# A Novel Concise Adaptive Neural Control for a Class of Nonlinear MIMO Systems with Unknown Time Delays

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Abstract—This note makes effort at the problem of robust adaptive control for a class of nonlinear MIMO systems with unknown time-delays. A concise adaptive neural control scheme is developed by using the Backstepping method, the Lyapunov-Krasovskii functional and a novel "MLN" technique. Unlike the existing literatures, the actual control laws are only composed of the state variables, the reference signals and their derivatives, independent of the designed virtual controls and the other intermediate variables. In addition, only n (the number of system outputs) neural networks are introduced to compensate the nonlinear uncertainties of the whole system. Thus, the outstanding advantage of the proposed scheme is that the control law with a concise structure is model-independent and easy to implement in practical engineering, due to less computational burden. The corresponding scheme guarantees uniform ultimate boundedness of all the signals in the closed-loop system, and the tracking errors can converge to a arbitrary small neighborhood of zero. Finally, a simulation example illustrates the effectiveness of the proposed scheme.

*Index Terms*—robust adaptive control, neural networks, MIMO systems, time delays

## I. INTRODUCTION

Recently, the development of adaptive neural control algorithms for uncertain nonlinear systems has been a focus of engineering interest as well as theoretical significance. Many positive results have shown that semiglobal uniform ultimate boundedness (SGUUB) of the closed-loop adaptive control system can be achieved and the output of the system is proven to converge to a small neighborhood of the desired trajectory, refer to [1-3] and the references therein for details. In [4], direct adaptive neural network (NN) control was proposed for a class of affine nonlinear systems with unknown nonlinearities. The scheme could avoid the controller singularity problem completely, by using a special property of the affine term. With the help of the Lyapunov-Krasovskii thermo, the control scheme was extended to control uncertain nonlinear systems with unknown time delays [5]. The extensively significant results on adaptive neural control also have been reported in [6, 7].

For multi-input multi-output (MIMO) nonlinear systems, the control task is very difficult due to the state

and input interconnections among various subsystems, which often severely limits system performance, even leading instability. Therefore, there exist relatively few research results available for nonlinear MIMO systems [8-10], comparing with the vast amount of results on control design for single-input, single-output (SISO) systems. In these results, the tracking control of nonlinear MIMO systems was addressed. Unlike the other literatures, this algorithm introduced NNs to approximate and compensate for both unknown functions and the uncertain time-delay function bounds, and simulation results have illustrated the effectness of the corresponding scheme. However, the aforementioned algorithm suffered from two major problems as expounded in [11]: the first one is the well known "explosion of complexity", which is inherent in the conventional backstepping technique [12]. It is caused by the repeated differentiations of virtual controls, which is impossible to implement in practice and leads to a complicated algorithm with heavy computational burden, especially for the high-order nonlinear system. The second problem is the so-called "curse of dimensionality", i.e., to satisfy the approximation requirement of high-order uncertain nonlinear system, the number of NNs and that of parameters to be updated online in the previous adaptive schemes is very large. In order to solve these problems, the dynamic surface control (DSC) technique [6, 13] and the minimal learning parameters (MLP) algorithm [14, 15] were extended to NNs-based adaptive control for nonlinear MIMO system in [16-18].

It is worth noting that the impact of the above problems has been minimized by the merit of the DSC technique and MLP algorithm, whereas not been solved absolutely. In these algorithms, there exist some intermediate control variables and dynamic surface of first order to be computed online, especially for the highorder system. This still require large computing time, even become unacceptably in the real-time control engineering. In addition, one concerned that only one online-learning parameter is induced to compensate the whole nonlinear uncertainties of the subsystem (for MIMO system). In this technique note, motivated by the above-mentioned observations, a novel kind of concise adaptive neural control scheme, which is performed by using the Lyapunov-Krasovskii functional, is developed for a class of nonlinear MIMO systems with unknown time delays. The nonlinear uncertainties and unknown state time delays at each step are delivered to the next step, without being compensated as in the conventional adaptive neural control. In the sequel, the uncertainties of the whole system are dealt with by introducing a radialbasis-function (RBF) NNs in the final step.

The main contributions of this note can be summarized as follows: 1) In the proposed scheme, the problems of "explosion of complexity" and "curse of dimensionality" are solved from the root causes, different from DSC and MLP. The number of online learning NNs is reduced to only *n*, which is equal to the number of the systems outs and independent of the system orders. The intermediate controls would not appear in the control scheme. That will lead to a much simpler controller with less computational burden. 2) The adaptive law proposed in this note is merely dependent on the state variables, the reference signals and their *m*th order derivatives. With the special property and structure of our algorithm, the potential controller singularity problem existing in may adaptive control algorithm is avoided.

## **II. PROBLEM FORMULATION**

In this note, we solve the adaptive control problem of the following nonlinear MIMO system with unknown time delays.

$$\begin{aligned} & \left( \dot{x}_{j,i_j} = f_{j,i_j}(\overline{x}_{j,i_j}) + g_{j,i_j}(\overline{x}_{j,i_j}) x_{j,i_j+1} + h_{j,i_j}(\overline{x}_{\tau,j,i_j}), \\ & i_j = 1, 2, \cdots, m_j - 1 \\ & \dot{x}_{j,m_j} = f_{j,m_j}(X, \overline{u}_{j-1}) + g_{j,m_j}(X, \overline{u}_{j-1}) u_j + h_{j,m_j}(X_{\tau}) \\ & y_j = x_{j,1}, \ j = 1, 2, \cdots, n \end{aligned}$$

where,  $X = [x_1^T, \dots, x_n^T]^T \in R^{n \times m}$  with  $x_j = [x_{j,1}, \dots, x_{j,m_j}]^T$ denotes the matrix of state variables,  $y = [y_1, \dots, y_n]^T \in R^n$  is the system output.  $\overline{x}_{j,i_j} = [x_{j,1}, \dots, x_{j,i_j}]^T \in R^{i_j}$ ,  $\overline{u}_j = [u_1, \dots u_j]^T$ .  $x_{\tau_{j,i_j}} = x_{j,i_j}(t - \tau_{j,i_j})$  with  $\tau_{j_{i_j}}$  as unknown time delays of the states.  $\overline{x}_{\tau_{j,i_j}} = [x_{\tau_{j,1}}, \dots, x_{\tau_{j,i_j}}]^T$ ,  $X_\tau = [\overline{x}_{\tau_{1,m_1}}^T, \dots, \overline{x}_{\tau_{m,m_n}}^T]^T$  are the vector of delayed state variables.  $f_{j,i_j}(\cdot)$ ,  $g_{j,i_j}(\cdot)$  and  $h_{j,i_j}(\cdot)$ are all unknown nonlinear continuous functions. For  $t \in [-\tau_{j,i_j}, 0]$ ,  $x_{j,i_j}(t)$  are assumed to be smooth and bounded. **Remark1:** Comparing with [9], e.g. the control gain function  $g_{j,m_j}(\cdot)$  includes all state variables and the inputs of the previous subsystems. Obviously, the plant (1) describes a class of nonlinear MIMO systems with more general form.

The following assumptions on system (1) are introduced.

**Assumption1:** The unknown virtual control gain functions are confined within a certain range such that

$$0 < \underline{g}_{j} \le g_{j,i_{j}}(\cdot) \le \overline{g}_{j,i_{j}} < \infty$$
<sup>(2)</sup>

where  $\underline{g}_{j}$  and  $\overline{g}_{j,i_{j}}$  are the positive constant lower and upper bound parameters.

**Remark2:** Assumption is reasonable for  $g_{j,i_j}(\cdot)$  being away from zero is the controllable condition of (1), which is made in current literatures. It should be mentioned that

 $\underline{g}_{j}$  and  $\overline{g}_{j,i_{j}}$  are only required for analytical purpose, which is not necessarily known in the proposed scheme.

**Assumption2:** The reference signals  $y_{dj}(t)$ ,  $j = 1, \dots, n$  and their time derivatives up to the  $m_j$ th order are continuous and bounded.

**Assumption3:** The unknown smooth function  $h_{j,i_j}(\bar{x}_{r_j,i_j})$  satisfies the following inequality:

$$|h_{j,i_j}(\overline{x}_{\tau j,i_j})| \le \sum_{l=1}^{i_j} Q_{j,l}^{j,i_j}(x_{\tau j,l})$$
 (3)

where  $Q_{j,l}^{j,i_j}(\cdot)$  are the positive functions for  $l = 1, \dots, i_j$ .

The control objective is to develop a novel concise adaptive neural tracking controller such that 1) all states of the uncertain nonlinear MIMO system (1) are SGUUB, and 2) the tracking error  $z_j = y_j - y_{dj}$  can be rendered arbitrary small.

In this note, RBF NNs are introduced to compensate all the systems uncertainties. As pointed out in [9, 16], universal approximation results indicate that, given a desired level of accuracy  $\delta$ , approximation to that level of accuracy can be guaranteed by making *l* sufficiently large. Therefore, the NNs  $W^TS(Z)$  can approximate any given real continuous function f(Z) with f(0)=0, which is written as (4).

$$f(Z) = W^{\mathrm{T}}S(Z) + \delta(Z), \quad \forall Z \in \Omega_{Z} \subset \mathbb{R}^{q}$$
(4)

where, l > 1 is the number of the NN nodes.  $\delta(Z)$  is the approximation error with unknown upper bound  $\overline{\delta}$  .  $W = [w_1, w_2, \dots, w_l]^T$  is the weight vector,  $S(Z) = [s_1(Z), s_2(Z), \dots, s_l(Z)]^T$  is a vector of RBF basic functions with the form of Gaussian functions defined by (5).  $\mu_i = [\mu_{i,1}, \mu_{i,2}, \dots, \mu_{i,q}]^T$  is the center of the receptive fields,  $\eta$  is the width of the Gaussian function, and  $\zeta$  is gain coefficient.

$$s_i(Z) = \zeta \exp\left[-\frac{(Z-\mu_i)^{\mathrm{T}}(Z-\mu_i)}{2\eta^2}\right], \ i = 1, 2, \cdots, l$$
 (5)

In order to design the novel adaptive law, Lemma1 will be explored. **Lemma1:** [19, 20] Consider the RBF NNs described in (4), let  $\phi = \frac{1}{2} \min_{i \neq j} ||\mu_i - \mu_j||$ . Then one may take an upper bound of ||S(Z)|| as (6).

$$\|S(Z)\| \le \sum_{k=0}^{\infty} 3q(k+2)^{q-1} \exp(-2\phi^2 k^2/\eta) := S^*$$
 (6)

The limited value  $s^*$  is independent of z and t.

#### III. DESIGN OF CONCISE ADAPTIVE NEURAL CONTROL

In this section, we develop a concise adaptive neural controller for uncertain nonlinear MIMO system (1) with Assumptions 1-3. The backstepping design procedure contains  $(j, m_j)$ ,  $j=1,\dots,n$  steps. At each step, the Lyapunov- Krasovskii functional is constructed to compensate for the unknown time delays, and the nonlinear functions  $\mathcal{F}_{j,i}(\cdot)$  and  $F_{j,i}(\cdot)$  shall be induced to described the nonlinear uncertainties, which is component in an intermediate control function  $\alpha_{j,i}$  being delivered to

the next step. In the  $j,m_j$  step, the RBF NNs is exploited to approximate the uncertainties of the whole *j*th subsystem, and the actual control  $u_j$  is derived.

To illustrate the design synthesis, the notion (7) is useful, with the properties of relationship as (8).

$$K_{i,p}^{j} = \sum_{1 \le l_{i} < l_{2}, \dots < l_{p} \le i} (k_{j,l_{i}} k_{j,l_{2}} \dots k_{j,l_{p}}), \ l = 1, 2, \dots, i, \ p \le i \quad (7)$$

$$\begin{cases} K_{i-1,1}^{j} + k_{j,i} = K_{i,1}^{j} \\ K_{i-1,p}^{j} + k_{j,i} K_{i-1,p-1}^{j} = K_{i,p}^{j} \\ k_{j,i} K_{i-1,i-1}^{j} = K_{i,i}^{j} \end{cases}$$
(8)

Throughout this note,  $\|\cdot\|$  is Euclidean norm of a vector;  $\lambda_{\max}(\cdot)$  denotes the largest eigenvalue of a square matrix. (c) is the estimate of (.), and the estimate error (z) = (z) - (.).

For notation simplicity, let  $f_{j,i} = f_{j,i}(\cdot)$ ,  $g_{j,i} = g_{j,i}(\cdot)$ ,  $h_{i,i} = h_{i,i}(\cdot)$  where  $j = 1, \dots, n, i = 1, \dots, m_j$ .

# A. Controller Design

The following coordinate transformation (9) is useful to design concise adaptive laws.

$$\begin{cases} z_{j,1} = x_{j,1} - y_{dj} \\ z_{j,i} = x_{j,i} - \alpha_{j,i} \end{cases}, \quad j = 1, 2, \dots, n, \quad i = 1, 2, , m_j$$
(9)

Furthermore,  $\alpha_{j,i}$  is the intermediate control laws at each step and is chosen as follows:

$$\begin{cases} \alpha_{j,2} = -k_{j,1}z_{j,1} + \dot{y}_{dj} - F_{j,1}(x_{j,1}, y_{dj}) \\ \alpha_{j,i+1} = -k_{j,i}z_{j,i} + y_{dj}^{(i)} - \sum_{p=1}^{i-1} K_{i-1,p}^{j} [x_{j,i+1-p} - y_{dj}^{(i-p)}] - F_{j,i}(\overline{x}_{j,i}, \overline{y}_{dj}^{(i-1)}) \end{cases}$$
(10)

where,  $k_{j,i} > 0$  are design parameters,  $K_{i-1,p}^{j}$  have been defined in (7).  $F_{j,i}(\cdot)$  denotes the previous *i* nonlinear uncertainties of the *j*th subsystem, which will be specified in each step.

Using the similar operation, the desired control laws  $u_j^*$  are derived.

$$u_{j}^{*} = -k_{j,i} z_{j,m_{j}} + y_{dj}^{(m_{j})} - \sum_{p=1}^{m_{j}-1} K_{m_{j}-1,p}^{j} [x_{j,m_{j}+1-p} - y_{dj}^{(m_{j}-p)}] - F_{j,m_{j}}(\overline{x}_{j,m_{j}}, \overline{y}_{dj}^{(m_{j}-1)})$$
(11)

**Step**  $(j, 1 \ (1 \le j \le n))$ . For the first differential equation of the *j*th subsystem, one has

$$\dot{z}_{j,1} = f_{j,1}(x_{j,1}) + g_{j,1}(x_{j,1})x_{j,2} + h_{j,1}(x_{r,j,1}) - \dot{y}_{dj} \qquad (12)$$

With Assumption 3, completing the square gives

$$z_{j,l}h_{j,l}(x_{rj,l}) \le \frac{1}{2}z_{j,l}^2 + \frac{1}{2}[Q_{j,l}^{j,l}(x_{rj,l})]^2$$
(13)

To deal with the delay term in (12), consider the Lyapunov-Krasovskii functional as follows (14).

 $V_{j,1} = \frac{1}{2} z_{j,1}^{2} + V_{Uj,1}$ (14) with  $V_{Uj,1} = \int_{t-\tau_{j}}^{t} \frac{1}{2} \left[ Q_{j,1}^{j,1}(x(s)) \right]^{2} ds$ 

Take time derivative of (14) alone (12), (13), it is easy to obtain (15).

$$\dot{V}_{j,1} \leq z_{j,1} (f_{j,1} - \dot{y}_{dj} + \frac{1}{2} z_{j,1} + g_{j,1} \alpha_{j,2}) + z_{j,1} g_{j,1} z_{j,2} + \frac{1}{2} [Q_{j,1}^{j,1} (x_{j,1})]^2$$

$$\leq z_{j,1} (g_{j,1} F_{j,1}(\cdot) - \dot{y}_{dj} + g_{j,1} \alpha_{j,1}) + z_{j,1} g_{j,1} z_{j,2} + [1 - 2 \tan^2(\frac{z_{j,1}}{\eta_{j,1}})] U_{j,1}$$
(15)

where,  $F_{j,1}(\cdot)$  is defined in (16),  $U_{j,1} = \frac{1}{2} [Q_{j,1}^{j,1}(x_{j,1})]^2$ . As pointed out in (8),  $\frac{1}{z} \tanh^2(\frac{z}{n})$  is well defined at z = 0 and

can be approximated by a RBF NNs.

$$F_{j,1}(\cdot) = g_{j,1}^{-1}(x_{j,1})[f_{j,1} + \frac{2}{z_{j,1}} \tanh^2(\frac{z_{j,1}}{\eta_{j,1}}) + \frac{1}{2}z_{j,1}] \quad (16)$$

Thus, substituting (10) into (15) results in

$$\dot{V}_{j,1} \leq -[k_{j,1}\underline{g}_j - (\overline{g}_{j,1} - 1)^2 \dot{y}_{dj}^2] z_{j,1}^2 + z_{j,1}g_{j,1}z_{j,2} + \frac{1}{4} + [1 - 2\tanh^2(\frac{z_{j,1}}{\eta_{j,1}})]U_{j,1}$$
(17)

The second error variable  $z_2$  shall be presented.

$$z_{j,2} = x_{j,2} - \dot{y}_{dj} + K_{1,1}^{j}(x_{j,1} - y_{dj}) + \mathcal{F}_{j,1}(\cdot)$$
(18)

where  $\mathcal{F}_{j,1}(\cdot) = F_{j,1}(x_{j,1}, y_{dj})$ . Stop (*i*, *i* (for *i*-2, *m*, 1))

**Step**  $(j, i \text{ (for } i=2, ..., m_j-1))$ . A similar procedure is employed recursively for each step (j, i). The (i)th error variable  $z_{j,i}$  is

$$z_{j,i} = x_{j,i} - y_{dj}^{(i-1)} + \sum_{p=1}^{i-1} K_{i-1,p}^{j} [x_{j,i-p} - y_{dj}^{(i-1-p)}] + \mathcal{F}_{j,i-1}(\cdot)$$
(19)

where,  $\mathcal{F}_{j,i-1}(\cdot) = \mathcal{F}_{j,i-1}(\overline{x}_{j,i-1}, \overline{y}_{dj}^{(i-2)})$  is constructed to represent the nonlinear uncertainties of the previous (j,i-1) differential equations.

Using (1), then the derivative of  $z_{j,i}$  is

$$\dot{z}_{j,i} = f_{j,i} + g_{j,i}x_{j,i+1} - y_{dj}^{(i)} + \sum_{p=1}^{i-1} K_{i-1,p}^{j} [f_{j,i-p} + g_{j,i-p}x_{i+1-p} - y_{dj}^{(i-p)}] + \sum_{p=1}^{i-1} \frac{\partial \mathcal{F}_{j,i-1}}{\partial x_{j,p}} (f_{j,p} + g_{j,p}x_{j,p+1}) + \sum_{p=0}^{i-2} \frac{\partial \mathcal{F}_{j,i-1}}{\partial y_{dj}^{(p)}} \dot{y}_{dj}^{(p)} + h_{j,i} + \sum_{p=1}^{i-1} K_{j-1,p}^{j} h_{j,i-p} + \sum_{p=1}^{i-1} \frac{\partial \mathcal{F}_{j,i-1}}{\partial x_{j,p}} h_{j,p}$$
(20)

With Assumption 3, the inequalities (21) can be obtained.

$$z_{j,j}h_{j,i}(\overline{x}_{\tau,j,i}) \leq \sum_{k=1}^{i} \frac{1}{2} z_{j,i}^{2} + \sum_{k=1}^{i} \frac{1}{2} [\mathcal{Q}_{j,k}^{j,i}(x_{\tau,j,k})]^{2}$$

$$z_{j,i} \sum_{p=1}^{i-1} K_{i-1,p}^{j}h_{j,i-p} + z_{j,i} \sum_{p=1}^{i-1} \frac{\partial \mathcal{F}_{j,i-1}}{\partial x_{j,p}} h_{j,p}$$

$$\leq \sum_{p=1}^{i-1} \sum_{k=1}^{p} \frac{1}{2} z_{j,i}^{2} [K_{i-1,p}^{j}]^{2} + \sum_{p=1}^{i-1} \sum_{k=1}^{p} \frac{1}{2} z_{j,i}^{2} [\frac{\partial \mathcal{F}_{j,i-1}}{\partial x_{j,p}}]^{2} + \sum_{p=1}^{i-1} \sum_{k=1}^{p} [\mathcal{Q}_{j,k}^{j,p}(x_{\tau,j,k})]^{2}$$
(21)

Consider the Lyapunov-Krasovskii functional (22), whose time derivative alone (10), (20), (23) is derived in (24).

(22)

 $V_{j,i} = \frac{1}{2} z_{j,i}^2 + V_{Uj,i}$ 

with

$$\begin{aligned} V_{Uj,i} &= \sum_{k=1}^{i} \int_{t-\tau_{j,k}}^{t} \frac{1}{2} \Big[ \mathcal{Q}_{j,k}^{j,i}(x(s)) \Big]^2 \, ds + \sum_{p=1}^{i-1} \sum_{k=1}^{p} \int_{t-\tau_{j,k}}^{t} \Big[ \mathcal{Q}_{j,k}^{j,p}(x(s)) \Big]^2 \, ds \\ \dot{V}_{Uj,i} &= U_{j,i} - \sum_{k=1}^{i} \frac{1}{2} \Big[ \mathcal{Q}_{j,k}^{j,i}(x_{\tau,j,k}) \Big]^2 - \sum_{p=1}^{i-1} \sum_{k=1}^{p} \Big[ \mathcal{Q}_{j,k}^{j,p}(x_{\tau,j,k}) \Big]^2 \ (23) \\ \text{where, } U_{j,i} &= \sum_{k=1}^{i} \frac{1}{2} \Big[ \mathcal{Q}_{j,k}^{j,i}(x_{j,k}) \Big]^2 + \sum_{p=1}^{i-1} \sum_{k=1}^{p} \Big[ \mathcal{Q}_{j,k}^{j,p}(x_{j,k}) \Big]^2 \ . \\ \dot{V}_{j,i} &\leq z_{j,1} \Bigg[ f_{j,i} + g_{j,i} \alpha_{j,i+1} - y_{dj}^{(i)} + \sum_{p=1}^{i-1} K_{i-1,p}^{j} (f_{j,i-p} + g_{j,i-p} x_{j,i+1-p} - y_{dj}^{(i-p)}) \\ &+ \sum_{p=1}^{i-1} \frac{\partial \mathcal{F}_{j,i-1}}{\partial x_{j,p}} (f_{j,p} + g_{j,p} x_{j,p+1}) + \sum_{p=0}^{i-2} \frac{\partial \mathcal{F}_{j,i-1}}{\partial y_{dj}^{(p)}} \dot{y}_{dj}^{(i)} \end{aligned}$$

$$+ \sum_{k=1}^{i} \frac{1}{2} z_{j,i} + \sum_{p=1}^{i-1} \sum_{k=1}^{p} \frac{1}{2} z_{j,i} (K_{i-1,p}^{j})^{2} + \sum_{p=1}^{i-1} \sum_{k=1}^{p} \frac{1}{2} z_{j,i} (\frac{\partial \mathcal{F}_{j,i-1}}{\partial x_{j,p}})^{2} \right]$$

$$+ z_{j,i} g_{j,i} z_{j,i+1} + \sum_{k=1}^{i} \frac{1}{2} \left[ \mathcal{Q}_{j,k}^{j,i} (x_{j,k}) \right]^{2} - \sum_{p=1}^{i-1} \sum_{k=1}^{p} \left[ \mathcal{Q}_{j,k}^{j,p} (x_{j,k}) \right]^{2}$$

$$\leq z_{j,1} \left[ g_{j,i} F_{j,i} (\cdot) - y_{dj}^{(i)} + g_{j,i} \sum_{p=1}^{i-1} K_{i-1,p}^{j} (x_{j,i+1-p} - y_{dj}^{(i-p)}) + g_{j,i} \alpha_{j,i+1} \right]$$

$$+ z_{j,i} g_{j,i} z_{j,i+1} + \left[ 1 - 2 \tanh^{2} \left( \frac{z_{j,i}}{\eta_{j,i}} \right) \right] U_{j,i}$$

$$(24)$$

where,  $F_{j,i}(\cdot) = F_{j,i}(\overline{x}_{j,i}, \overline{y}_{dj}^{(i-1)})$  is defined in (25).

$$\begin{aligned} F_{j,i}(\cdot) &= g_{j,i}^{-1} \left\{ f_{j,i} + \sum_{p=1}^{i-1} K_{i-1,p}^{j} \left[ f_{j,i-p} + g_{j,i-p} x_{j,i+1-p} - y_{dj}^{(i-p)} - g_{j,i}(x_{j,i+1-l} - y_{dj}^{(i-l)}) + \frac{\partial \mathcal{F}_{j,i-1}}{\partial x_{j,p}} (f_{j,p} + g_{j,p} x_{j,p+1}) \right] \\ &+ \sum_{p=0}^{i-2} \frac{\partial \mathcal{F}_{j,i-1}}{\partial y_{dj}^{(p)}} \dot{y}_{dj}^{(p)} + \sum_{k=1}^{i} \frac{1}{2} z_{j,i} + \sum_{p=1}^{i-1} \sum_{k=1}^{p} \frac{1}{2} z_{j,i} \left[ K_{i-1,p}^{j} \right]^{2} \\ &+ \sum_{p=1}^{i-1} \sum_{k=1}^{p} \frac{1}{2} z_{j,i} \left[ \frac{\partial \mathcal{F}_{j,i-1}}{\partial x_{j,p}} \right]^{2} + \frac{2}{z_{j,i}} \tanh^{2} (\frac{z_{j,i}}{\eta_{j,i}}) U_{j,i} \end{aligned} \end{aligned}$$

$$(25)$$

Substituting (10) into (25), the inequality below can be obtained easily.

$$\dot{V}_{j,i} \leq -[k_{j,i}\underline{g}_j - (\overline{g}_{j,i} - 1)^2 y_{dj}^{(i)2}] z_{j,i}^2 + z_{j,i} g_{j,i} z_{j,i+1} + \frac{1}{4} + [1 - 2 \tanh^2(\frac{z_{j,i}}{\eta_{i,i}})] U_{j,i}$$
(26)

Now, considering (8) derives the following (27), which is very useful for the design procedure.

$$\begin{aligned} k_{j,i}(x_{j,i} - y_{dj}^{(i-1)}) + k_{j,i} \sum_{p=1}^{i-1} K_{i-1,p}^{j} [x_{j,i-p} - y_{dj}^{(i-1-p)}] \\ + \sum_{p=1}^{i-1} K_{i-1,p}^{j} [x_{j,i+1-p} - y_{dj}^{(i-p)}] \\ = K_{i,1}^{j} (x_{j,i} - y_{dj}^{(i-1)}) + k_{j,i} K_{i-1,i-1}^{j} (x_{j,1} - y_{dj}) + K_{i-1,1}^{j} (x_{j,i} - y_{dj}^{(i-1)}) \\ + \sum_{p=1}^{i-2} [(k_{j,i} K_{i-1,p}^{j} + K_{i-1,p+1}^{j}) (x_{j,i-p} - y_{dj}^{(i-1-p)})] \\ = \sum_{p=1}^{i} [K_{i,p}^{j} (x_{j,i+1-p} - y_{dj}^{(i-p)})]$$
(27)

It follows from (9), (10), (19) and (27) that

$$z_{j,i+1} = x_{j,i+1} - y_{kj}^{(i)} + \sum_{p=1}^{i-1} K_{i-1,p}^{j} (x_{j,i-p} - y_{dj}^{(i-1-p)}) + \mathcal{F}_{j,i-1}(\cdot)] + \sum_{p=1}^{i-1} K_{i-1,p}^{j} (x_{j,i+1-p} - y_{dj}^{(i-p)}) + F_{j,i}(\cdot)$$

$$= x_{j,i+1} - y_{dj}^{(i)} + \sum_{p=1}^{i} K_{i,p}^{j} [x_{j,i+1-p} - y_{dj}^{(i-p)}] + \mathcal{F}_{j,i}(\cdot)$$
(28)

where  $\mathcal{F}_{j,i}(\cdot) = \mathcal{F}_{j,i}(\overline{x}_{j,i}, \overline{y}_{dj}^{(i-1)}) = k_{j,i}\mathcal{F}_{j,i-1}(\cdot) + F_{j,i}(\cdot)$ .

**Step**  $(j, m_j)$ . In this step, the desired actual control (11) is firstly derived by the similar operation in step (j, i), then a RBF NNs is induced to compensate the sum of the unknown nonlinear terms.

Different from the step (j, i), the unknown functions  $f_{j,m_j}(\cdot), g_{j,m_j}(\cdot), h_{j,m_j}(\cdot)$  contain all state variables and the inputs of the previous subsystems. Thus, we select the Lyapunov-Krasovskii functional  $V_{j,m_j} = \frac{1}{2} z_{j,m_j}^2 + V_{Uj,m_j}$ . Using the desired actual control (11), the corresponding time derivative is

$$\dot{V}_{j,m_{j}} \leq -[k_{j,m_{j}}\underline{g}_{j} - (\overline{g}_{j,m_{j}} - 1)^{2} y_{dj}^{(m_{j})2}]z_{j,m_{j}}^{2} + \frac{1}{4} + [1 - 2 \tanh^{2}(\frac{z_{j,m_{j}}}{\eta_{j,m_{i}}})]U_{j,m_{j}}$$
(29)

with

$$U_{j,m_j} = \sum_{j=1}^{n} \sum_{k=1}^{m_j} \frac{1}{2} \Big[ \mathcal{Q}_{j,k}^{j,m_j}(x_{j,k}) \Big]^2 + \sum_{p=1}^{m_j-1} \sum_{k=1}^{p} \Big[ \mathcal{Q}_{j,k}^{j,p}(x_{j,k}) \Big]^2$$

 $F_{j,m_j}(\cdot)$  is constructed as (30)

$$F_{j,m_{j}}(\cdot) = g_{j,m_{j}}^{-1} \left\{ f_{j,m_{j}} + \sum_{p=1}^{m_{j}-1} K_{m_{j}-1,p}^{j} \left[ f_{j,m_{j}-p} + g_{j,m_{j}-p} x_{j,m_{j}+1-p} - y_{dj}^{(m_{j}-p)} - g_{j,m_{j}}(x_{j,m_{j}+1-l} - y_{dj}^{(m_{j}-l)}) + \frac{\partial \mathcal{F}_{j,m_{j}-1}}{\partial x_{j,p}}(f_{j,p} + g_{j,p} x_{j,p+1}) \right] + \sum_{p=0}^{m_{j}-2} \frac{\partial \mathcal{F}_{j,m_{j}-1}}{\partial y_{dj}^{(p)}} \dot{y}_{dj}^{(p)} + \sum_{j=1}^{n} \sum_{k=1}^{m_{j}} \frac{1}{2} z_{j,m_{j}} + \sum_{p=1}^{m_{j}-1} \sum_{k=1}^{p} \frac{1}{2} z_{j,m_{j}} [K_{m_{j}-1,p}^{j}]^{2} + \sum_{p=1}^{m_{j}-1} \sum_{k=1}^{p} \frac{1}{2} z_{j,m_{j}} [\frac{\partial \mathcal{F}_{j,m_{j}-1}}{\partial x_{j,p}}]^{2} + \frac{2}{z_{j,m_{j}}} \tanh^{2}(\frac{z_{j,m_{j}}}{\eta_{j,m_{j}}}) U_{j,m_{j}} \right\}$$

$$(30)$$

Substituting  $z_{j,m_j} = x_{j,m_j} - \alpha_{j,m_j}$  into (11), we obtain (31) by (27).

$$u_{j}^{*} = y_{dj}^{(m_{j})} - \sum_{p=1}^{m_{j}} K_{m_{j},p}^{j} [x_{j,m_{j}+1-p} - y_{dj}^{(m_{j}-p)}] - \mathcal{F}_{j,m_{j}}(\cdot)$$
(31)

According to (4) and Lemma1, a RBF NNs  $\hat{\mathcal{F}}_{j,m_j}(\cdot)$ with input vector  $\overline{Z}_j = [X^T, \overline{u}_{j-1}^T, \overline{y}_{dj}^{(m_j-1)T}]^T \in \Omega^{(n \times m+j-1+m_j) \times 1}$ , where  $\Omega^{(n \times m+j-1+m_j) \times 1}$  is a compact set, is introduced to approximate the  $\mathcal{F}_{j,m_j}(\cdot)$  with  $W_j$  being ideal constant weights. Considering Lemma1, we choose the following actual control law (32), the weights online learning law is designed as (33).

$$u_{j} = y_{dj}^{(m_{j})} - \sum_{p=1}^{m_{j}} K_{m_{j},p}^{j} [x_{j,m_{j}+1-p} - y_{dj}^{(m_{j}-p)}] - \hat{W}_{j}^{\mathsf{T}} S_{j}(\overline{Z}_{j}) \quad (32)$$
$$\dot{\hat{W}}_{j} = \Gamma_{j} [S_{j}(\overline{Z}_{j}) - \sigma_{j} \hat{W}_{j}] \quad (33)$$

where  $\Gamma_j = \Gamma_j^T > 0$ ,  $\sigma_j > 0$  are the design matrixes.

# C. Stability Analysis

In this section, we state the main result in this note as follows.

**Theorem1.** Consider the closed-loop system consisting of the nonlinear MIMO time-delay system (1) satisfying Assumption 1-3, the controller (32) and the concise adaptive law (33). For all initial conditions satisfying  $\sum_{j=1}^{n} \sum_{i=1}^{m_j} V_{j,i} + \sum_{j=1}^{n} \frac{1}{2} g_{j,m_j} \tilde{W}_j^{\mathsf{T}} \Gamma_j^{-1} \tilde{W}_j \le 2\Delta$ , with any  $\Delta > 0$ , one can tune the controller parameters  $k_{j,1}, \dots k_{j,m_j}$ ,  $\Gamma_j$  and  $\sigma_j$  such that all the signals in the system are semi-global uniformly ultimately bounded (SGUUB).

*Proof:* Consider the following Lyapunov function candidate.

$$V = \sum_{j=1}^{n} \sum_{i=1}^{m_j} \left(\frac{1}{2} z_{j,i}^2 + V_{Uj,i}\right) + \sum_{j=1}^{n} \frac{1}{2} g_{j,m_j} \tilde{W}_j^{\mathrm{T}} \Gamma_j^{-1} \tilde{W}_j$$
(34)

Form Young's inequality, it is worth mentioning that

$$z_{j,i}g_{j,i}z_{j,i+1} \leq \overline{g}_{j,i}z_{j,i}^{2} + \frac{1}{4}\overline{g}_{j,i}z_{j,i+1}^{2},$$

$$\delta_{j}z_{j,m_{j}}g_{j,m_{j}} \leq \overline{g}_{j,m_{j}}^{2}z_{j,m_{j}}^{2} + \frac{\overline{\delta}_{j}^{2}}{4},$$

$$-\tilde{W}_{j}^{T}S_{j}(\overline{Z}_{j})z_{j,m_{j}}g_{j,m_{j}} + \tilde{W}_{j}^{T}S_{j}(\overline{Z}_{j})z_{j,1}g_{j,m_{j}}$$

$$\leq \overline{g}_{j,m_{j}}S_{j}^{*2}z_{j,m_{j}}^{2} + \overline{g}_{j,m_{j}}S_{j}^{*2}z_{j,1}^{2} + \frac{\overline{g}_{j,m_{j}}}{2}\widetilde{W}_{j}^{T}\widetilde{W}_{j}$$
(35)

Based on (17), (26), (29), and (35), the time derivative of (34) is

$$\begin{split} \dot{V} &\leq -\sum_{j=1}^{n} [k_{j,1} \underline{g}_{j} - (\overline{g}_{j,1} - 1)^{2} \dot{y}_{dj}^{2} - \overline{g}_{j,1} - \overline{g}_{j,m_{j}} S_{j}^{*2}] z_{j,1}^{2} \\ &- \sum_{j=1}^{n} \sum_{i=2}^{m_{j}-1} [k_{j,i} \underline{g}_{j} - (\overline{g}_{j,i} - 1)^{2} y_{dj}^{(i)2} - \frac{5\overline{g}_{j,i}}{4}] z_{j,i}^{2} \\ &- \sum_{j=1}^{n} [k_{j,m_{j}} \underline{g}_{j} - (\overline{g}_{j,m_{j}} - 1)^{2} y_{dj}^{(m_{j})2} - \frac{\overline{g}_{j,m_{j}}}{4} - \overline{g}_{j,m_{j}} S_{j}^{*2} - \overline{g}_{j,m_{j}}^{2}] z_{j,1}^{2} (36) \\ &- \sum_{j=1}^{n} (\frac{\sigma_{j} \underline{g}_{j}}{2} - \frac{\overline{g}_{j,m_{j}}}{2}) \widetilde{W}_{j}^{T} \widetilde{W}_{j} + \sum_{j=1}^{n} (\frac{m_{j}}{4} + \frac{\overline{\delta}_{j}^{2}}{4} + \frac{\sigma_{j} \overline{g}_{j,m_{j}}}{2} \|W\|^{2}) \\ &+ \sum_{j=1}^{n} \sum_{i=1}^{m_{j}} [1 - 2 \tanh^{2}(\frac{z_{j,i}}{\eta_{j,i}})] U_{j,i} \\ &\text{Letting,} \end{split}$$

$$\begin{aligned} k_{j,1} &= \underline{g}_{j}^{-1} [\alpha_{j0} + (\overline{g}_{j,1} - 1)^{2} \dot{y}_{dj}^{2} + \overline{g}_{j,1} + \overline{g}_{j,m_{j}} S_{j}^{*2} ], \\ k_{j,i} &= \underline{g}_{j}^{-1} [\alpha_{j0} + (\overline{g}_{j,i} - 1)^{2} y_{dj}^{(i)2} + \frac{\overline{Sg}_{j,i}}{4} ], \\ k_{j,m_{j}} &= \underline{g}_{j}^{-1} [\alpha_{j0} + (\overline{g}_{j,m_{j}} - 1)^{2} y_{dj}^{(m_{j})2} + \frac{\overline{g}_{j,m_{j}}}{4} + \overline{g}_{j,m_{j}} S_{j}^{*2} + \overline{g}_{j,m_{j}}^{2} ] \\ \alpha_{j} &= \min \left\{ 2\alpha_{j,0}, \ \frac{\sigma_{j} - 1}{\lambda_{\max}(\Gamma_{j}^{-1})} \right\}, \ \varepsilon_{j} &= \frac{m_{j}}{4} + \frac{\overline{\delta}_{j}^{2}}{4} + \frac{\sigma_{j}\overline{g}_{j,m_{j}}}{2} \|W\|^{2} \\ \alpha &= \min_{1 \leq j \leq n} \{\alpha_{j}\}, \ \varepsilon = \sum_{j=1}^{n} \varepsilon_{j} \end{aligned}$$

with  $\alpha_{i0}$  being positive constant, (36) finally becomes

$$\dot{V} \le -\alpha V + \varepsilon + \sum_{j=1}^{n} \sum_{i=1}^{m_j} [1 - 2 \tanh^2(\frac{z_{j,i}}{\eta_{j,i}})] U_{j,i}$$
(37)

Thus, by (37) the SGUUB stability follows immediately from the same line used in the proof of Theorem1 in [9, 16]. The proof is completed.

**Remark3:** It can be observed from (32) and (33) that the proposed control scheme is concise and with less computational burden. Different from the current literatures, the actual control law and the adaptive law are constructed only by the state variables, the reference signals and their derivatives, independent of the designed virtual control and the other intermediate variables. Only n(equal to the number of system outputs) neural networks are introduced to compensate for the sum of all the uncertainties, regardless of the number of system orders. Thus, the problems "explosion of complexity" and "curse of dimensionality" are circumvented from the root causes. That is a novel "minimum learning networks (MLN)" technique.

#### **IV. SIMULATION EXAMPLE**

In this section, a simulation example is presented to illustrate the effectiveness of the proposed control scheme. For comparison, we consider the following uncertain nonlinear MIMO time-delay system, which has been employed in [16].

$$\begin{cases} \dot{x}_{1,1} = -x_{1,1} + (1 + \sin^{2}(x_{1,1}))x_{1,2} + x_{r1,1}^{2} \\ \dot{x}_{1,2} = x_{1,1}x_{1,2} + x_{2,1} + x_{2,2} \\ + (1 + \sin^{2}(x_{1,1}) + 0.5\cos^{2}(x_{2,2}))u_{1} + x_{r1,2} \\ \dot{x}_{2,1} = -x_{2,1} + x_{2,2} + x_{r2,1} \\ \dot{x}_{2,2} = (x_{1,2} + x_{2,1})x_{2,2} - x_{1,1}u_{1} \\ + (2 + \sin^{2}(u_{1}) - \sin(x_{2,1}x_{2,2} - x_{1,1}))u_{2} + x_{r1,1}x_{r2,2} \end{cases}$$
(38)

where  $x_{\tau_{j,i}} = x_{j,i}(t - \tau_{j,i})$ , for j = 1, 2, i = 1, 2, and  $\tau_{1,1} = 2, \tau_{1,2} = 1.5, \tau_{1,2} = 0.5, \tau_{1,2} = 1$ .

The reference signals are assumed to be (39). In the simulation, letting the signals  $y_{dj}^{(i)}$ , i = 0,1,2 pass through a first order tilter  $\tau_i \dot{y}_{rj}^{(i)} + y_{rj}^{(i)} = y_{dj}^{(i)}$ ,  $\tau_i = e^{-5t} +0.001$ , the  $y_{rj}^{(i)}$  is for the control design.

$$y_{d1}(t) = 0.5(\sin(t) + \sin(0.5t)),$$
  

$$y_{d2}(t) = 0.5\sin(t) + \sin(0.5t)$$
(39)

According to the control scheme in this note, the concise adaptive neural controller and the adaptive laws are as follows.

$$u_{1} = \ddot{y}_{r1} - (k_{1,1} + k_{1,2})(x_{1,2} - \dot{y}_{r1}) - k_{1,1}k_{1,2}(x_{1,1} - y_{r1}) - W_{1}^{T}S_{1}(Z_{1})$$

$$u_{2} = \ddot{y}_{r2} - (k_{2,1} + k_{2,2})(x_{2,2} - \dot{y}_{r2}) - k_{2,1}k_{2,2}(x_{2,1} - y_{r2}) - \hat{W}_{2}^{T}S_{2}(\overline{Z}_{2})$$

$$\dot{\hat{W}}_{1} = \Gamma_{1} \Big[ S_{1}(\overline{Z}_{1})z_{1,1} - \sigma_{1}\hat{W}_{1} \Big], \quad \overline{Z}_{1} = [X^{T}, y_{r1}, \dot{y}_{r1}]$$

$$\dot{\hat{W}}_{2} = \Gamma_{2} \Big[ S_{2}(\overline{Z}_{2})z_{2,1} - \sigma_{2}\hat{W}_{2} \Big], \quad \overline{Z}_{2} = [X^{T}, u_{1}, y_{r1}, \dot{y}_{r1}]$$
(40)

In this simulation, the initial condition is  $x_{1,1}(t_0) = 0.5, x_{1,2}(t_0) = 0.1, x_{2,1}(t_0) = 0.3, x_{2,2}(t_0) = -0.15$ . The corresponding control parameters are taken as  $k_{1,1} = k_{1,2} = 30, k_{2,1} = k_{2,2} = 20, \Gamma_1 = \text{diag}\{1.0\}, \Gamma_2 = \text{diag}\{0.5\},$ 

 $\sigma_1 = 0.5$ ,  $\sigma_2 = 0.2$ . The RBF NNs in (40) includes 25 nodes, with centers  $\mu_p$  spaced in  $[-2.5, 2.5]^{\times 6}$  for  $u_1$  and  $[-2.5, 2.5]^{\times 7}$  for  $u_2$ , and widths  $\eta = 5$ .



Figure 1. (a) The first reference signal  $y_{d1}$  (solid line) and the subsystem output  $y_1$  (dashed line); (b) The second reference signal  $y_{d2}$  (solid line) and the subsystem output  $y_2$  (dashed line).

Simulation results are shown in Figs. 1-3. Fig. 1(a) and 1(b) present the response curves of system outputs  $y_1$ ,  $y_2$  and the reference signals  $y_{d1}$ ,  $y_{d2}$ . Fig.2 show that the control efforts of the concise adaptive controllers are in a

reasonable range. Fig. 3 illustrates the  $L_2$  norms of the NNs weights adaptation. Comparing with the results [16], the results illustrate the control performance of the proposed scheme.



Figure 2. (a) The control  $u_1$  of the first subsystem; (b) the control  $u_2$  of the second subsystem.



Figure 3.  $L_2$  norms of the NNs weights adaptation:  $\|\hat{W}_1\|$  (solid line) and  $\|\hat{W}_2\|$  (dashed line).

# V. CONCLUSION

In this note, the tracking control problem of uncertain nonlinear MIMO system with unknown time delays is addressed. A novel concise adaptive neural control scheme is developed with the "MLN" technique, which obtain some advantages: a concise structure and ease to implementation in control engineering, due to its computational burden. Both the problems of "explosion of complexity" and the "curse of dimensionality" are circumvented from the root causes, which are everpresented in the current literatures on approximationbased adaptive control.

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