}) \\ \text{s.t.}\ abx_1 + Jbx_2x_3 + Kax_2x_3 \le V_0 \end{cases} \tag{1}$$

where $x_{1,0}$-$x_{3,0}$ are the initial value of *h*, $t_1$, $h_1$, respectively; $q_{cr}$ is optimization object; $q_0$ is the initial value of the buckling load; $C_1$-$C_3$ are the undetermined coefficients which can be obtained by fitting the software analysis results; $V_0$ is the volume of rectangular plate before optimization; *K* is the quantity of transverse stiffeners; *J* is the quantity of longitudinal stiffeners.

## III. OPTIMIZATION RESULT ANALYSIS

There are two instability patterns of thin stiffened rectangular plate under normal compression, which are global instability and local instability surrounded by stiffeners. For the rectangular plate with overcrowded stiffeners, because the area surrounded by stiffeners is small, the global instability takes place easily rather than the local instability firstly. For the rectangular plate with sparse stiffeners, due to the bigger area surrounded by stiffeners, the local instability of thin plate happens more easily than the global instability. In this situation, the reinforce effect of stiffeners is not obvious [17]. In this paper, the optimization limits of *h*, $t_1$, $h_1$ were $0.001\text{mm} \le h \le 25\text{mm}$, $0.001\text{mm} \le t_1 \le 60\text{mm}$, $0.001\text{mm} \le h_1 \le 60\text{mm}$, respectively; The initial optimization value of *h*, $t_1$, $h_1$ was 10 mm, 5 mm, 20 mm, respectively.

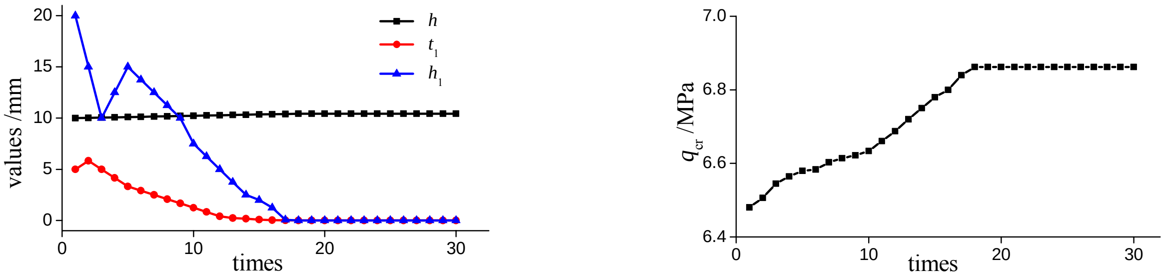
### A. 3 Longitudinal Stiffeners, 1 Transverse Stiffeners

The 3 longitudinal stiffeners and 1 transverse stiffeners were arranged on the back of rectangular plate. The geometrical properties before and after optimization were shown in Table I and the variation processes of variables and objective were shown in Fig.2. Based on the optimization results analysis, the buckling capacity of stiffened rectangular plate under uniform normal compression was improved rarely. The variables $t_1$, $h_1$ converged to the lower limits of the dimensional constraints, while the variable *h* increased correspondingly. The local instability took place in the thin plate firstly due to the too sparse stiffeners. Furthermore, the result was to make the structure as a no-stiffener plate.

TABLE I.
COMPARISON OF MECHANICS PROPERTY OF RECTANGULAR PLATE WITH 3 LONGITUDINAL STIFFENERS AND 1 TRANSVERSE STIFFENERS BEFORE AND AFTER OPTIMIZATION WITH NO-STIFFENER PLATE

| Items | Thickness of plate $h$/ mm | Width of stiffener $t_1$/ mm | Height of stiffener $h_1$/ mm | Buckling load $q_{cr}$/ MPa |
|---|---|---|---|---|
| Before optimization | 10.000 | 5.000 | 20.000 | 6.482 |
| After optimization | 10.400 | 0.001 | 0.001 | 6.862 |
| Theoretical value* | 10.410 | - | - | 6.892 |

*The theoretical value of no-stiffener plate in the same volume which can be looked up in the tables of the Reference 2. The same as in the following tables.



a) Variation processes of variables b) Variation processes of objective

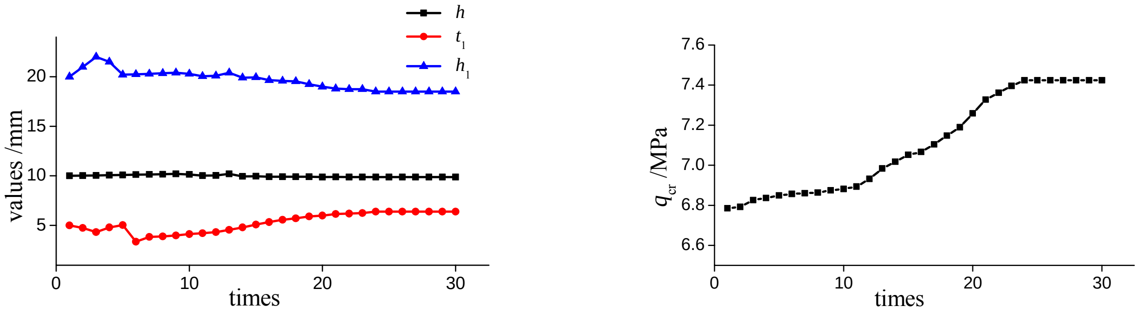
Fig.2 Variation process of variables and objective of rectangular plate with 3 longitudinal stiffeners and 1 transverse stiffeners

### *B. 5 Longitudinal Stiffeners, 2 Transverse Stiffeners*

The 5 longitudinal stiffeners and 2 transverse stiffeners were arranged uniformly on the back of rectangular plate, and the variation processes of variables and objective were presented in Fig.3, and the geometrical properties before and after optimization were shown in Table II. Through analysis of the optimization results, the buckling capacity of stiffened rectangular plate under uniform normal compression was improved evidently. The variables $h$, $t_1$, $h_1$ converged to certain value within the dimensional constraints, and the density of longitudinal stiffeners was bigger than that of transverse stiffeners. And the ratio of the height to the width of the stiffener was about 2.5. Meanwhile, the global instability took place under the normal compression and the reinforce effect of stiffeners was obvious.

TABLE II.
COMPARISON OF MECHANICS PROPERTY OF RECTANGULAR PLATE OF 5 LONGITUDINAL STIFFENERS AND 2 TRANSVERSE STIFFENERS BEFORE AND AFTER OPTIMIZATION WITH NO-STIFFENER PLATE

| Items | Thickness of plate $h$/ mm | Width of stiffener $t_1$/ mm | Height of stiffener $h_1$/ mm | Buckling load $q_{cr}$/ MPa |
|---|---|---|---|---|
| Before optimization | 10.000 | 5.000 | 20.000 | 6.785 |
| After optimization | 9.880 | 6.400 | 17.500 | 7.433 |
| Theoretical value | 10.750 | - | - | 7.076 |



a) Variation processes of variables b) Variation processes of objective

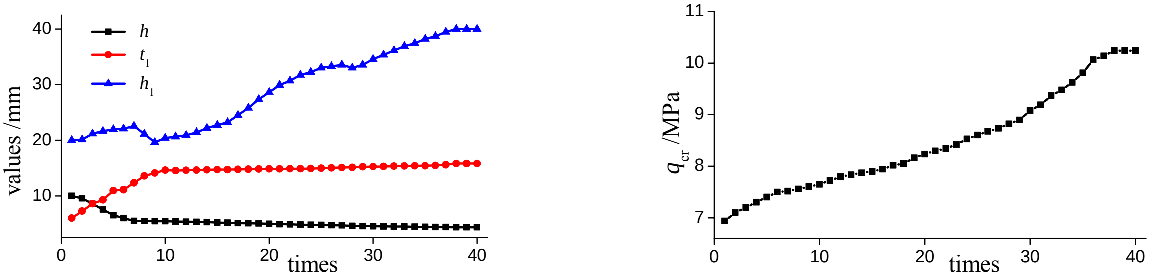
Fig.3 Variation process of variables and objective of rectangular plate with 5 longitudinal stiffeners and 2 transverse stiffeners

### *C. 7 Longitudinal Stiffeners, 3 Transverse Stiffeners*

The 7 longitudinal stiffeners and 3 transverse stiffeners were arranged uniformly on the back of rectangular plate, and the variation processes of variables and objective were presented in Fig.4. Through analysis of the optimization results shown in Table III, the buckling capacity of stiffened rectangular plate under uniform normal compression was improved greatly. The variables $h$, $t_1$, $h_1$ converged to certain value within the dimensional constraints, while the variable $h$ got much smaller. And the densities of longitudinal stiffeners and transverse stiffeners were similar. In the meantime, the stiffener frame was nearly square, which made the vertical and horizontal non-deformability corresponding and was supportive to improve buckling capacity under normal compression. And the ratio of the height to the width of the stiffener was about 2.5.Therefore, the reinforce effect was remarkable.

TABLE III.
COMPARISON OF MECHANICS PROPERTY OF RECTANGULAR PLATE OF 7 LONGITUDINAL STIFFENERS AND 3 TRANSVERSE STIFFENERS BEFORE AND AFTER OPTIMIZATION WITH NO-STIFFENER PLATE

| Items | Thickness of plate $h$/ mm | Width of stiffener $t_1$/ mm | Height of stiffener $h_1$/ mm | Buckling load $q_{cr}$/ MPa |
|---|---|---|---|---|
| Before optimization | 10.000 | 5.000 | 20.000 | 6.936 |
| After optimization | 4.367 | 15.500 | 40.000 | 10.253 |
| Theoretical value | 11.080 | - | - | 7.159 |



a) Variation processes of variables b) Variation processes of objective

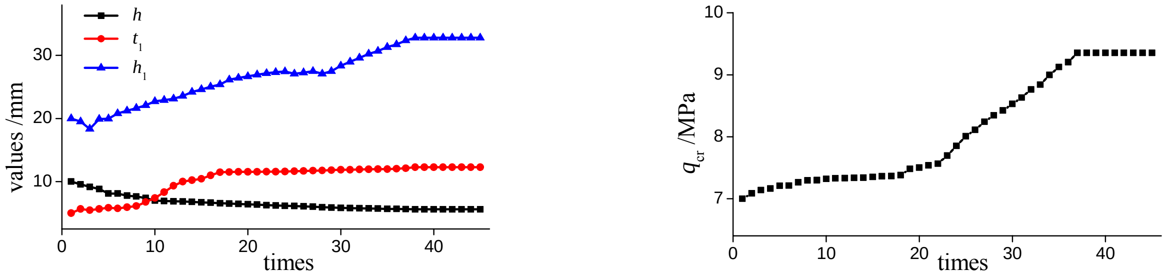
Fig.4 Variation process of variables and objective of rectangular plate with 7 longitudinal stiffeners and 3 transverse stiffeners

### *D. 7 Longitudinal Stiffeners, 5 Transverse Stiffeners*

The 7 longitudinal stiffeners and 5 transverse stiffeners were arranged uniformly on the back of rectangular plate, and the variation processes of variables and objective were presented in Fig.5. Through analysis of the optimization results shown in Table Ⅳ, the buckling capacity of stiffened rectangular plate under uniform normal compression was also improved obviously. The variables $h$, $t_1$, $h_1$ converged to certain value within the dimensional constraints, while $h$ also got much smaller. And the densities of longitudinal stiffeners and transverse stiffeners were similar and suitable. Hence, the reinforce effect of stiffeners was remarkable.

TABLE IV.
COMPARISON OF MECHANICS PROPERTY OF RECTANGULAR PLATE OF 7 LONGITUDINAL STIFFENERS AND 5 TRANSVERSE STIFFENERS BEFORE AND AFTER OPTIMIZATION WITH NO-STIFFENER PLATE

| Items | Thickness of plate $h$/ mm | Width of stiffener $t_1$/ mm | Height of stiffener $h_1$/ mm | Buckling load $q_{cr}$/ MPa |
|---|---|---|---|---|
| Before optimization | 10.000 | 5.000 | 20.000 | 7.003 |
| After optimization | 5.700 | 12.300 | 32.800 | 9.367 |
| Theoretical value | 11.420 | - | - | 7.236 |



a) Variation processes of variables b) Variation processes of objective

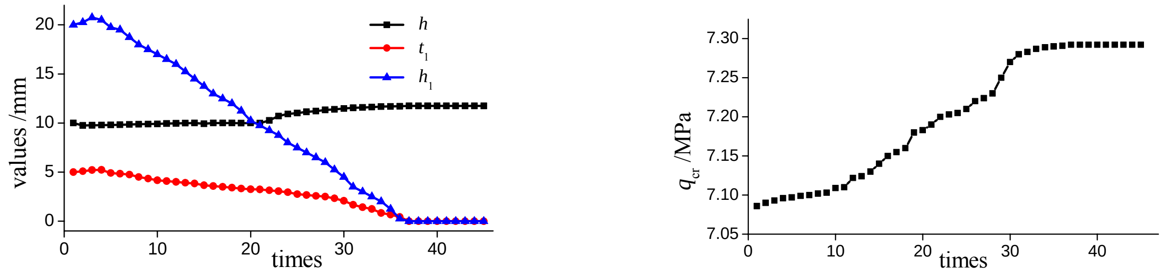
Fig.5 Variation process of variables and objective of rectangular plate with 7 longitudinal stiffeners and 5 transverse stiffeners

### *E. 11 Longitudinal Stiffeners, 5 Transverse Stiffeners*

The 11 longitudinal stiffeners and 5 transverse stiffeners were arranged uniformly on the back of rectangular plate, and the variation processes of variables and objective were presented in Fig.6. Through analysis of the optimization results shown in Table Ⅴ, the buckling capacity was improved a little. The variables $t_1$, $h_1$ converged to the lower limits of the dimensional constraints, while $h$ increased correspondingly. And the result was to make the volume of stiffeners decrease but the thickness of the plate increase, leading the structure to be a rectangular plate without stiffeners. Consequently, for the too overcrowded stiffeners, the reinforce effect of stiffeners was not remarkable.

TABLE V.
COMPARISON OF MECHANICS PROPERTY OF RECTANGULAR PLATE OF 11 LONGITUDINAL STIFFENERS AND 5 TRANSVERSE STIFFENERS BEFORE AND AFTER OPTIMIZATION WITH NO-STIFFENER PLATE

| Items | Thickness of plate $h$/ mm | Width of stiffener $t_1$/ mm | Height of stiffener $h_1$/ mm | Buckling load $q_{cr}$/ MPa |
|---|---|---|---|---|
| Before optimization | 10.000 | 5.000 | 20.000 | 7.086 |
| After optimization | 11.740 | 0.001 | 0.001 | 7.293 |
| Theoretical value | 11.75 | - | - | 7.302 |



a) Variation processes of variables b) Variation processes of objective

Fig.6 Variation process of variables and objective of rectangular plate with 11 longitudinal stiffeners and 5 transverse stiffeners

## IV. CONCLUSION

Based on APDL and ANSYS commands, we set up the optimization model of stiffened rectangular plate, and executed the serial linear programming optimization procedure, obtained the following conclusions.

Whether the stiffeners are too sparse or too overcrowded, the width and the height of stiffeners converge to the lower limits of the dimensional constraints, leading the structure to be a rectangular no-stiffener plate. Only when the plate has an appropriate density, the normal buckling capacity of stiffened rectangular plate will be significantly improved after optimization. While the stiffener frame is nearly square, it will make the vertical and horizontal non-deformability corresponding, which will be supportive to improve buckling capacity under normal compression. As a result, for the rectangular plate with determined structure sizes, the density of stiffeners should be regulated in a reasonable scope.

Moreover, changing the dimensional constraints of stiffeners has no influence on optimization results. And the ratio of the height to the width of the stiffener should be about 2.5, which could improve the normal buckling capacity of stiffened rectangular plate.

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