Stochastic Stability Analysis and Control of Networked Control Systems with Multiple Markovian Processes

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Abstract—**A novel stochastic system model with multiple Markovian processes for networked control system (NCS) is developed as a two chains Markovian jump linear system. By using an active sampling method, random time delays are reshaped into short time delays which are less than one sampling period such that the structure of the augmented discrete-time model becomes straightforward. Based on Lyapunov approach, sufficient conditions for the stochastic stability of the networked control system are derived and mode-dependent controller is designed in terms of linear matrix inequalities (LMIs) to overcome the adverse influences of stochastic time delays and packet dropouts encountered in network-based distributed real-time control. Gridding approach is introduced to guarantee the solvability of the LMIs with finite jump modes. A numerical example is given to illustrate the effectiveness of the proposed method.**

Index Terms—**networked control system, stochastic stability, markov chain, time delay, packet dropout**

I. INTRODUCTION

Networked control systems (NCSs) are distributed feedback control systems closed via a shared band limited digital communication network, which connects sensor nodes, actuator nodes and controller nodes together to exchange information and control signals [1-3]. Due to the advantages of low installation cost, reduced wiring, easy maintenance and good system flexibility, NCSs have been widely used in manufacturing systems, monitoring system, vehicle highway systems, aircraft systems, teleportation of robots, intelligent control systems of vehicles. Despite lots of advantages the network brings to the control system, potential issues such as network induced time delays and packet dropouts arise that may degrade system performance and even cause system instability [4-6].

Many researchers have studied stability criteria and stabilizing controller design method for networked control systems in the presence of network induced delays and packet dropouts. The stability problem of NCSs with short time delays is studied and the constant stabilizing state feedback gain is obtained [7]. Time-based time delay analysis of the NCS is provided to explain how it affects network systems and an adaptive Smith predictor control scheme is designed [8]. A switched system approach was used to study the stability of networked control systems and optimal gain is calculated for stabilizing controller design [9]. Time delay is considered in an independent layer to design a stabilizing controller based on model predictive control approach [10]. Uncertain long-delayed systems are considered and a robust controller is designed on a unique structure [11]. The maximum allowable delay bounds are obtained for the stability of NCSs and are used as the basic parameters for a scheduling method for NCSs [12]. With the development of stochastic control theory, time delays and packet dropouts are considered as stochastic parameters of NCS models in many research works. Time delays are considered as independent distributed random variable [13]. The network-induced random delays are modeled as Markov chains such that the closed-loop system is a jump linear system with one mode [14-17]. The dropout process is modeled as an identically independently distributed process of NCS [18]. NSCs with packet dropouts are modeled as discrete Markov jump system [19-22]. It is noticed that time delays and packet dropouts are considered separately in the papers mentioned above. The stochastic stability problem of NCSs is only driven by one Markov chain. So far, the stability synthesis for the NCSs with time delays and packet dropouts as a Markovian jump system with two Markov chains has not been fully investigated.

In this paper, the stochastic stability problem of NCSs with random time delays and packet dropouts is investigated. By considering delay characteristics and dropout process simultaneously, the state-feedback

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closed-loop NCS is modeled as a discrete-time jump system characterized by two different Markov chains. An active time-varying sampling method [23] is proposed in this paper to make sure time delay always shorter than one sampling period. Based on the Lyapunov stability theory, sufficient conditions for the stochastic stabilization of the NCS with packet loss and time-varying delays are obtained and the mode-dependent stabilizing controller for the closed-loop NCS is designed in the linear matrix inequalities (LMIs) formulation via the Shur complement theory. A "gridding" approach [24] is introduced to obtain the finite combination of time delays and packet dropouts which ensures the feasibility of the constructed LMIs.

The remainder of this paper is organized as follows. In section 2, the description of problem is presented and the NCS model by using active time-varying sampling method is proposed. In section 3, sufficient conditions for stochastic stability of the NCSs are provided and the mode-dependent stabilizing controller is designed in section 4. Simulations and results are present in section 5 and conclusions are given in section 6.

Notation: The notation used throughout the paper is fairly standard. $\boldsymbol{A}^{\boldsymbol{T}}$ represents the transpose of matrix $\boldsymbol{A}$, the notation $\boldsymbol{P} > \boldsymbol{0}\ (\boldsymbol{P} < \boldsymbol{0})$ means that P is positive definite (negative definite), $\boldsymbol{I}$ and $\boldsymbol{0}$ represent identity matrices and zero matrices with appropriate dimensions, respectively. * denotes the entries of matrices implied by symmetry. $\lambda_{\max}(\boldsymbol{P})(\lambda_{\min}(\boldsymbol{P}))$ denotes the maximal (minimal) eigenvalue of matrix $\boldsymbol{P}$; $diag\{\cdots\}$ stands for a block-diagonal matrix; $E[\bullet]$ stands for the mathematical expectation; $\|\bullet\|$ denotes the standard norm. Matrices, if not explicitly stated, are assumed to have appropriate dimensions.

II. Problem Formulation

Consider a linear time-invariant plant described by

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the input vector. $\boldsymbol{A}, \boldsymbol{B}$ are constant matrices of appropriate dimensions.

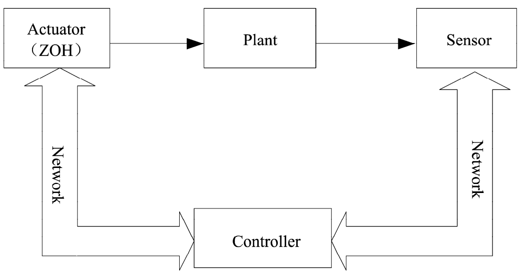

Figure 1. The structure of NCS

We consider the case where sensor nodes, actuator nodes and controller nodes are sharing use of network to exchange data as shown in Fig. 1. Actuator accepts the digital signal from sensor via some feedback calculated by controller. Data collisions and communication failure will inevitably happen because of limited capacity of transmission channel. Thus, time delays and packet dropouts should be taken into account to design a network controller for NCS. If constant sampling period is adopted, time delays may be shorter or longer than one sampling period which will make NCS modeling more complicated.

In the following, the sampling period will be set time varying to make sure time delay is shorter than one sampling period. In order to achieve this goal, sensor is assumed both time-driven and event-driven. Actuator and controller are event-driven. Suppose time axis is partitioned into equidistant small intervals and the length of each interval is l. Define t_k as the kth updating instant of actuator, and assume that total transmission delay from sensor to actuator of the updating signal at the instant t_k is τ_k. Then the next sampling instant can be selected as

$$s_{k+1} = \begin{cases} s_k + al & t_k \in [s_k + (a-1)l,\ s_k + al) \\ s_k + \tau & t_k \geq s_k + \tau \end{cases} \tag{2}$$

where s_k is the kth sampling instant, τ is the allowable maximum delay from sensor to actuator ($\tau = \eta l$, η is the bound positive integer of delay), a is a positive integer and $0 < a < \eta$.

Fig. 2 shows the sampling and updating conditions of NCS. if the transmission time of sampled signal at time s_k is less than τ, the actuator will be updated by the signal and the sensor will be driven to do the next sampling, which is called an effective sampling instant because the signal at this sampling instant is successfully transmitted from sensor to actuator, such as s_1 and s_2 marked in Fig. 2; if the signal sampled at time s_k has not arrived before the maximal allowable updating time $s_k + \tau$, which means the total transmission delay is out of τ, the signal will be discarded and the sensor will adopt time-driven mode, such as s_3 marked in Fig. 2; if packet dropout happened to the sampled signal, which can be seen as a long delay packet, the time-driven mode will adopted by sensor to do the next sampling, such as s_6 marked in Fig. 2.

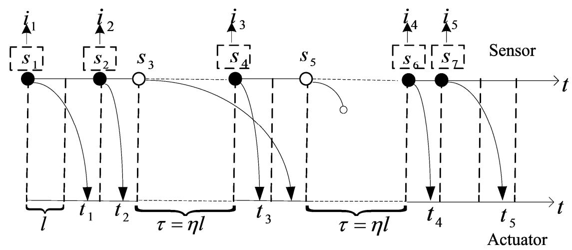

Figure 2. Packet transmission conditions of NCS

Let us denote the effective sampling instant as i_m and assume $U = \{i_1, i_2, i_3, \ldots\}$, which have been marked in Fig. 2; correspondingly, updating instants are denoted as $S = \{t_1, t_2, t_3, \ldots\}$. Between two updating instants, actuator operates in a zero order hold (ZOH) fashion, meaning that the value of control signal remains constant during the interval $[t_k, t_{k-1})$. Therefore, we have

$$u(t) = u(i_k) = \boldsymbol{K(i_k)} x(i_k), \quad t_k \le t < t_{k+1} \tag{3}$$

with t_{k+1} being the next updating instant of the ZOH after t_k.

Note that state-feedback control gains in (3) are not constant but varying with every updating interval $[t_k, t_{k-1})$ to ensure the stability of the time varying sampled system.

Based on the selection principle of sampling instant proposed above, the time delay between sensor and actuator is ensured to be shorter than one sampling interval. Suppose h_k as the length of interval between two successive effective sampling instants i_k and i_{k+1}, which is denoted as effective sampling period at sampling instant i_k. Thus, the discrete time representation of (1) can be described as follows

$$x(i_{k+1}) = \boldsymbol{\Phi_k} x(i_k) + \boldsymbol{\Gamma_0}(\tau_k, h_k) u(i_k) + \boldsymbol{\Gamma_1}(\tau_k, h_k) u(i_{k-1}) \tag{4}$$

where $\boldsymbol{\Phi_k} = e^{Ah_k}$,

$$\boldsymbol{\Gamma_0}(\tau_k, h_k) = \int_0^{h_k - \tau_k} e^{As} \boldsymbol{B} ds, \ \boldsymbol{\Gamma_1}(\tau_k, h_k) = \int_{h_k - \tau_k}^{h_k} e^{As} \boldsymbol{B} ds.$$

Let us introduce a new augmented state $z(k) = \left[x(i_k) \quad u(i_{k-1}) \right]^T$. Therefore, from (3) and (4), we can get the following augmented closed-loop system

$$z(k+1) = \boldsymbol{\Psi_k} z(k) \tag{5}$$

where

$$\boldsymbol{\Psi_k} = \begin{bmatrix} \boldsymbol{\Phi_k} + \boldsymbol{\Gamma_0}(\tau_k, h_k) \boldsymbol{K}(i_k) & \boldsymbol{\Gamma_1}(\tau_k, h_k) \\ \boldsymbol{K}(i_k) & 0 \end{bmatrix} \tag{6}$$

Remark1: It is important to note that i_k in (6) refers to the *effective sampling instant*. Only packets sampled at that instant successfully transmits from sensor to actuator. Since time varying sampling principle is adopted based on (2), ZOH will be updated only once between two *effective sampling instants* by control signal. Thus, during any sampling interval, only two control signals will be acted on the plant: $u(i_{k-1})$ and $u(i_k)$.

III. STOCHASTIC STABILITY OF NCS

Define d_k as the number of dropped packet between two successive *effective updating instants* i_k and i_{k+1}, then we can get $i_{k+1} - i_k = d_k + 1$. If assume the bound of consecutive dropped packets is d, we can conclude that d_k takes value from a finite set $\Omega = \{0, 1, \cdots, d\}$.

In section 2, we defined the allowable maximum delay from sensor to actuator as τ. Based on the selection principle of sampling instants proposed above, it can be supposed that as soon as the packet reaches actuator and the time delay is less than τ, the next sampling event is driven at sensor, and if the time delay is longer than τ, the packet will be discarded and the sensor will be driven to do the next sampling. Therefore, the bound of τ_k ($\tau = al$) is τ ($\tau = \eta l$) and a takes value from a finite set $\mathrm{M} = \{1, 2, \cdots, \eta\}$, and then τ_k takes value from the finite set $\mathrm{T} = \{1l, 2l, \cdots, \eta l\}$.

In this paper, we assume that random delays τ_k and packet dropouts d_k are two independent Markov chains that take values in Ω and M with the following transition probabilities

$$\begin{aligned} \omega_{mi} &= \Pr(\tau_{k+1} = il \mid \tau_k = ml), \quad \forall i, m \in \mathrm{M} \\ \lambda_{nj} &= \Pr(d_{k+1} = j \mid d_k = n), \quad \forall j, n \in \Omega \end{aligned} \tag{7}$$

where $\omega_{mi}, \lambda_{nj} \ge 0$, and

$$\sum_{i=1}^{\eta} \omega_{mi} = 1, \ \sum_{j=0}^{d} \lambda_{nj} = 1 \tag{8}$$

The transition probability matrixes are defined by

$$\begin{aligned} \boldsymbol{\Theta} &= \begin{bmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1\eta} \\ \omega_{21} & \omega_{22} & \cdots & \omega_{2\eta} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{\eta 1} & \omega_{\eta 2} & \cdots & \omega_{\eta\eta} \end{bmatrix}, \\ \boldsymbol{\Pi} &= \begin{bmatrix} \lambda_{01} & \lambda_{02} & \cdots & \lambda_{0d} \\ \lambda_{10} & \lambda_{12} & \cdots & \lambda_{1d} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{d0} & \lambda_{d1} & \cdots & \lambda_{dd} \end{bmatrix} \end{aligned} \tag{9}$$

Remark2: Random delays and packet dropouts are two independent Markov chains, the value of next state is only related with the value of current state. In other words, if we know the values of time delays τ_k and packet dropouts d_k at instant i_k, then we can obtain the value of τ_{k+1} and d_{k+1} at instant i_{k+1}.

Based on the above assumptions, the *effective sampling period* h_k at *effective sampling instant* i_k is also a variable related to two Markov process, which can be written into:

$$h_k = \tau_k + \tau d_k \tag{10}$$

Therefore, the values of $\boldsymbol{\Phi_k}$, $\boldsymbol{\Gamma_0}(\tau_k, h_k)$ and $\boldsymbol{\Gamma_1}(\tau_k, h_k)$ are finally determined by τ_k and d_k, system (5) can be seen as a discrete-time Markovian jump linear system with finite jump modes varying in a finite set which is combined by sets T and Ω. Let us define a compact set I to denote jump modes of (5), then we can get $I = \{1, 2, 3 ..., \eta * (d+1)\}$. Define $\hat{A}(m, n)$ as the jump modes determined by $\tau_k = m$ and $d_k = n$, $\boldsymbol{K}(m,n)$ as the mode-dependent state feedback controller gain, then augmented system (5) can be transformed into a discrete-time Markovian jump linear system

$$z(k+1) = \hat{\boldsymbol{A}}(m,n)z(k), \quad \forall m \in \mathrm{M}, n \in \Omega \tag{11}$$

where

$$\hat{\boldsymbol{A}}(m,n) = \begin{bmatrix} \boldsymbol{\Phi}(m,n) + \boldsymbol{\Gamma_0}(m,n)\boldsymbol{K}(m,n) & \boldsymbol{\Gamma_1}(m,n) \\ \boldsymbol{K}(m,n) & \boldsymbol{0} \end{bmatrix} \tag{12}$$

Remark3: By introducing the "gridding" approach, the original time-varying discrete system is transformed into a discrete-time Markovian jump linear system with finite jump modes. Therefore, the stability problem of NCS can be managed with the help of the Markovian jump linear system control theory and the controller gain will be designed to be dependent on τ_k and d_k from the finite sets T and Ω.

Definition 1: The system (11) is stochastically stable if for every initial state $z_0 = z(0)$ and initial distributions $\tau_0 = \tau(0) \in \mathrm{T}$ and $d_0 = d(0) \in \Omega$, there exists a finite matrix $\boldsymbol{Q} > 0$ such that

$$E(\sum_{k=0}^{\infty} \|z(k)\|^2 \mid z_0, \tau_0, d_0) < z_0^T \boldsymbol{Q} z_0$$

With Definition 1, the necessary and sufficient conditions on the stochastic stability of closed-loop system (11) can be obtained.

Theorem 1: If there exists symmetric positive definite matrices $\boldsymbol{X}(m,n) > 0, m \in M, n \in \Omega$ satisfying

$$\begin{bmatrix} \boldsymbol{X}(m,n) & \boldsymbol{X}(m,n)\boldsymbol{U}(m,n) \\ \boldsymbol{U^T}(m,n)\boldsymbol{X^T}(m,n) & \boldsymbol{\Lambda} \end{bmatrix} > 0, \quad m = 1,2,\ldots,\eta,\ n = 0,1,2,\ldots,d \tag{13}$$

where

$$\begin{aligned} &\boldsymbol{U}(m,n) = [\sqrt{\omega_{m1}\lambda_{n0}}\hat{\boldsymbol{A}}^{\boldsymbol{T}}(1,0)\cdots\sqrt{\omega_{m\eta}\lambda_{nd}}\hat{\boldsymbol{A}}^{\boldsymbol{T}}(\eta,d)] \\ &\boldsymbol{\Lambda} = diag\{\boldsymbol{X}(1,0),\cdots,\boldsymbol{X}(\eta,d)\} \end{aligned} \tag{14}$$

Then the system (11) is stochastically stable.

Proof: Consider the following form of the Lyapunov function:

$$V(k) = z^T(k)\boldsymbol{P}(m,n)z(k) \tag{15}$$

where

$$\boldsymbol{P}(m,n) = \boldsymbol{X^{-1}}(m,n) \tag{16}$$

Then we have

$$\begin{aligned} E(\Delta V) &= E(V(k+1) - V(k)) \\ &= E(z^T(k+1)\boldsymbol{P}(\tau_{k+1}, d_{k+1})z(k+1) \mid z(k), \tau_k = m, d_k = n) \\ &\quad - z^T(k)\boldsymbol{P}(m,n)z(k) \\ &= \sum_{j=0}^{d}\sum_{i=1}^{\eta}\lambda_{nj}\omega_{mi}z^T(k)\hat{\boldsymbol{A}}^{\boldsymbol{T}}(i,j)\boldsymbol{P}(i,j)\hat{\boldsymbol{A}}(i,j)z(k) \\ &\quad - z^T(k)\boldsymbol{P}(m,n)z(k) \\ &= z^T(k)[\sum_{j=0}^{d}\sum_{i=1}^{\eta}\lambda_{nj}\omega_{mi}\hat{\boldsymbol{A}}^{\boldsymbol{T}}(i,j)\boldsymbol{P}(i,j)\hat{A}(i,j) - \boldsymbol{P}(m,n)]z(k) \\ &= z^T(k)V(m,n)z(k) \end{aligned} \tag{17}$$

where
$V(m,n) = \sum_{j=0}^{d}\sum_{i=1}^{\eta}\lambda_{nj}\omega_{mi}\hat{A}^T(i,j)\boldsymbol{P}(i,j)\hat{A}(i,j) - \boldsymbol{P}(m,n)$.

If $V(m,n) < 0$, then

$$\begin{aligned} E(\Delta V(k)) &= E(V(k+1) - V(k)) \\ &\le -\lambda_{\min}(-E(m,n))z^T(k)z(k) \\ &\le -\alpha z^T(k)z(k) = -\alpha \|z(k)\|^2 \end{aligned} \tag{18}$$

where $\alpha = \inf\{\lambda_{\min}(-E(m,n)), m \in M, n \in \Omega\} > 0$. From the previous inequality(18), for any integer $M \ge 1$, we have

$$E(V(z(M+1)) - E(V(z_0)) \le -\alpha E\{\sum_{k=0}^{M}\|z(k)\|\}$$

which implies

$$E\{\sum_{k=0}^{\infty}\|z(k)\|\} \le \frac{1}{\alpha}E(V(z_0)) = \frac{1}{\alpha}z_0^T\boldsymbol{P}(\tau_0, d_0)z_0$$

Thus, from Definition 1, if $V(m,n) < 0$, the system (11) is stochastically stable.

Define $\boldsymbol{H}(m,n) = diag\{\boldsymbol{P}(m,n), \boldsymbol{I_{10}}, \ldots, \boldsymbol{I_{\eta d}}\}$, and pre-multiply and post-multiply (13) by $\boldsymbol{H}(m,n)$, we get

$$\begin{bmatrix} \boldsymbol{P}(m,n) & \boldsymbol{U}(m,n) \\ U^T(m,n) & \boldsymbol{\Lambda} \end{bmatrix} > 0, \quad m = 0,1,2,\ldots,\eta,\ n = 0,1,2,\ldots,d \tag{19}$$

By Schur complement, (19) is equivalent to $V(m,n) < 0$, Therefore, the stochastic stability is obtained. This completes the proof.

IV. Stochastic Stabilizing Controller Design

This part will study the design of stabilizing controller for NCS.

Let us denote the following matrixes:

$$\begin{aligned} &\tilde{A}(m,n) = \begin{bmatrix} \boldsymbol{\Phi}(m,n) & \boldsymbol{\Gamma_1}(m,n) \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \\ &\tilde{\boldsymbol{B}}(m,n) = \begin{bmatrix} \boldsymbol{\Gamma_0}(m,n) \\ \boldsymbol{I} \end{bmatrix}, \\ &\tilde{\boldsymbol{K}}(m,n) = \begin{bmatrix} \boldsymbol{K}(m,n) & \boldsymbol{0} \end{bmatrix} \end{aligned} \tag{20}$$

Then, system (11) can be rewritten into the following form:

$$z(k+1)=[\tilde{\boldsymbol{A}}(m,n)+\tilde{\boldsymbol{B}}(m,n)\tilde{\boldsymbol{K}}(m,n)]z(k) \quad (21)$$

Theorem 2: If there exists symmetric positive definite matrices $\boldsymbol{G}(m,n)$ and $\boldsymbol{V}(m,n)$, matrices $\boldsymbol{R}(m,n)$ ($\forall m\in M, n\in\Omega$) satisfying

$$\begin{bmatrix} \boldsymbol{G}(m,n) & \boldsymbol{0} & \boldsymbol{\Xi_1} \\ * & \boldsymbol{V}(m,n) & \boldsymbol{\Xi_2} \\ * & * & \boldsymbol{Z} \end{bmatrix} > 0, \forall m\in M, n\in\Omega \quad (22)$$

and

$$\boldsymbol{\Xi_1}=\begin{bmatrix} \sqrt{\omega_{m1}\lambda_{n0}}\begin{bmatrix}\boldsymbol{\Phi}(1,0)\boldsymbol{G}(m,n)+\boldsymbol{\Gamma_0}(1,0)\boldsymbol{R}(m,n)\\ \boldsymbol{R}(m,n)\end{bmatrix}^T \\ \vdots \\ \sqrt{\omega_{m\eta}\lambda_{nd}}\begin{bmatrix}\boldsymbol{\Phi}(\eta,d)\boldsymbol{G}(m,n)+\boldsymbol{\Gamma_0}(\eta,d)\boldsymbol{R}(m,n)\\ \boldsymbol{R}(m,n)\end{bmatrix}^T \end{bmatrix}^T$$

$$\boldsymbol{\Xi_2}=\begin{bmatrix} \sqrt{\omega_{m1}\lambda_{n0}}\begin{bmatrix}\boldsymbol{\Gamma_1}(1,0)\boldsymbol{V}(m,n)\\ \boldsymbol{0}\end{bmatrix}^T \\ \vdots \\ \sqrt{\omega_{m\eta}\lambda_{nd}}\begin{bmatrix}\boldsymbol{\Gamma_1}(\eta,d)\boldsymbol{V}(m,n)\\ \boldsymbol{0}\end{bmatrix}^T \end{bmatrix}^T \quad (23)$$

$$\boldsymbol{Z}=diag\{diag\{\boldsymbol{G}(0,0),\boldsymbol{V}(0,0)\},\ \ldots,diag\{\boldsymbol{G}(\eta,d),\boldsymbol{V}(\eta,d)\}\}$$

Then the system (5) is stochastically stable and the mode-dependent state feedback controller is given by

$$\boldsymbol{K}(m,n)=\boldsymbol{R}(m,n)\boldsymbol{G}^{-1}(m,n), \quad m\in M, n\in\Omega \quad (24)$$

Proof: Assume that there exist symmetric positive definite matrices $\boldsymbol{G}(m,n)$ and $\boldsymbol{V}(m,n)$, matrices $\boldsymbol{R}(m,n)$ ($\forall m\in M, n\in\Omega$) such that (22) is satisfied. From (24) we get

$$\boldsymbol{R}(m,n)=\boldsymbol{K}(m,n)\boldsymbol{G}(m,n), \quad m\in M, n\in\Omega \quad (25)$$

Replacing $\boldsymbol{R}(m,n)$ in (23) by $\boldsymbol{K}(m,n)\boldsymbol{G}(m,n)$, and applying the Schur complement formula, we get

$$\begin{bmatrix}\boldsymbol{G}(m,n) & \\ & \boldsymbol{V}(m,n)\end{bmatrix}-\begin{bmatrix}\boldsymbol{G}(m,n) & \\ & \boldsymbol{V}(m.n)\end{bmatrix} \bar{U}(m,n)\boldsymbol{Z}^{-1}\bar{U}^T(m,n)\begin{bmatrix}\boldsymbol{G}(m,n) & \\ & \boldsymbol{V}(m.n)\end{bmatrix}^T>0 \quad (26)$$

where

$$\bar{U}(m,n)=[\sqrt{\omega_{m1}\lambda_{n0}}\hat{\boldsymbol{A}}^T(1,0)\cdots\sqrt{\omega_{m\eta}\lambda_{nd}}\hat{\boldsymbol{A}}^T(\eta,d)]$$
$$=\begin{bmatrix} \sqrt{\omega_{m1}\lambda_{n0}}\begin{bmatrix}(\boldsymbol{\Phi}(1,0)+\boldsymbol{\Gamma_0}(1,0)K(1,0))G(m,n)\\ K(1,0)G(m,n)\end{bmatrix}^T \\ \vdots \\ \sqrt{\omega_{m\eta}\lambda_{nd}}\begin{bmatrix}(\boldsymbol{\Phi}(\eta,d)+\boldsymbol{\Gamma_0}(\eta,d)K(\eta,d))\boldsymbol{G}(m,n)\\ K(\eta,d)\boldsymbol{G}(m,n)\end{bmatrix}^T \end{bmatrix}^T$$
$$=\begin{bmatrix} \sqrt{\omega_{m1}\lambda_{n0}}(\tilde{\boldsymbol{A}}(1,0)+\tilde{\boldsymbol{B}}(1,0)\tilde{\boldsymbol{K}}(1,0)) \\ \vdots \\ \sqrt{\omega_{m\eta}\lambda_{nd}}(\tilde{\boldsymbol{A}}(\eta,d)+\tilde{\boldsymbol{B}}(\eta,d)\tilde{\boldsymbol{K}}(\eta,d)) \end{bmatrix}^T \quad (27)$$

Let us denote

$$\boldsymbol{X}(m,n)=\begin{bmatrix}\boldsymbol{G}(m,n) & \\ & \boldsymbol{V}(m,n)\end{bmatrix} \quad (28)$$

then (26) can be written into

$$\boldsymbol{X}(m,n)-\boldsymbol{X}(m,n)\bar{U}(m,n)\boldsymbol{Z}^{-1}\bar{U}^T(m,n)\boldsymbol{X}^T(m,n)>0, \quad \forall m\in M, n\in\Omega \quad (29)$$

notice that

$$\begin{aligned}\boldsymbol{Z}&=diag\{diag\{\boldsymbol{G}(0,0),\boldsymbol{V}(0,0)\},\ \ldots,diag\{\boldsymbol{G}(\eta,d),\boldsymbol{V}(\eta,d)\}\}\\ &=diag\{\boldsymbol{X}(0,0),\cdots\boldsymbol{X}(\eta,d)\}\\ &=\boldsymbol{\Lambda}\end{aligned} \quad (30)$$

Therefore, according to Theorem 1, the system (5) is stochastically stable and the mode-dependent state feedback controller gain is given by $\boldsymbol{K}(m,n)=\boldsymbol{R}(m,n)\boldsymbol{G}^{-1}(m,n)$. This completes the proof.

Remark4: The conditions in Theorem 2 are a set of linear matrix inequalities (LMI) with normal format. The state feedback controller gain $\boldsymbol{K}(m,n)$ can be obtained by solving the LMIs directly without any transformation. The obtained controller gains are mode-dependent which is determined by time delay and packet dropout of that sampling period.

V. NUMERICAL EXAMPLE

To illustrate the effectiveness of the proposed methods for stochastic stability problem of NCS, simulation results are presented in this section. Consider the following system

$$\dot{x}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u \quad (31)$$

where u is the control input for the continuous-time linear plant. According to the driven mode and active time-varying sampling method proposed in this paper, the value of the state feedback controller for discrete-time plant during one effective sampling period should be $u(t)=u(i_k)=\boldsymbol{K}(\boldsymbol{i_k})x(i_k), \quad t_k\le t<t_{k+1}$.

Suppose the length of gridded equidistant small interval l is 0.05 ms ,the maximum time delays $\tau = 0.15ms$ and τ_k takes value from $\mathrm{T} = \{0.05ms, 0.10ms, 0.15ms\}$; for simplicity, we suppose the bound of consecutive dropped packets $d = 2$ and $d_k \in \{0,1,2\}$, and their transition probability matrices are given by

$$\boldsymbol{\Theta} = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.5 & 0.3 & 0.2 \\ 0.5 & 0.3 & 0.2 \end{bmatrix}, \boldsymbol{\Pi} = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.5 & 0.4 & 0.1 \\ 0.6 & 0.3 & 0.1 \end{bmatrix} \quad (32)$$

Table 1 shows the result of the mode-dependent controller gains determined by two Markovian jumping parameters.

TABLE I.
THE MODE-DEPENDENT CONTROLLER GAINS

	$\tau_1 = 0.01s$	$\tau_2 = 0.02s$	$\tau_3 = 0.03s$
$d_1 = 0$	$K(1,0)$	$K(2,0)$	$K(3,0)$
$d_2 = 1$	$K(1,1)$	$K(2,1)$	$K(3,1)$
$d_3 = 2$	$K(1,2)$	$K(2,2)$	$K(3,2)$

To obtain state feedback controller gains, we can use the Matlab LMI Control Toolbox to solve the LMI feasible problem presented in Theorem 2. The results are as follows:

$$\begin{aligned} \boldsymbol{K(1,0)} &= [\text{-}3.5641 \quad \text{-}3.7316]; \\ \boldsymbol{K(2,0)} &= [\text{-}4.2266 \quad \text{-}4.5611]; \\ \boldsymbol{K(3,0)} &= [\text{-}4.5710 \quad \text{-}5.1117]; \\ \boldsymbol{K(1,1)} &= [\text{-}2.3304 \quad \text{-}2.7167]; \\ \boldsymbol{K(2,1)} &= [\text{-}2.4199 \quad \text{-}2.9573]; \\ \boldsymbol{K(3,1)} &= [\text{-}2.4674 \quad \text{-}3.1717]; \\ \boldsymbol{K(1,2)} &= [\text{-}1.5520 \quad \text{-}2.0967]; \\ \boldsymbol{K(2,2)} &= [\text{-}1.5638 \quad \text{-}2.2365]; \\ \boldsymbol{K(3,2)} &= [\text{-}1.5650 \quad \text{-}2.3721]; \end{aligned} \quad (33)$$

The state trajectories of NCS with the feedback control law proposed in this paper. It can be seen that the networked control system is stochastically stable even if there exist time delays and packet dropouts.

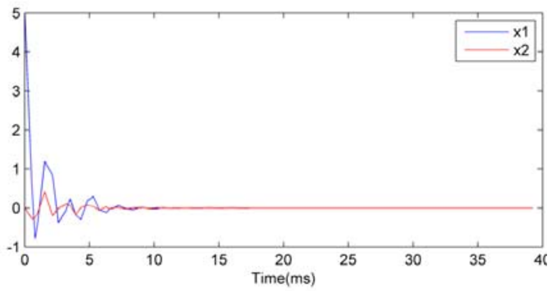

Figure 3. States trajectory of NCS

VI. CONCLUSIONS

This paper studies the stability problem of networked control system by modeling the random time delays and packet dropouts as two Markov chains. Active time varying sampling period strategy is proposed to make sure the time delay between sensor and actuator is shorter than one sampling interval, which conducts the closed-loop NCS as a Markovian jump linear system. Sufficient conditions of stochastic stability for the jump linear systems are given in terms of a set of LMIs. To solve the LMIs for obtaining feedback gains, the "gridding approach" is adopted to guarantee the LMIs set for the jump linear system with finite jump modes. It is shown the state feedback gain is mode-dependent. Numerical examples illustrate the effectiveness of the proposed strategy for the stochastic stabilizing controller over NCS.

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