

Modeling and Analysis for CPS Physical Entities Based on Spatio-Temporal Petri Net

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Abstract—CPS is a kind of multidimensional complex system fused with computing system, physical environment and network environment. CPS achieves the target of real-time perception and dynamic control of physical environment by the organic integration of 3C operations and depth of their cooperation. However, CPS also faces enormous challenges. Since CPS is a kind of large, heterogeneous, distributed and real-time feedback system which is integrated with multiple heterogeneous subsystem via the network, its system complexity is above the general information system. And the introduction of physical entities control system brings tremendous difficulties to development as well as its system complexity. This paper conducts a detailed analysis of the characteristics of the physical aspects of CPS and studies CPS's physical entities of the properties and their location changes process in-depth. Then we propose a formal modeling method of physical entities in CPS by constructing a Spatial-Temporal Petri net model introducing space factor into the traditional Timed Petri net, which not only able to describe logical and time-level the behavior of physical entities, but also the changes of state caused by the position change of the physical entity. At last an example of robot control system proves the effectiveness of spatial and temporal Petri net models.

Index Terms—CPS, Time Petri net, physical entities, spatial factor, Spatial-Temporal Petri net

I. INTRODUCTION

CPS is a new generation of distributed real time feedback system. It integrates computation, communication and control, is capable of sensing and actuating the physical world, bridges the physical world and cyber world^[1]. The instinctive difference of CPS and other normal information systems is that CPS includes the concept of control system^[2], which makes it possible to change the physical environment by the help of physical entities. As B. Borzoo proposed, CPS is an embedded system, which includes massive sensors and actuators deployed in the physical world^[3]; The computation process cooperates with physical process closely.

Although CPS is very powerful, it also faces great difficulties. CPS integrated with wireless sensor system, next-generation network^[4] and network control system. Unprecedented complex system structure and the

introduction of physical entities control system^[2] are the key problems to model CPS. As a integration of calculation process and physical process^[5-7], CPS contains three basic operations: computing, communication and control, which makes CPS become a complex system with powerful functions. CPS also has a high real-time property and a feature of time, space combination. The above mentioned contents prove CPS has a great challenge in system modeling and verification. Error modeling in system will directly affect the reliability of CPS, and even endanger the safety of life and property. Consequently, a more comprehensive modeling approach for modeling and analysis CPS's characteristics is needed. With the gradual deepening research of CPS, there is an urgent need for a rigorous and reliable modeling approach. So an attempt research for CPS modeling approach is necessary.

Petri net^[8] is a mathematical model for researching information systems and their relationship. It was proposed by C.A Petri in his doctoral dissertation "Kommunikation mit Automaten" in 1962. Petri net describe the dynamics of systems by using token flow, which is an integrated graphical and mathematical modeling tool, not only characterize structures of systems with intuitive graphs, but also analyze properties of structures of systems by introduction of mathematical methods, which provides an effective means for charactering and researching information systems with distributed, concurrent, and asynchronous properties.

The application of classical Petri net is limited because it can't describe spatial aspect of systems. To resolve this problem, P.Merlin proposed the concept of Timed Petri net^[10]. Based on classical Petri net, Timed Petri net associated every transition with time span to represent the span of the transition. The introduction of time factor makes Petri net capable to analysis the time aspect performance of systems and expands its application. Timed Petri net has been widely used to analyze and verify real time systems.

Also, Timed Petri net can be also used to describe the time aspect performance of systems by the introduction of time factor. But it can't be used to model CPS, because that Timed Petri net lacks the ability on spatial aspect, it can't be used to describe the positional transition of

physical entities^[11]. This paper studies the fundamentals of spatial property and time property of CPS, by introduction of spatial property, proposed Spatio-Temporal Petri net, which can be used to analyze the consistency of time and space of CPS, and apply to the modeling of CPS physical entities space.

II. RELATED WORKS

CPS connected the physical entities together through the channels of the network to form into a network system using physical entities as nodes. Physical entities in the system can not only autonomously sense the information of environment which transmitted to process unit at any time and space, but also receive the real-time feedbacks from process unit and perform the corresponding operations to achieve the goal of adjust the physical environment.

In order to achieve the goal of real-time feedback, the physical entities within the CPS must ensure the consistency of task execution time and space in their environment. It means system must have the ability to assign the correct physical entities to move to the specified location within the specified time, and complete tasks in a timely manner.

The major work of this paper is to expand the basic research for the CPS in the temporal and spatial consistency, and set up formal model to reflect the temporal and spatial characteristics of CPS.

Due to CPS connected computing systems with physical world, CPS has a combination of characteristics of distributed and embedded systems, which not only give CPS powerful functionality, but also bring enormous difficulties to model CPS. Nowadays there is no suitable formal methods can characterize the composition and running of the CPS.

Existing formal methods cannot meet the modeling requirements of CPS, but previous studies set the foundation for setting up method of modeling CPS.

In 1991, Wang Yi added a delay operator to Calculus of Communication System (CSS), expanded CCS into Timed Calculus of Communication System (TCCS)^[12]. In 1994, He Ji-feng introduce time factor into Communicating Sequential Processes (CSP), proposed Hybrid Communicating Sequential Processes (HCSP)^[13]. In 2003, Round et al introduce time factor into Pi-Calculus, proposed Phi-Calculus system^[14].

In 1994, Alur et al^[15] proposed the concept of timed automata (TA), introduced a number of real-valued clock into model to describe state transitions containing the time constraints, which achieved the behavior formal model of systems on time property.

The above-mentioned formal methods are mainly concentrated in the level of system performance on time property, which provide a good reference for modeling CPS on time property. However, due to the lack of ability of describing systems on spatial property, they are unable to meet the modeling requirements of the CPS. In response to the issue, this paper expands the time Petri nets by introducing spatial factor, and establishes Spatio-

Temporal Petri net model to describe the spatial and temporal consistency of CPS.

III. MODELING ANALYSIS FOR CPS PHYSICAL ENTITIES

A. Classification of Physical Entities

CPS retrieves information of the physical environment by sensors and changes the physical environment by actuators executing instructions. So CPS physical entities include 2 categories^[14]:

- Sensing devices. In order to retrieve the information of the physical environment, CPS need massive sensors deployed in the physical world to selectively collect the required information of the physical world, e.g. temperatures, moisture etc. The information is processed and transferred to information processing unit via networks. And sensors refresh information periodically to make the system access to the latest physical environmental changes.
- Actuating devices. CPS information processing unit analyzes the information transferred from the sensors and sends specific control instructions to the actuators via networks. The actuators perform operations according to the instruction to change the physical environment, e.g. rising the temperature, reducing the moisture etc.

Figure 1 is the graph of CPS physical structure. Among it, the sensors are normally deployed statically in the physical world to sense the information of their nearby environment. And the actuators are dynamical in their position. They need to move to the destination along a specific path to execute operation. The main reason to construct the Spatio-Temporal Petri net is describing the movement process of actuators.

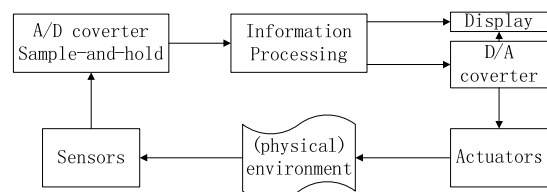


Figure 1. Physical structure of CPS

B. Modeling of Physical Entities

Each physical entity must contain the following basic properties:

- A unique identification to distinguish with other physical entities.
- Specific spatial and non-spatial property. Spatial property means the coordinate, which is in 2 dimensions form (x, y) , of the physical entity under system's coordinate

system. Non-spatial property means the state of the entity, e.g. stop, moving, accelerating, and decelerating.

- Sensors are capable of sensing the information of the environment, and transfer the information to the processing unit by networks.
- Actuators are capable of receiving the control information from the processing unit by networks and move to destination to change the environment correctly according to the instruction.

Definition 1 A physical entity class is represented as a quaternion

$$EC = (ECid, Prop, Op, Map)$$

ECid represents the ID of physical entity class; *Prop* represents collection of the physical entity class properties; *Op* represents collection of the physical entity class behavior; *Map* represent the map from the set of attributes to data types, that is $Prop \rightarrow DataTypes$, in addition to containing a variety of basic data types such as integer, Boolean, float, *DataTypes* also includes the data type of coordinates like (x, y) .

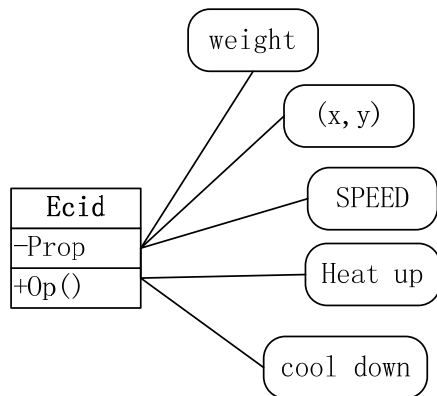


Figure 2. Physical entity class

Every physical entity belongs to a physical entity class; every physical entity corresponds to an instance of the physical entity class.

Definition 2 A physical entity is represented as a triple $E = (Eid, EC, ST)$

Eid represents ID of physical entity; *EC* represents entity classes corresponding to each entity; *ST* represents Spatio-Temporal Petri net model of physical entities, which is used to describe the behavior of physical entities as well as the position transition process. Spatio-Temporal Petri net definition will be explained in detail in the next section.

Eid
EC
ST

Figure 3. Physical entity

Every physical entity in this paper belongs to a same Spatio-Temporal Petri model and is a resource of it in form of token.

IV. SPATIO-TEMPORAL PETRI NET MODEL

A. Timed Petri Net

In this paper, timed Petri net is the basis of setting up Spatio-Temporal Petri net, so we first give the definition of time Petri Net:

Definition 3 Timed Petri Net is a quintuple

$$TPN = (S, T; F, M, I)$$

S is poor and non-empty set of places, which can be represented as circular nodes. *T* is the poor and non-empty set of transitions, expressed as square nodes. *F* is the set of directed arcs between places and transitions,

$$F \subseteq (S \times T) \cup (T \times S), \quad dom(F) \cup cod(F) = S \cup T,$$

$$dom(F) = \{x \in S \cup T \mid \exists y \in S \cup T : (x, y) \in F\}$$

$$cod(F) = \{x \in S \cup T \mid \exists y \in S \cup T : (y, x) \in F\}$$

dom(F) represents the collection of directed arcs pointed to by the places. *cod(F)* represent the collection of directed arcs from transitions to places.

M is used to identify the distribution of the token in each place during transmission process; $(S, T; F, M)$ is an original Petri net, $S \cup T \neq \emptyset$, $S \cap T = \emptyset$, for any transition $t \in T$, if $\forall s \in S : s \in \bullet t \rightarrow M(s) \geq 1$, it identifies that transmission *t* can occurred at *M*, wrote as $M[t >$; if $M[t >$, then *M'* is the new identification after *t* occurred, for $\forall s \in S$:

$$M'(s) = \begin{cases} M(s) - 1, & \text{if } s \in \bullet t - t \bullet \\ M(s) + 1, & \text{if } s \in t \bullet - \bullet t \\ M(s), & \text{others} \end{cases}$$

$\bullet t$ and $t \bullet$ represent the set of input places and output places respectively.

I is a function of Time interval over the collection of transmission, $I : T \rightarrow R \times (R \cup \{\infty\})$, *R* represents the set of real numbers.

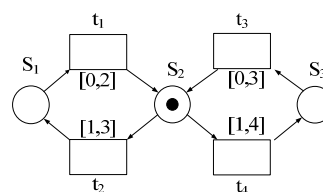


Figure 4. A time Petri net Σ

Figure 4 shows a simple time Petri nets Σ . only place S1 has one token in the initial stage in Σ .

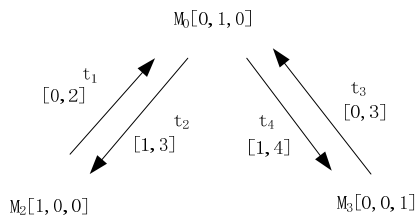


Figure 5. Σ 's reachable marking graph

Figure 5 shows the order which the various transmissions occur in Σ , and marked the transfer of the token during the transition process.

B. Construct the Spatio-Temporal Petri Net

Constructing the spatial-temporal Petri nets model for CPS physical entity must address the following issues:

- It is able to represent the physical entities in which the coordinate range, which is the most basic signs of the Spatial-Temporal Petri net introduced spatial factors. Physical entity is different from the calculation unit, its position is not fixed over time, and actuators must follow the instructions to reach the destination to perform the operation. Spatial-Temporal Petri net must find a way to describe the coordinates of the location of physical entities.
- It is able to indicate the existence of physical entities, which form of abstraction for system resources in Spatial-Temporal Petri net. Therefore, physical entity can be represented by token. In order to distinguish between physical entities and other resources such as information resources, Spatial-Temporal Petri net can use different types of tokens. Different types of physical entities can also be distinguished by different types of tokens.
- It is able to describe the moving process of the physical entity correctly. A physical entity is in the form of token. The moving process of the physical entity can be described by the transfer of the token. Spatial-Temporal Petri net must have an ability to describe the process of the concurrent movement of multiple physical entities.

Analyzed the above three issues, the Spatio-Temporal Petri net model can be retained by expanding the Timed Petri net.

Definition 4 A Spatio-Temporal Petri net is a seven tuple $STPN = (S, TU; FU, MU, I, W, A)$

S and I are the same as Timed Petri Net; $TU = \bigcup_{i=1}^n Ti$, Ti is the collection of transition corresponding to a type of token^[17,18]. For $\forall t \in Ti$, transition t only fires the i -th type of token.

$FU = \bigcup_{i=1}^n Fi$, Fi represent the corresponding arc collection

to Ti ; $MU = \bigcup_{i=0}^n Mi$, Mi corresponds to Ti , it represent

the distribution of transition of the i -th class token in each place^[18]; n is the number of token types; W is a map function defined over the running track on the set of transitions, mainly for the physical entity, because the physical entities move are not absolute straight; A is a map function defined over the spatial location, described by two-dimensional coordinate, where the space position is divided into points, lines, surfaces.

Point entities	Line entities	Plane entities
$[x, y]$	$[(x_1, y_1), (x_2, y_2)]$	$[(x_1, y_1), (x_2, y_2)]$

Figure 6. Physical entities classification

Spatio-Temporal Petri net model has the following characteristics:

- The physical environment and the physical entity are abstracted into formal models, which can correctly describe the moving process of physical entities in their environment.
- It has a variety of different transition modes, and respectively represents the transition process of a variety of resources. The resources can be information resources, and may be physical entity resources.
- Physical entity is treated as resources, described as tokens, which are classified by distinguishing between the physical entity and other resources, each token representing different resource types.

The next section, a case is given to demonstrate how to use the spatial-temporal Petri net to model for CPS physical entity.

V. CASE STUDY

A. Application Scenarios

This section takes a robot control system for example, the, and uses Spatio-Temporal Petri net to model the physical entities of the robot control system.

The robot control system scenario shown in Figure 5:

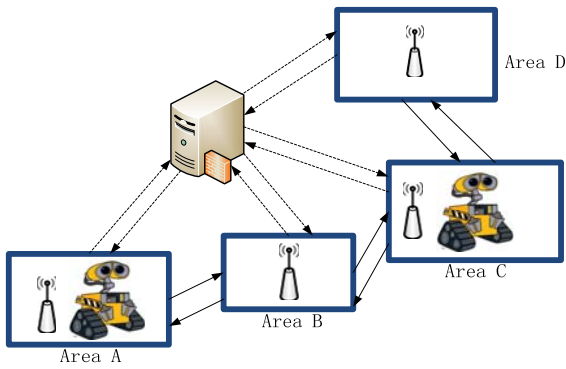


Figure 7. Robot control system

Figure 7 describes a robot control system includes five components: robots, sensors, information processing device, information transmission lines and robot movement route. Workflow of the robot system is divided into the following sections:

- (1) Sensors sense the physical environment and then transmit information to the information processing device through the information transmission line.
- (2) The information processing device sent the corresponding instructions to each robot of each area through the information transmission line after analyzing the information sent by sensors.
- (3) If the target area exists a robot, robots in other areas needn't move to the area, otherwise selecting a robot in other areas to move to the target area according to the situation.

B. Instance Modeling

Sensors and robots in the robot control system belonging sensor entity class and robot entity class as physical entities. Using the formal modeling definition of physical entities in 3.2 section, sensors and robots can be defined as follows:

$$\begin{aligned} & (Sensor, \{Area\}, Op, \{Coordinate\}) \\ & (Robot, \{Area, Move, Stop, Speed, Direction\}, \\ & Op, \{Coordinate, Boolean, Boolean, Integer, Integer\}). \end{aligned}$$

Area represents the geographical location of physical entities; Coordinate represents the data type; Area A ~ Area D represent Coordinate position. Robot class and Sensor class can be defined as follows:

$$\begin{aligned} & (Robot1, Robot, ST) \\ & (Sensor1, Sensor, ST) \end{aligned}$$

ST represents the Spatio-Temporal Petri net model of the robot control system. Two different tokens are used in the model to distinguish between sensors and robot. Rob and Sen respectively donate the robots and sensors' distribution of each position. Sensors' positions are fixed, so there are no movements of sensors in the model. In order to describe the transmission process of the environment information and control instructions, defining two new token, respectively denoting the environment information and control instructions.

Inf and Ins in the model respectively represent the distribution of the changing process information and control instructions at each location. According to the above, Spatio-Temporal Petri net model of physical entities in this case consists of three kinds of transition type. This example focuses on recounting the moving process of physical entities, so the specific behavior of the physical entities does not discussed in this case.

The robot control system contains five places, respectively donating an information processing device and four geographical locations, S_1 represent information processing device, $S_2 \sim S_5$ respectively represent Area A ~ Area D ; The initial distribution of robot tokens is $Rob(0,1,0,1,0)$; The initial distribution of sensors tokens is $Sen(0,1,1,1,1)$; The initial distribution of information tokens is $Inf(0, \infty, \infty, \infty, \infty)$; The initial distribution of control instructions is $Ins(\infty, 0, 0, 0, 0)$.

According to the above, Spatio-Temporal Petri net model of physical entity in this case is as follows:

In Figure 8, \blacktriangle , \blacksquare , \blacklozenge , \bullet respectively represent robots, sensors, control instruction, information; InfoProcess represent the position of information processing apparatus. MS_i represent the command of moving to area S_i . $\neg Rob(S_i)$ is used to judge that whether the number of robots token is zero at area S_i . $Send(S_i)$ represent the environment information which sensor send to the information processing device; $Send(Cod_i)$ represent the information processing device transfers the control commands to each area , i represent the number of control instruction sequence; $Ins(S_i)-1$ represent that the number of control the instruction tokens in S_i is decremented by 1. In Figure 6 $t_1 \sim t_4$ belong to the information transitions, t_5 belongs to the control instruction transition. $t_6 \sim t_{15}$ belong to robot transition. The each kind of transition connects the places with corresponding arcs.

TABLE I.
TRANSITION ROUTES

transition	router	domain
t_6	$y = x^2$	$x \in [0, 100]$
t_7	$y = x^2$	$x \in [100, 0]$
t_8	$y = x$	$x \in [0, 50]$
t_9	$y = x$	$x \in [50, 0]$
t_{10}	$y = 1/2x$	$x \in [0, 80]$
t_{11}	$y = 1/2x$	$x \in [80, 0]$

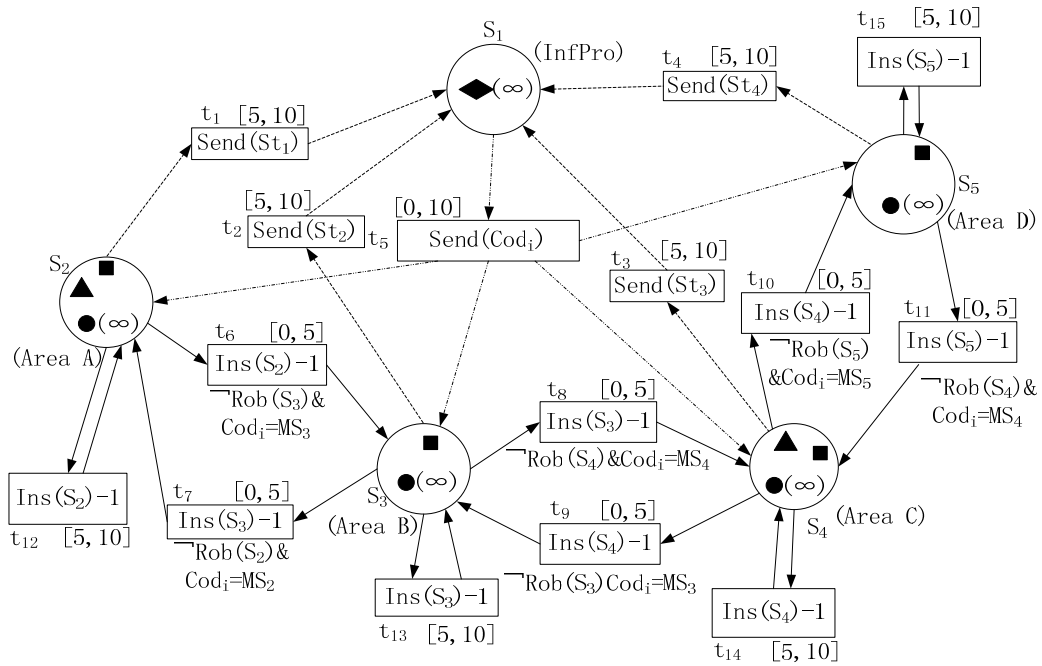


Figure 8. Spatio-Temporal Petri net model of the physical entities

In order to ensure the correctness of the control command, $S_2 \sim S_5$ all have an empty position transition. That is to say the number of robot tokens in S_i is not changed, but the number of control command token is decremented by 1.

Transition $t_6 \sim t_{11}$ belong to robot position changes. Assume that the robot moves as Table I.

t_6, t_9, t_{10} respectively have the same transition route with t_7, t_8, t_{11} , so each couple has the same router function. But two elements of each couple have the opposite direction, so the domain of their function is opposite. Both ends of the domain represent the space coordinates of start and end points.

VI. CONCLUSION

CPS is emerging distributed real-time feedback system integrating a variety of physical entities with physical environment closely, sensing the general information of physical environment, which brought great challenge for modeling and verification of CPS.

In this paper, a Spatio-Temporal Petri net model is proposed for the space-time consistency problem during the process of position transition of physical entities. The CPS physical entities are modeled by Spatio-Temporal Petri net and their moving processes are described in the section of case study, which prove the effectiveness of Spatio-Temporal Petri net model. This approach has some place to be further optimized and perfected. The future work is studying the wrong mobile and specific operations, improving Spatio-Temporal Petri net model and design the method of model verification.

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