Multicriteria Decision-making Method using Cosine Similarity Measures for Reduct Fuzzy Sets of Interval-valued Fuzzy Sets

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***Abstract*—This paper introduces optimistic, neutral and pessimistic reduct fuzzy sets of an interval-valued fuzzy set, optimistic, neutral and pessimistic cosine similarity measures for the reduct fuzzy sets. A new decision-making method is proposed by means of three weighted cosine similarity measures depending on optimistic, neutral, and pessimistic points to reduce cognitive dissonance in multiple criteria decision analysis. We give the measures of optimism, neutralism, and pessimism to further determine suitability for alternative rankings through choosing optimistic, neutral, and pessimistic weighted cosine similarity measures. Finally, an illustrative example is conducted to validate the feasibility and applicability of the proposed method.**

***Index Terms*—cosine similarity measure, interval-valued fuzzy set, reduct fuzzy set, multicriteria decision making**

I. INTRODUCTION

In many real-world situations, the decision maker cannot provide deterministic alternative values because the decision information given by decision makers is often imprecise or uncertain due to a lack of data, time pressure, or the decision makers' limited attention and information processing capabilities. This kind of uncertainty in multicriteria decision making can be modeled using fuzzy set theory and is ideally suited for solving these problems. Bellman and Zadeh [1] first proposed the fuzzy decision-making model. Since then, great numbers of studies on fuzzy multicriteria decision problems have most often been performed in a fuzzy environment [2-6]. In addition, because it may be difficult for decision makers to exactly quantify their opinions as a number in the interval [0, 1], it is more suitable to represent this degree of certainty by an interval. Therefore, Zadeh [7] first proposed the concept of an interval-valued fuzzy set (IVFS). IVFSs are suitable for capturing imprecise or uncertain decision information. After that, IVFSs have been applied to multicriteria decision-making problems [8, 9]. On the other hand, optimism and pessimism, concepts developed by Scheier and Carver [10], are fundamental constructs that reflect how people respond to their perceived environment and how they form subjective judgments. Although theories differ in their specifics, a common idea is that optimists and pessimists diverge in their explanations and predictions of future events. Recently, Chen [11] presented a new method to reduce cognitive dissonance and to relate optimism and pessimism in multicriteria decision analysis in an interval-valued fuzzy decision environment. However, the similarity measures depending on optimism, neutralism, and pessimism for subjective judgments that accompanies the decision making process have not been studied in an interval-valued fuzzy decision environment.

The similarity measure is one of important tools for the degree of similarity between objects. Functions expressing the degree of similarity of items or sets are used in physical anthropology, numerical taxonomy, ecology, information retrieval, psychology, citation analysis, and automatic classification. In fact, the degree of similarity or dissimilarity between the objects under study plays a role. In the query expansion, various term-term similarity measures based on the collocation have been suggested to select the additional search terms. The cosine similarity measure [12] is often used for this purpose. Recently, Ye [13] proposed cosine similarity measures and their applications of pattern recognitions in intuitionistic fuzzy environment. Then Ye [14] proposed the Dice similarity measure based on the reduct intuitionistic fuzzy sets of interval-valued intuitionistic fuzzy sets and its multicriteria decision-making method with the dispositional optimism, neutralism, and pessimism desired by the decision maker. However, the attention has not been given to the cosine similarity measures on the dispositional optimism, neutralism, and pessimism the decision maker desire on multicriteria decision-making problems in an interval-valued fuzzy decision environment.

In this paper we propose optimistic, neutral, and pessimistic reduct fuzzy sets of an interval-valued fuzzy set, optimistic, neutral and pessimistic cosine similarity measures for the reduct fuzzy sets, and optimistic, neutral, and pessimistic weighted cosine similarity measures for the reduct fuzzy sets to relate the optimism, neutralism, and pessimism the decision maker desire in an interval-valued fuzzy environment. A new decision-making

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method is developed by means of the three weighted cosine similarity measures which utilize optimistic, neutral, and pessimistic points to deal with difficult decision-making problems in some cases of multiple criteria decision analysis. We give the measure of optimism, neutralism, and pessimism to further determine suitability for alternative rankings through optimistic, neutral, and pessimistic weighted cosine similarity measures. The proposed decision-making method can solve multicriteria decision problems in which the performance rating values are expressed in IVFSs. Finally, an illustrative example is conducted to validate the feasibility and applicability of the current method.

II. Preliminaries

In this section, we introduce some basic concepts and definitions related to fuzzy sets, interval-valued fuzzy sets, and a cosine similarity measure for fuzzy sets, which will be needed in the following analysis.

Definition 1. Zadeh [15] defined a fuzzy set A in the universe of discourse X as follows:

$$A=\left\{\left\langle x,\mu_A(x)\right\rangle \mid x\in X\right\} \tag{1}$$

which is characterized by membership function $\mu_A(x)$: $X\rightarrow[0, 1]$, where $\mu_A(x)$ indicates the membership degree of the element x to the set A.

In fuzzy sets theory, it is often difficult for an expert to exactly quantify his or her opinion as a number in interval [0, 1]. Therefore, it is more suitable to represent this degree of certainty by an interval. From such point of view, Zadeh further proposed the concept of an interval-valued fuzzy set (IVFS).

Definition 2. An IVFS A in the universe of discourse X is given by Zadeh [7]:

$$A=\left\{\left\langle x,\left[\mu_A^-(x),\mu_A^+(x)\right]\right\rangle \mid x\in X\right\} \tag{2}$$

where $\mu_A^-(x): X\rightarrow[0,1]$ and $\mu_A^+(x): X\rightarrow[0,1]$ are called a lower limit of membership degree and a upper limit of membership degree of the element x to the set A, respectively, with the condition $0\le\mu_A^-(x)\le\mu_A^+(x)\le1$.

A cosine similarity measure for fuzzy sets [12] is defined as the inner product of two vectors divided by the product of their lengths. This is nothing but the cosine of the angle between the vector representations of the two fuzzy sets.

Assume that $A = (\mu_A(x_1), \mu_A(x_2), \ldots, \mu_A(x_n))$ and $B = (\mu_B(x_1), \mu_B(x_2), \ldots, \mu_B(x_n))$ are two fuzzy sets in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. A cosine similarity measure (angular coefficient) between A and B can be defined as follows [12]:

$$C_F(A,B)=\frac{\sum_{i=1}^{n}\mu_A(x_i)\mu_B(x_i)}{\sqrt{\sum_{i=1}^{n}\mu_A^2(x_i)}\sqrt{\sum_{i=1}^{n}\mu_B^2(x_i)}} \tag{3}$$

where $0\le C_F(A, B)\le1$.

III. Reduct Fuzzy Sets of Interval-valued Fuzzy Sets

Let A be an IVFS in a universe of discourse $X = \{x\}$. The concept of reduct fuzzy set of the interval-valued fuzzy set A is proposed as follows.

Definition 3. Let α, $\beta\in[0, 1]$ and $\alpha+\beta=1$. The vector $W = (\alpha, \beta)$ is called an opinion weighting vector. Then,

$$A_w=\left\{\left\langle x,\alpha\mu_A^-(x)+\beta\mu_A^+(x)\right\rangle \mid x\in X\right\} \tag{4}$$

is called the weighted reduct fuzzy set of the interval-valued fuzzy set A with respect to the opinion weighting vector W.

By adjusting the value of α and β, an interval-valued fuzzy set can be converted into a reduct fuzzy set a decision maker desires. Specifically, let $\alpha=1$ and $\beta=0$, $\alpha=0$ and $\beta=1$, and $\alpha=\beta=0.5$, respectively. We will have three reduct fuzzy sets of A, i.e., pessimistic reduct fuzzy set A_-, optimistic reduct fuzzy set A_+ and neutral reduct fuzzy set A_N. They are defined respectively as follows:

$$A_-=\left\{\left\langle x,\mu_A^-(x)\right\rangle \mid x\in X\right\} \tag{5}$$

$$A_+=\left\{\left\langle x,\mu_A^+(x)\right\rangle \mid x\in X\right\} \tag{6}$$

$$A_N=\left\{\left\langle x,(\mu_A^-(x)+\mu_A^+(x))/2\right\rangle \mid x\in X\right\} \tag{7}$$

An interval-valued fuzzy set is changed to fuzzy sets by computing the reduct fuzzy sets. Therefore, based on the cosine measure of fuzzy sets, we can propose three cosine similarity measures for the reduct fuzzy sets of IVFSs in the next section.

IV. Cosine Similarity Measures for Reduct Fuzzy Sets of IFSs

Assume that there are two IVFSs A and B in $X = \{x_1, x_2,\ldots, x_n\}$. Based on the of the cosine measure for fuzzy sets, three cosine similarity measure between the reduct fuzzy sets of IVFSs A and B are proposed respectively as follows:

$$C_+(A,B)=\frac{\sum_{i=1}^{n}\mu_A^+(x_i)\mu_B^+(x_i)}{\sqrt{\sum_{i=1}^{n}(\mu_A^+(x_i))^2}\sqrt{\sum_{i=1}^{n}(\mu_B^+(x_i))^2}} \tag{8}$$

$$C_N(A,B)=\frac{\sum_{i=1}^{n}\left\{[\mu_A^-(x_i)+\mu_A^+(x_i)][\mu_B^-(x_i)+\mu_B^+(x_i)]\right\}}{\sqrt{\sum_{i=1}^{n}[\mu_A^-(x_i)+\mu_A^+(x_i)]^2}\sqrt{\sum_{i=1}^{n}[\mu_B^-(x_i)+\mu_B^+(x_i)]^2}} \tag{9}$$

$$C\_(A,B)=\frac{\sum_{i=1}^{n}\mu_A^-(x_i)\mu_B^-(x_i)}{\sqrt{\sum_{i=1}^{n}(\mu_A^-(x_i))^2}\sqrt{\sum_{i=1}^{n}(\mu_B^-(x_i))^2}} \tag{10}$$

where $C_+(A, B)$, $C_N(A, B)$, and $C\_(A, B)$ are the optimistic cosine similarity measure, neutral cosine similarity measure, and pessimistic cosine similarity measure for the reduct fuzzy sets of IVFSs A and B. These cosine similarity measures are within the values between 0 and 1.

If we consider the weights of x_i, three weighted cosine similarity measure between the reduct fuzzy sets of IFSs A and B are proposed as follows:

$$W_+(A,B)=\frac{\sum_{i=1}^{n}\omega_i^2\mu_A^+(x_i)\mu_B^+(x_i)}{\sqrt{\sum_{i=1}^{n}(\omega_i\,\mu_A^+(x_i))^2}\sqrt{\sum_{i=1}^{n}(\omega_i\mu_B^+(x_i))^2}} \tag{11}$$

$$W_N(A,B)=\frac{\sum_{i=1}^{n}\left\{\omega_i^2[\mu_A^-(x_i)+\mu_A^+(x_i)][\mu_B^-(x_i)+\mu_B^+(x_i)]\right\}}{\sqrt{\sum_{i=1}^{n}\left\{\omega_i\,[\mu_A^-(x_i)+\mu_A^+(x_i)]\right\}^2}\sqrt{\sum_{i=1}^{n}\left\{\omega_i\,[\mu_B^-(x_i)+\mu_B^+(x_i)]\right\}^2}} \tag{12}$$

$$W\_(A,B)=\frac{\sum_{i=1}^{n}\omega_i^2\mu_A^-(x_i)\mu_B^-(x_i)}{\sqrt{\sum_{i=1}^{n}(\omega_i\mu_A^-(x_i))^2}\sqrt{\sum_{i=1}^{n}(\omega_i\mu_B^-(x_i))^2}} \tag{13}$$

where the weight of x_i is $\omega_i \in [0, 1]$, i =1, 2, …, n, and $\sum_{i=1}^{n}\omega_i=1$, and $W_+(A, B)$, $W_N(A, B)$, and $W\_(A, B)$ are the optimistic weighted cosine similarity measure, neutral weighted cosine similarity measure, and pessimistic weighted cosine similarity measure, respectively, for the reduct fuzzy sets of IVFSs A and B. These cosine similarity measures are within the values between 0 and 1.

If we take $\omega_i = 1/n$, i =1, 2, …, n, then there are $W_+(A, B) = C_+(A, B)$, $W_N(A, B) = C_N(A, B)$, and $W\_(A, B) = C\_(A, B)$.

V. Decision Making Method Based on the Cosine Similarity Measures

For a multicriteria decision-making problem, the evaluations of each alternative with respect to each criterion for the fuzzy concept ‘‘excellence” are given using IVFSs. Suppose that there exists a set of alternatives $A = \{A_1, A_2, . . ., A_m\}$. Each alternative is assessed on n criteria, which are denoted by $C = \{C_1, C_2, …, C_n\}$. The preference value of a criterion C_j (j = 1, 2, …, n) on an alternative A_i (i = 1, 2, …, m) is an IVFS $d_{ij}=[\mu_{ij}^-(C_j),\mu_{ij}^+(C_j)]$ (i = 1, 2, …, m; j = 1, 2, …, n) given by the decision maker or expert according to some evaluated criteria. Thus we can obtain an interval-valued fuzzy decision matrix $D = (d_{ij})_{m\times n}$, which is defined as the following form:

$$D=\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c}\begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array}\\ \begin{bmatrix} [\mu_{11}^-,\mu_{11}^+] & [\mu_{12}^-,\mu_{12}^+] & \cdots & [\mu_{1n}^-,\mu_{1n}^+] \\ [\mu_{21}^-,\mu_{21}^+] & [\mu_{22}^-,\mu_{22}^+] & \cdots & [\mu_{2n}^-,\mu_{2n}^+] \\ \vdots & \vdots & \vdots & \vdots \\ [\mu_{m1}^-,\mu_{m1}^+] & [\mu_{m2}^-,\mu_{m2}^+] & \cdots & [\mu_{mn}^-,\mu_{mn}^+] \end{bmatrix}\end{array} \tag{14}$$

In multicriteria decision-making environments, the concept of ideal point has been used to help the identification of the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct to evaluate alternatives. Therefore, we define an ideal interval-valued fuzzy set for each criterion in the ideal alternative A^* as $d_j^* = [1, 1]$, j = 1, 2, …, n.

The weight vector of criteria for the different importance of each criterion is given as the fuzzy weight vector $\omega = (\omega_1, \omega_2, …, \omega_n)$, where an weight $\omega_j \geq 0$ and $\sum_{j=1}^{n}\omega_j=1$. Thus three weighted cosine similarity measures between the reduct fuzzy sets of an alternative A_i and the ideal alternative A^* represented by the IVFSs are given as the follows:

$$W_+(A^*,A_i)=\frac{\sum_{j=1}^{n}\omega_j^2\mu_B^+(C_j)}{\sqrt{\sum_{j=1}^{n}\omega_j^2}\sqrt{\sum_{j=1}^{n}(\omega_j\mu_{A_i}^+(C_j))^2}} \tag{15}$$

$$W_N(A^*,A_i)=\frac{\sum_{j=1}^{n}\left\{\omega_j^2\left[\mu_B^-(C_j)+\mu_B^+(C_j)\right]\right\}}{\sqrt{\sum_{j=1}^{n}\omega_j^2}\sqrt{\sum_{j=1}^{n}\left\{\omega_j[\mu_B^-(C_j)+\mu_B^+(C_j)]\right\}^2}} \tag{16}$$

$$W\_(A^*,A_i)=\frac{\sum_{j=1}^{n}\omega_j^2\mu_B^-(C_j)}{\sqrt{\sum_{j}^{n}\omega_j^2}\sqrt{\sum_{j=1}^{n}(\omega_j\mu_{A_i}^-(C_j))^2}} \tag{17}$$

where $W_+(A, B)$, $W_N(A, B)$, and $W\_(A, B)$ are the optimistic weighted cosine similarity measure, and neutral weighted cosine similarity measure, and pessimistic weighted cosine similarity measure. These weighted cosine similarity measures are within the values between 0 and 1. Then, in the decision-making process the choosing measure depends on the optimistic or neutral or pessimistic nature of the decision maker.

These weighted cosine similarity measures provide the global evaluation for each alternative regarding all the criteria. From Eqs. (15)-(17), the larger the value of the weighted cosine similarity measure, the better the alternative. Through choosing one of three weighted cosine similarity measures, the ranking order of all the alternatives can be determined and the best alternative can be easily identified as well.

The advantages of the proposed decision-making method are mainly twofold. Firstly, we need not treat interval-valued fuzzy sets directly in decision making but only deal with the related reduct fuzzy sets after choosing certain weighted cosine similarity measure. This makes our method simpler and easier for application in practical problems. Secondly, there are three kinds of weighted cosine similarity measures that can be used to find the optimal choice, hence the proposed method has great flexibility.

As pointed out in Feng et al. [16], many decision making problems are essentially humanistic and subjective in nature; hence there actually does not exist a unique or uniform criterion for decision making in an imprecise environment. This choosing feature makes the proposed method not only efficient, but more appropriate for many practical applications.

VI. Illustrative Example

The following practical example involves a supplier selection problem in a supply chain discussed in Chen [11]. The authorized decision maker in a small enterprise attempts to reduce the supply chain risk and uncertainty to improve customer service, inventory levels, and cycle times, which results in increased competitiveness and profitability. The decision maker considers various criteria involving (i) C_1: performance (e.g., delivery, quality, and price); (ii) C_2: technology (e.g., manufacturing capability, design capability, and ability to cope with technology changes); and (iii) C_3: organizational culture and strategy (e.g., feeling of trust, internal and external integration of suppliers, compatibility across levels, and functions of the buyer and supplier). Using the supplier rating system, the decision maker evaluates five suppliers, $A = \{A_1, A_2, \ldots, A_5\}$, based on three criteria, $C = \{C_1, C_2, C_3\}$. The decision matrix for the lower extreme $\mu_{ij}^{-}(C_j)$ and upper extreme $\mu_{ij}^{+}(C_j)$ of the membership degrees for the suppliers $A_i \in A$ with respect to the criterion $C_j \in C$ is given below:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} [0.21,0.36] & [0.43,0.60] & [0.51,0.76] \\ [0.03,0.84] & [0.33,0.78] & [0.48,0.71] \\ [0.81,0.92] & [0.01,0.76] & [0.33,0.58] \\ [0.32,0.68] & [0.15,0.34] & [0.36,0.97] \\ [0.59,0.87] & [0.31,0.64] & [0.14,0.63] \end{bmatrix} \end{array}$$

The fuzzy weight vector of the three criteria is given as $\omega = (0.30, 0.23, 0.47)$ in Chen [11]. The decision-making process of this problem depends on the optimistic or neutral or pessimistic nature of the decision maker, which is as the follows.

If we deal with this problem by the decision rule of the optimistic weighted cosine similarity measure, then using Eq. (15), we can obtain the following values of the optimistic weighted cosine similarity measure:

$W_1(A^*, A_1) = 0.9669$, $W_2(A^*, A_2) = 0.9973$, $W_3(A^*, A_3) = 0.9785$, $W_4(A^*, A_4) = 0.9624$, and $W_5(A^*, A_5) = 0.9891$.

From the optimistic point of view, therefore, the alternatives can be ranked as

$$A_2 > A_5 > A_3 > A_1 > A_4$$

which implies that the optimal alternative is A_2.

If we deal with this problem by the decision rule of the neutral weighted cosine similarity measure, using Eq. (16), we can obtain the following values of the neutral weighted cosine similarity measure:

$W_1(A^*, A_1) = 0.9638$, $W_2(A^*, A_2) = 0.9926$, $W_3(A^*, A_3) = 0.9475$, $W_4(A^*, A_4) = 0.9670$, and $W_5(A^*, A_5) = 0.9580$.

Therefore, from the neutral point of view the alternatives can be ranked as

$$A_2 > A_4 > A_1 > A_5 > A_3$$

which implies that the optimal alternative is also A_2.

If we deal with this problem by the decision rule of the pessimistic weighted cosine similarity measure, by using Eq. (17), we can obtain the following values of the pessimistic weighted cosine similarity measure:

$W_1(A^*, A_1) = 0.9587$, $W_2(A^*, A_2) = 0.8783$, $W_3(A^*, A_3) = 0.8415$, $W_4(A^*, A_4) = 0.9756$, and $W_5(A^*, A_5) = 0.8249$.

Therefore, from the pessimistic point of view the alternatives can be ranked as

$$A_4 > A_1 > A_2 > A_3 > A_5$$

which implies that the optimal alternative is A_4.

As a choosing approach, one can use different rules in the above decision making problem and in general the final optimal decision will change accordingly. Through choosing one of three weighted cosine similarity measures, the ranking order of all the alternatives can be determined and the best alternative can be easily identified as well.

Many decision making problems are essentially humanistic and subjective in nature [16]; hence there actually does not exist a unique or uniform criterion for decision making in an imprecise environment. This choosing feature makes the proposed method not only efficient, but more appropriate for many practical applications.

VII. Conclusion

In this study, we have proposed the reduct fuzzy sets of IVFSs and three cosine similarity measures of reduct fuzzy sets, and three weighted cosine similarity measures depending on the optimistic, neutral, and pessimistic natures. Then a multicriteria decision-making method was proposed based on the weighted cosine similarity measures in an interval-valued fuzzy decision environment. The decision-making process depends on the optimistic or neutral or pessimistic nature of the decision maker. Through choosing one of three weighted cosine similarity measures, the ranking order of all the alternatives can be determined and the best alternative can be easily identified as well. The feasibility and effectiveness of the proposed multicriteria decision-making methods that consider optimism, neutralism, and pessimism were illustrated by an illustrative example. We conclude that the proposed method gives desirable alternative ranking results. Furthermore, we demonstrated that this choosing feature makes the proposed method more appropriate for many practical applications of decision making in an imprecise environment.

To extend this work, one can apply the cosine similarity measures of the reduct fuzzy sets of IVFSs to other practical applications, or discuss how to cope with decision making problems based the cosine similarity measures under incomplete information.

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