Compensated-Tracking-Errors-Based Adaptive Fuzzy Controller Design for Uncertain Nonlinear System with Minimal Parameterization

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***Abstract*—This paper addresses an adaptive fuzzy controller design for a class of nonlinear system. The nonlinear system is in the framework of strict-feedback form. The system funtions are unknown, and external disturbances meet the triangular bound assumption. Takagi-Sugeno (T-S) type fuzzy logic systems are used to approximate the uncertain nonlinear functions. The control objective is to steering the system's output to track a given signal. The closed-loop control system is proven to be uniformly ultimately bounded, and the tracking error converge to a neighborhood of zero through choosing appropriate parameters. Compensated tracking errors, not tracking errors, are employed to construct the controller, such that the proposed design avoids the repeated differential of virtual control law completely. Furthermore, the adaptive law is in the sense of minimal parameterization. Namely, the number of adaptive law is equal to the order of the nonlinear system. The simulation results show the effectiveness and usage of the proposed strategy.**

***Index Terms*—adaptive control, backstepping control, fuzzy system, tracking error, uncertain nonlinear system**

I. Introduction

In recent years, adaptive control for uncertain nonlinear systems has received much attention, and many significant developments were achieved [1-2]. As a breakthrough in nonlinear control, adaptive backsteping control approach was introduced to achieve global stability [2]. Backstepping is a powerful tool for the controller design for nonlinear systems in or transformable to the parameter strict-feedback form, where $x \in \Re^n$ is the state, $u \in \Re$ is the control input, and $\theta \in \Re^p$ is an unknown constant vector. The adaptive backstepping approach utilizes stabilizing functions $\bar{\alpha}_i$ and tuning functions τ_i for $i = 1, \cdots, n$. Calculation of these quantities require the partial derivatives $\partial\bar{\alpha}_{i-1}/\partial x_j$ and $\partial\bar{\alpha}_{i-1}/\partial\hat{\theta}_l$.

However, these schemes can only suitable for the systems with known dynamic models, or with the unknown parameters appearing linearly with respect to known nonlinear functions. Furthermore, conventional adaptive control methodology cannot incorporate human operators' experiences, which are in the form of linguistic descriptions. Fortunately, fuzzy logic can use not only the sensor's digital data, but also the operator's language information. Hence, fuzzy systems can be applied to those systems which are ill-defined or too complex to have a mathematical model.

Therefore, analytical studies of nonlinear control, using fuzzy logic system [3], have become the popular tool to tackle the uncertainties in a dynamical system (see [4-8] and references therein). The adaptive fuzzy controller design [6] is proposed for a class of affine-type nonlinear system. Controller with H_∞ tracking performance was studied in [9] for canonical strict-feedback system. The authors in [11] gave the adaptive backsteping design when not all the states are available. However, the backstepping approach brings out the problem of "explosion of terms" in [9]-[12]. This problem is caused by the repeated differentiations of virtual input. This problem also appears in the other designs which use other kinds of approximator to construct the unknown system dynamics, such as wavelet [13] and neural networks [14]. To overcome the problem of explosion of complexity inherent in adaptive fuzzy backstepping design, the authors in [15] proposed a command filter backstepping (CFB) control design method. The methodology therein is useful for the system with exactly known dynamics.

Motivated by the aforementioned observations, in this paper, compensated-tracking-error-based adaptive fuzzy backstepping control approach is proposed for a class of strict-feedback nonlinear system. The system dynamics are completely unknown. Takagi-Sugeno (T-S) fuzzy logic systems are used to model the unknown nonlinear system functions. The boundedness of all the signals in the closed-loop system is guaranteed. Compensated tracking errors are used to formulate the adaptive fuzzy controller. The proposed design avoids the repeated differential of virtual control law efficiently. Hence, the proposed controller is of simple structure. Furthermore, compared to the approach in [8], the adaptive law is in the sense of minimal parameterization. That is, the

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number of adaptive law is equal to the order of the system. Finally, simulation researches are carried out.

II. Problem Statement and Preliminaries

This section will present some descriptions of problem formulations and some useful preliminaries.

A. Problem Statement

Consider a class of n-th order single-input-single-output nonlinear system in strict-feedback form as follows

$$\begin{cases} \dot{x}_1 = x_2 + f(x_1) + d_1, \\ \dot{x}_2 = x_3 + f(x_1, x_2) + d_2, \\ \cdots \\ \dot{x}_{n-1} = x_n + f_{n-1}(x_1, \cdots, x_{n-1}) + d_{n-1}, \\ \dot{x}_n = u + f_n(x_1, \cdots, x_n) + d_n, 1 \le i \le n, \end{cases} \tag{1}$$

where $x = [x_1, \cdots, x_n] \in \Re^n$ is the state vector with initial condition $x(0) = x_0$, the first state x_1 is considered as the scalar output, and u is the scalar control signal. The functions $f_i(x_1, \cdots, x_i) : \Re^i \to \Re$ are assumed to be unknown and satisfy the following assumption. External disturbances d_i are unknown smooth functions that satisfy the following growth conditions.

Assumption 1 (Triangular bounds): There exist (not necessarily known) parameter values $\psi_i^* \ge 0$ and smooth functions $p_i(x_1, \cdots, x_i)$, such that for all $x \in \Re^n$ and $t \in \Re^+$,

$$|\Delta_i(x,t)| \le \phi_i^* p_i(x_1, \cdots x_i), 1 \le i \le n-1. \tag{2}$$

Our objective is trajectory tracking. Therefore, we assume there is a desired trajectory $x_{1c}(t) : \Re^+ \to \Re$. Specific assumptions related to this desired trajectory will be stated in subsequent sections. The objective of the control design are to specify a control signal $u(t)$ to steer $x(t)$ from any initial conditions to track the reference input $x_{1c}(t)$, to achieve boundedness of all signals and states defined in the control law, and to achieve boundedness for the system states $x_i(t)$ from $i = 2, \cdots, n.$

B. Useful Lemmas

To proceed, the following simple lemmas play an important role in the manipulations of our main results on adaptive fuzzy controller design.

Lemma 1 (Young's inequality): [10] For scalar time functions $x(t) \in \Re$ and $y(t) \in \Re$, it holds that

$$2xy \le \frac{1}{\omega} x^2 + \omega y^2 \tag{3}$$

for any $\omega > 0$.

Lemma 4 IF there exists

$$u = \frac{AB^2}{|A|B + \varepsilon} \tag{4}$$

where u is control input, $A, B \ne 0, A, B \in \Re$, and $\varepsilon > 0$, then $Au + |A|B \le \varepsilon$ will always holds.

Proof: Substitute (4) into the left side of the inequality, and we have

$$Au + |A|B \le \frac{|A|B\varepsilon}{|A|B + \varepsilon} \le \frac{|A|B\varepsilon + \varepsilon^2}{|A|B + \varepsilon} \le \varepsilon \tag{5}$$

C. Descriptions of T-S Fuzzy System

Generally, fuzzy logic system consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine, and the defuzzifier. The knowledge base contain a group of IF-THEN rules. Especially, T-S fuzzy rules [16] are a set of linguistic statements in the following form

R_j : IF x_1 is F_1^j and x_2 is F_2^j and $\cdots$ and x_n is F_n^j ,

THEN $y_j = a_0^j + a_1^j x_1 + \cdots + a_n^j x_n, j = 1, 2, \cdots, K,$

where x_i are the input variables, $a_i^j, i = 0, 1, \cdots, n$ are the unknown constants to be adapted, y_j is the output variable of the fuzzy system, and F_i^j are fuzzy sets associated with membership functions $\mu_{F_i^j}(x_i)$. Together with singleton fuzzifier and center-average defuzzifier, and product inference, the crisp output of T-S fuzzy system can be expressed as follows

$$y(x) = \frac{\sum_{j=1}^{K} y_j \left[\prod_{i=1}^{n} \mu_{F_i}^j(x_i) \right]}{\sum_{j=1}^{K} \left[\prod_{i=1}^{n} \mu_{F_j}^j(x_i) \right]} = \sum_{j=1}^{K} \zeta_j(x) y_j, \tag{6}$$

where

$$y_j = a_0^j + a_1^j x_1 + \cdots a_n^j x_n , \tag{7}$$

$$\zeta_j(x) = \frac{\prod_{i=1}^{n} \mu_{F_i}^i(x_i)}{\sum_{j=1}^{K} \left[\prod_{i=1}^{n} \mu_{F_i}^j(x_i) \right]} , \tag{8}$$

which is called fuzzy basis function. From universal approximation theorem [16], it is well known that T-S fuzzy logic system (6) is capable of uniformly approximating any well-defined nonlinear function over a compact set U_c to any degree of accuracy with triangular or Gaussian membership function. Due to their approximation capability, we can assume that the nonlinear system in (1) can be approximated by the above T-S fuzzy logic systems. Next, similar to the process in [16], (6) can be easily written as

$$y(x) = \zeta(x) A_z^0 + \zeta(x) A_z^1 x + \delta , \tag{9}$$

where

$$\zeta(x)=\left[\zeta_1(x),\zeta_2(x),\cdots\zeta_K(x)\right],$$

$$X=\left[x_1,x_2,\cdots,x_n\right]^{\mathrm{T}},$$

$$A_z^0=\left[a_0^1,a_0^2,\cdots,a_0^K\right]^{\mathrm{T}},$$

$$A_z^1=\begin{bmatrix} a_1^1 & a_2^1 & \cdots & a_n^1 \\ a_1^2 & a_2^2 & \cdots & a_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ a_1^K & a_2^K & \cdots & a_n^K \end{bmatrix}.$$

III. Adaptive Fuzzy Controller Design Based on Compensated Tracking Error

In this section, we will incorporate backstepping method into the adaptive fuzzy control design for n^{th}-order system, which is described by the equation (1). The detailed design procedure is described in the following steps.

Step 1: Firstly, we define two tracking errors for the state x_1 respectively as follows

$$\tilde{x}_1=x_1-x_{1c}, \tag{10}$$

$$\bar{x}_1=\tilde{x}_1-\zeta_1, \tag{11}$$

where x_{1c} is the desired trajectory, $\tilde{x}_1$ is tracking error, and $\bar{x}_1$ is compensated tracking error. Because $f_1(x_1)$ is an unknown continuous function, we will construct T-S fuzzy system with input vector x_1 to approximate the system function $f_1(x_1)$. Then, similar to section II-C, $f_1(x_1)$ can be expressed as

$$\begin{aligned} f_1(x_1)&=\zeta_1(x_1)A_{z1}+\delta_1 \\ &=\zeta_1(x_1)A_{z1}^0+\zeta_1(x_1)A_{z1}^1x_1+\delta_1 \\ &=\zeta_1(x_1)A_{z1}^1\bar{x}_1+\zeta_1(x_1)\left(A_{z1}^0+A_{z1}^1x_{1c}\right) \\ &\quad+\zeta_1(x_1)A_{z1}^1\xi_1+\delta_1, \end{aligned} \tag{12}$$

where A_{z1},A_{z1}^0,A_{z1}^1 are matrices with unknown elements, ξ_1 will be defined later. Then, we obtain

$$\dot{\bar{x}}_1=x_2+\zeta_1(x_1)A_{z1}^1\bar{x}_1+\Omega_1, \tag{13}$$

where Ω_1 is an introduced variable for simplicity and will be discussed as follows

$$\begin{aligned} \Omega_1&=\zeta_1(x_1)(A_{z1}^0+A_{z1}^1x_{1c})+\zeta_1(x_1)A_{z1}^1\xi_1+\delta_1 \\ &\quad+p_1^*\phi_1(x_1)-\dot{x}_{1c} \\ &\le\left\|\zeta_1(x_1)\right\|\left\|A_{z1}^0+A_{z1}^1x_{1c}\right\|+\left\|\zeta_1(x_1)\right\|\left\|A_{z1}^1\right\|\left\|\xi_1\right\| \\ &\quad+\left\|\delta_1\right\|+\left\|p_1^*\right\|\left\|\psi_1(x_1)\right\|+\left\|\dot{x}_{1c}\right\| \\ &\le\vartheta_1\psi_1(x_1), \end{aligned} \tag{14}$$

where $c_{\vartheta1}$ is a constant only for analytic purpose, the accurate value of which is not necessarily known, δ_1^* is the bound of approximation error, and

$$\begin{aligned} \vartheta_1&=\max\left\{\left\|A_1^0\right\|+\left|c_{\vartheta1}\right|\cdot\left|x_{1c}\right|,\left|c_{\vartheta1}\right|,\left|\delta_1^*\right|+\left|\dot{x}_{1c}\right|,\left\|p_1^*\right\|\right\}, \\ \psi_1(x_1)&=1+\left\|\zeta_1(x_1)\right\|+\left\|\phi_1(x_1)\right\|+\left\|\zeta_1(x_1)\right\|\cdot\left|\xi_1\right|, \end{aligned} \tag{15}$$

where $\|\bullet\|$ stands for Euclidean norm of vectors and induced norm of matrices. Next, we define

$$\dot{\xi}_1=-k_1\xi_1+(x_{2c}-x_{2c}^0), \tag{16}$$

$$x_{2c}^0=\alpha_1-\xi_2, \tag{17}$$

where ξ_2 will be define in Step 2, the signal x_{2c}^0 is filtered to produce the command signal x_{2c} and its derivative $\dot{x}_{2c}$, α_1 is virtual control input which will be discussed later, k_1 is a positive constant and chosen by designer. Such a filter will be defined later. By use of (16) and (17), the dynamics of the compensated tracking errors are described by

$$\dot{\bar{x}}_1=\dot{\tilde{x}}_1-\dot{\xi}_1=\zeta_1(x_1)A_1^1\bar{x}_1+\Omega_1+\bar{x}_2+\alpha_1+k_1\xi_1. \tag{18}$$

Choose Lyapunov candidate function as follows

$$V_1=\frac{1}{2}\bar{x}_1^2+\frac{1}{2}\Gamma_1^{-1}\tilde{\varsigma}_1^2, \tag{19}$$

where $\tilde{\varsigma}_1=\varsigma_1^*-\hat{\varsigma}_1$, and Γ_1 are positive constant, which will be chosen by designer. Note that, we use compensated tracking error, not tracking error in the conventional schemes, to formulate Lyapunov candidate function in our design. Essentially, we use compensated tracking errors to remove the repeated differentiation of virtual control laws. Then, the derivative of the Lyapunov candidate is given as follows.

$$\dot{V}_1=\bar{x}_1\zeta(x_1)A_1^1\bar{x}_1+\bar{x}_1\Omega_1+\bar{x}_1\bar{x}_2+\bar{x}_1\alpha_1+\bar{x}_1k_1\xi_1+\Gamma_1^{-1}\tilde{\varsigma}_1\dot{\hat{\varsigma}}_1 \tag{20}$$

We discuss some items in the above formulae. From Young's inequality in Lemma 1, we have

$$\begin{aligned} &\bar{x}_1\zeta_1(x_1)A_1^1\bar{x}_1+\bar{x}_1\Omega_1 \\ &\le\frac{c_{\vartheta1}^2}{2w_1}\bar{x}_1^2\zeta_1(x_1)\zeta_1^{\mathrm{T}}(x_1)+\frac{w_1}{2}\left\|A_1^m\right\|^2\bar{x}_1^{\mathrm{T}}\bar{x}_1+\chi_1^*\left|\bar{x}_1\right|\left|\psi_1(x)\right| \\ &\le\varsigma_1^*\frac{1}{2w_1}\bar{x}_1^2\zeta_1(x_1)\zeta_1^{\mathrm{T}}(x_1)+\varsigma_1^*\left|\bar{x}_1\right|\left|\psi_1(x_1)\right|+\frac{w_1}{2}\bar{x}_2^{\mathrm{T}}\bar{x}_2 \\ &\le\hat{\varsigma}_1\frac{1}{2w_1}\bar{x}_1^2\zeta_1(x_1)\zeta_1^{\mathrm{T}}(x_1)+\hat{\varsigma}_1^*\left|\bar{x}_1\right|\left|\psi_1(x_1)\right| \\ &\quad+\tilde{\varsigma}_1\frac{1}{2w_1}\bar{x}_1^2\zeta_1(x)\zeta_1^{\mathrm{T}}(x_1)+\tilde{\varsigma}_1\left|\bar{x}_1\right|\left|\psi_1(x)\right|+\frac{w_1}{2}\bar{x}_1^{\mathrm{T}}\bar{x}_1 \end{aligned} \tag{21}$$

We use the following virtual control law

$$\alpha_1=-k_1\tilde{x}_1-\frac{1}{2w_1}\hat{\varsigma}_1\bar{x}_1\zeta_1(x_1)\zeta_1^{\mathrm{T}}(x_1)-\frac{\hat{\varsigma}_1\hat{\vartheta}_1^2\psi_1^2(x_1)}{\left|\bar{x}_1\right|\hat{\varsigma}_1\psi_1(x_1)+\varepsilon_1} \tag{22}$$

with adaptive law

$$\dot{\hat{\varsigma}}_1 = \Gamma_1\left[\frac{1}{2w_1}\bar{x}_1^2\zeta_1(x_1)\zeta_1^{\mathrm{T}}(x_1) + |\bar{x}_1|\psi_1(x_1) - \sigma_1(\hat{\varsigma}_1 - \varsigma_1^0)\right] \tag{23}$$

where $\sigma_1 \geq 0$, $\varepsilon_1 \geq 0$, ς_1^0 are design constants. Furthermore, by completing squares, there exists the following inequality

$$\tilde{\varsigma}_1\left(\hat{\varsigma}_1 - \varsigma_1^0\right) \leq -\frac{1}{2}\tilde{\varsigma}_1^2 - \frac{1}{2}\left(\hat{\varsigma}_1 - \varsigma_1^0\right)^2 + \frac{1}{2}\left(\varsigma_1^* - \varsigma_1^0\right)^2 \tag{24}$$

Then, substituting (21)-(23) into (18) yields

$$\dot{V}_1 \leq -k_1\bar{x}_1{}^2 + \frac{w_1}{2}\bar{x}_1{}^2 - \frac{\sigma_1}{2}\tilde{\varsigma}_1{}^2 + \frac{\sigma_1}{2}(\varsigma_1^* - \varsigma_1^0) + \varepsilon_1 + \bar{x}_1\bar{x}_2 \tag{25}$$

We introduce

$$c_1 := \min\{2k_1 - w_1, \sigma_1\Gamma_1\},\ \varpi_1 := \frac{\sigma_1}{2}\left(\varsigma_1^* - \varsigma_1^0\right)^2 + \varepsilon_1, \tag{26}$$

then $\dot{V}$ can be further written as follows

$$\dot{V}_1 \leq -c_1V_1\left(\bar{x}_1, \tilde{\varsigma}_1\right) + \varpi_1 + x_1x_2. \tag{27}$$

Step i ($2 \leq i \leq n-1$): Similar to Step 1, we define two tracking errors for the state x_i respectively as follows

$$\tilde{x}_i = x_i - x_{ic}, \tag{28}$$

$$\bar{x}_i = \tilde{x}_i - \xi_i, \tag{29}$$

where x_{ic} is the desired trajectory, $\tilde{x}_i$ is tracking error, and $\bar{x}_i$ is compensated tracking error. Then we use T-S fuzzy system to approximate unknown function $f_i(x_i)$. The dynamics of tracking errors can be expressed as follows

$$\dot{\tilde{x}}_i = x_{i+1} + \zeta_i A_i^1\bar{x}_i + \Omega_i. \tag{30}$$

Next, we define

$$\dot{\xi}_i = -k_i\xi_i + (x_{(i+1)c} - x_{(i+1)c}^0), \tag{31}$$

$$x_{(i+1)c}^0 = \alpha_i - \xi_{(i+1)}, \tag{32}$$

where the signal $x_{(i+1)c}^0$ is filtered to produce the command signal $x_{(i+1)c}$ and its derivative $\dot{x}_{(i+1)c}$, α_i is virtual control input which will be discussed later, k_i is a positive constant and chosen by designer. Then we obtain

$$\dot{\bar{x}}_i = \zeta_i c_{\vartheta i}A_i^m\bar{x}_i + \Omega_i + \bar{x}_{i+1} + \alpha_i + k_i\xi_i. \tag{33}$$

Choose Lyapunov candidate function

$$V_i = V_{i-1} + \frac{1}{2}\bar{x}_i^2 + \frac{1}{2}\Gamma_i^{-1}\tilde{\varsigma}_i^2, \tag{34}$$

where $\tilde{\varsigma}_i = \varsigma_i^* - \hat{\varsigma}_i$, Γ_i is positive constant and chosen by designer. The derivative of V_i is given as follows

$$\dot{V}_i = \dot{V}_{i-1} + \bar{x}_i\zeta_i A_i^1\bar{x}_i + \bar{x}_i\Omega_i + \bar{x}_i\bar{x}_{i+1} + \bar{x}_i\alpha_i + \bar{x}_ik_i\xi_i + \Gamma_i^{-1}\tilde{\varsigma}_i\dot{\hat{\varsigma}}_i. \tag{35}$$

We use the following virtual control law

$$\alpha_i = -k_i\tilde{x}_i - \frac{1}{2w_i}\hat{\varsigma}_i\bar{x}_i\zeta_i(x_i)\zeta_i^T(x_i) - \bar{x}_{i-1} - \frac{\hat{\varsigma}_i^2\psi_i^2(x_i)}{|\bar{x}_i|\hat{\varsigma}_i\psi_i(x_i) + \varepsilon_i}, \tag{36}$$

where

$$\psi_i(x_i) = 1 + \left\|\zeta_i\left(x_i\right)\right\| + \left\|\phi_i\left(x_i\right)\right\| + \left\|\zeta_i\left(x_i\right)\right\|\cdot|\xi_i|, \tag{37}$$

$$\dot{\hat{\varsigma}}_i = \Gamma_{i1}\left[\frac{1}{2w_i}\bar{x}_i^2\zeta_i\left(x_i\right)\zeta_i^T\left(x_i\right) + \|x_i\|\psi_i\left(x_i\right) - \sigma_{i1}\left(\hat{\varsigma}_i - \varsigma_i^0\right)\right]. \tag{38}$$

From (36)-(38), we obtain

$$\dot{V}_i = -c_iV_i\left(\bar{x}_i, \tilde{\varsigma}_i\right) + \varpi_i + \bar{x}_i\bar{x}_{i+1}, \tag{39}$$

where

$$c_i := \min\{2k_i - w_i, \sigma_i\Gamma_i, 2c_{i-1}\}, \tag{40}$$

$$\varpi_i := \sum_{s=1}^{i-1}\varpi_s + \frac{\sigma_i}{2}\left(\varsigma_i^* - \varsigma_i^0\right)^2 + \varepsilon_i, \tag{41}$$

Step n: We define two tracking errors for the state x_n respectively as follows

$$\tilde{x}_n = x_n - x_{nc}, \tag{42}$$

$$\bar{x}_n = \tilde{x}_n - \xi_n, \tag{43}$$

The unknown function $f_n\left(x_n\right)$ is approximated by T-S fuzzy system. Then, we obtain

$$\dot{\tilde{x}} = u + \zeta_n\left(x_n\right)A_n^1\bar{x}_n + \Omega_n + k_n\xi_n. \tag{44}$$

Next, we define

$$\bar{x}_n = \tilde{x}_n - \xi_n, \tag{45}$$

$$\dot{\xi}_n = -k_n\xi_n + \left(u_c - u_c^0\right). \tag{46}$$

Furthermore, note that $u_c = u_c^0 = u.$ Then, we obtain

$$\dot{\bar{x}} = u + \zeta_nc_{\vartheta n}A_n^m\tilde{x} + \Omega_n + k_n\xi_n. \tag{47}$$

Choose Lyapunov candidate function

$$V_n = V_{n-1} + \frac{1}{2}\bar{x}_n^2 + \frac{1}{2}\Gamma_n^{-1}\tilde{\varsigma}_n^2, \tag{48}$$

where $\tilde{\varsigma}_n = \varsigma_n^* - \hat{\varsigma}_n$, Γ_n is positive constant and chosen by designer. The derivative of V_n is given as follows

$$\dot{V}_n = \dot{V}_{n-1} + \bar{x}_n\zeta_nA_n^1\bar{x}_n + \bar{x}_n\Omega_n + \bar{x}_nu + \bar{x}_nk_n\xi_n + \Gamma_n^{-1}\tilde{\varsigma}_n\dot{\hat{\varsigma}}_n. \tag{49}$$

We use the following control law

$$u=-k_n\tilde{x}_n-\frac{1}{2w_n}\hat{\varsigma}_n\bar{x}_n\zeta_n(x_n)\zeta_n^{\mathrm{T}}(x_n)-\bar{x}_{n-1} -\frac{\hat{\varsigma}_n^2\psi_n^2(x_n)}{|\bar{x}_n|\hat{\varsigma}_n\psi_n(x_n)+\varepsilon_n}, \quad (50)$$

where

$$\psi_n(x_n)=1+\|\zeta_n(x_n)\|+\|\phi_n(x_n)\|+\|\zeta_n(x_n)\|\cdot|\xi_n|, \quad (51)$$

$$\dot{\hat{\varsigma}}_n=\Gamma_n\left[\frac{1}{2w_n}\bar{x}_n^2\zeta_n(x_n)\zeta_n^{\mathrm{T}}(x_n)+\|x_n\|\psi_n(x_n) -\sigma_n(\hat{\varsigma}_n-\varsigma_n^0)\right] \quad (52)$$

We introduce

$$C:=\min\{2k_n-w_n,\sigma_n\Gamma_n,2c_{n-1}\} \quad (53)$$

$$M:=\sum_{i=1}^{n-1}\varpi_i+\frac{\sigma_n}{2}\left(\varsigma_n^*-\varsigma_n^0\right)^2+\varepsilon_n, \quad (54)$$

then, $\dot{V}_n$ will be rewritten into

$$\dot{V}_n\le -CV_n(\bar{x}_n,\hat{\varsigma}_n)+M. \quad (55)$$

The above equation (61) implies that

$$V_n(t)\le V_n(t_0)\mathrm{e}^{-C(t-t_0)}+\frac{C}{M} \le V_n(t_0)+\frac{C}{M},\quad \forall t\ge t_0. \quad (56)$$

As a result, all $\bar{x}_i$ and $\tilde{\varsigma}_i$ belong to the compact set $\left\{(\bar{x}_i,\tilde{\varsigma}_i)\middle|V_n(t)\le V_n(t_0)+\frac{M}{C}\right\}$. Namely, all the signals, i.e. $\bar{x}_i$ and $\tilde{\varsigma}_i$ in the closed-loop system are bounded. From (31), it is concluded that $x_{(i+1)c}-x_{(i+1)c}^0$ can be made arbitrarily small by well-defined command filter. Then, compensated tracking error ξ_i is bounded. When $x_{(i+1)c}-x_{(i+1)c}^0$ approaches zero, then $\xi_i\to 0$ and $\bar{x}_i\to\tilde{x}_i$ .Therefore, $\tilde{x}_i$ is bounded, since $\bar{x}_i$ is bounded. Namely, the tracking errors $\tilde{x}_i$ are UUB. Furthermore, appropriate choice of design parameters will make the ultimate error bound arbitrarily small.

IV. SIMULATION EXAMPLE

To illustrate the fuzzy adaptive control procedures, we consider the second-order nonlinear system

$$\begin{cases}\dot{x}_1(t)=x_2(t)+x_1\mathrm{e}^{-0.5x_1}=x_2(t)+f_1(x_1)\ ,\\ \dot{x}_2(t)=u(t)+x_1\sin(x_2^2)=u(t)+f_1(x_1,x_2)\ ,\end{cases} \quad (57)$$

where $f_1(x_1)$ and $f_1(x_1,x_2)$ are unknown nonlinear functions. The control objective is to guarantee that all the signals in the closed-loop system are bounded, and the output follows reference signal $x_{1c}=\sin(t)/2$. Choose membership functions of $x_1(t)$ and $x_2(t)$ as follows

$$\mu_{F_1^l}(x_1)=\exp\left[-\frac{(x_1-3+l)^2}{16}\right],l=1,2,\cdots,5 \quad (58)$$

$$\mu_{F_2^l}(x_2)=\exp\left[-\frac{(x_2-3+l)^2}{16}\right],l=1,2,\cdots,5 \quad (59)$$

where the membership functions of $x_1(t)$ and $x_2(t)$ are of the same structures, l denotes the number of membership functions. In this example, we use totally 5 rules to construct T-S fuzzy system for the unknown parts, namely, $f_1(x_1)$ and $f_1(x_1,x_2)$.

From the membership functions of $x_1(t)$ and $x_2(t)$, we define the fuzzy basis functions for unknown nonlinear functions $f_1(x_1)$ and $f_1(x_1,x_2)$ as follows

$$\zeta_{1j}=\frac{\exp\left[-\frac{(x_1-3+j)^2}{16}\right]}{\sum_{n=1}^{5}\exp\left[-\frac{(x_1-3+n)^2}{16}\right]} \quad (60)$$

$$\zeta_{2j}=\frac{\exp\left[-\frac{(x_1-3+j)^2}{16}\right]\exp\left[-\frac{(x_2-3+j)^2}{16}\right]}{\sum_{n=1}^{5}\left\{\exp\left[-\frac{(x_1-3+n)^2}{16}\right]\exp\left[-\frac{(x_2-3+n)^2}{16}\right]\right\}} \quad (61)$$

where $j=1,2,\cdots,5.$ We use the virtual control law

$$\alpha_1=-15\tilde{x}_1-\frac{1}{20}\hat{\lambda}_1\bar{x}_1\zeta_1(x_1)\zeta_1^T(x_1)\tilde{x}_1 -\frac{\hat{\varsigma}_1^2\psi_1^2(x_1)}{|\bar{x}_1|\hat{\varsigma}_1\psi_1(x_1)+0.5}, \quad (62)$$

with parameter adaptation laws

$$\dot{\hat{\varsigma}}_1=5\left[\frac{1}{20}\bar{x}_1^2\zeta_1(x_1)\zeta_1^T(x_1)+\|x_1\|\psi_1(x_1) -0.1(\hat{\varsigma}_1-0.01)\right], \quad (63)$$

and the following control law

$$u=-10\tilde{x}_2-\frac{1}{2w_2}\hat{\lambda}_2\bar{x}_2\zeta_2(x_2)\zeta_n^T(x_2)\tilde{x}_2-\bar{x}_1 -\frac{\hat{\varsigma}_2^2\psi_2^2(x_2)}{|\bar{x}_2|\hat{\varsigma}_2\psi_2(x_2)+0.5}, \quad (64)$$

with parameter adaptation laws

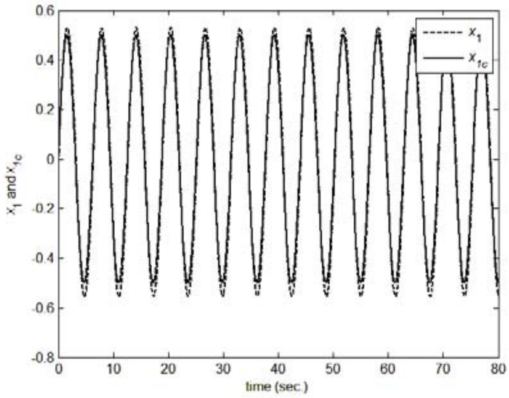

Fig. 1. Trajectories of x_1 and x_{1c}

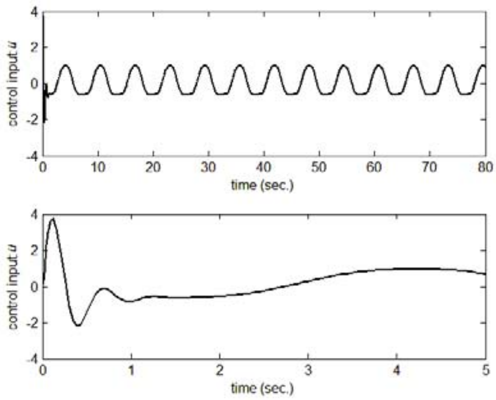

Fig. 2. Control input

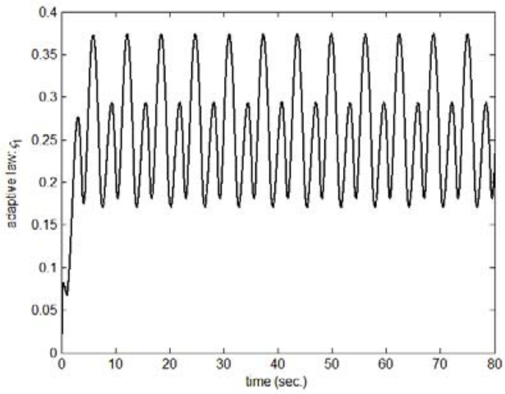

Fig. 3. Adaptive law: $\hat{\varsigma}_1$

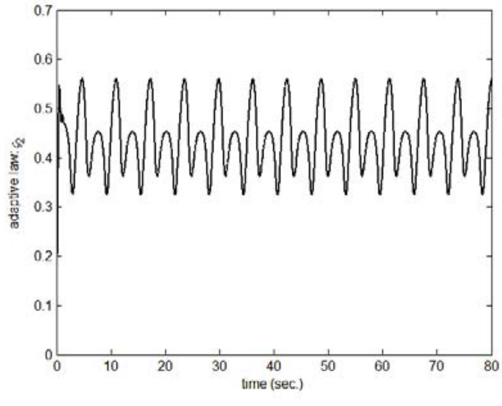

Fig. 4. Adaptive law: $\hat{\varsigma}_2$

$$\dot{\hat{\varsigma}}_2 = 5\left[\frac{1}{2}\bar{x}_2^2\zeta_2(x_2)\zeta_2^T(x_2)+\|x_2\|\psi_2(x_2) - 0.1(\hat{\varsigma}_2-0.02)\right]. \tag{65}$$

We use the following command filter

$$x_{2c}(t)=\frac{20}{20+s}\left[x_{2c}^0(t)\right]. \tag{66}$$

Simulation results in Fig. 1-4 show the effectiveness of the proposed adaptive control design. Fig. 1 shows that the tracking error converges to a small neighborhood around zero. Fig. 2 shows that the boundness of control input. The enlarged part, namely, the time response of the beginning5 seconds, shows that the control input is quite smooth without heavy chattering. Fig. 3-4 show the time histories of adaptive parameters $\hat{\varsigma}_1$ and $\hat{\varsigma}_2$ .From the figures, it can be concluded that all the signals in the closed-loop is UUB.

V. Conclusions

In this paper, adaptive tracking fuzzy control scheme is proposed for a class of nonlinear system in strict-feedback form. The system dynamics are completely unknown, and external disturbances satisfy triangular bounds. The proposed algorithm can guarantee the boundedness of all the signals in the closed-loop system. Compensated tracking errors, not tracking errors, are used to construct the controller. The proposed design avoids the repeated differential of virtual control law completely, which make the controller structure quite simple and easy to implement. Furthermore, the adaptive law achieves minimal parameterization. Numerical example is used to demonstrate the effectiveness of the control algorithm.

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