A New Way to Generate High-Dimensional Hyperchaos

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Abstract—In this paper we described the process of generating a high-dimensional hyperchaos in signal communication and computer chaotic encryption. We found out four new high-dimensional complex hyperchaotic systems and three regular ways to generate high-dimensional hyperchaotic systems. Then we revealed the relationship between the low-dimensional chaos and high-dimensional hyperchaos and studied the features of phase space maps and Lyapunov exponents maps of high-dimensional complex hyperchaotic system. The results of theoretical analysis and experiment show that the new high-dimensional complex hyperchaotic system have strong chaotic features.

Index Terms—hyperchaos, high-dimensional chaos, complex chaotic system, Lyapunov exponents, phase space

I. INTRODUCTION

There are three important scientific discover in the 20th century. They are the theory of relativity, the chaos phenomenon and the quantum mechanics. The chaotic phenomenon is widespread in signal communication. There often were some noises in the signal communicating equipments. Those noises were some uncertain messy outputs waveform. In the past, they were generally considered to be due to the circuit to generate self-excited oscillation and noise. In fact, in many cases, this time the circuit was in a chaotic state. Therefore, to understand the chaotic phenomena and the regularity of produced high-dimensional chaotic phenomena in signal communication has important significance for the design of signal communicating equipment. Another new field of chaotic application is chaotic encryption. It is a new research direction in the computer communication security. To chaotic encryption, the more parameters and the higher dimension in the chaotic system, the better security it has. So study the regularity of generating a high-dimensional hyperchaotic system has practical significance, as detailed in reference [1-9].

II. NEW DUFFING-CHEN CHAOTIC SYSTEM

A. The Form of the Duffing Chaotic System is:

$$\begin{align*}
\dot{x} &= y \\
y &= -dx - x^3 + e \cos wt
\end{align*}$$

(1)

The parameter d is damping ratio, the parameter e is period amplitude, and the parameter w is circular frequency, as detailed in reference [10]. The form of the Chen chaotic system is as the following:

$$\begin{align*}
x &= a(y - x) \\
y &= (c - a)x - xz + cy \\
z &= xy - bz
\end{align*}$$

(2)

The parameters of a–c in the system are real constants.

B. Dissipative Analysis

$$\begin{align*}
\nabla V &= \frac{\partial \dot{x}}{x} + \frac{\partial \dot{y}}{y} + \frac{\partial \dot{z}}{z} + \frac{\partial \dot{u}}{u} + \frac{\partial \dot{v}}{v} + \frac{\partial \dot{w}}{w} \\
&= -a + h - c - d
\end{align*}$$

(3)

When a-h+c+d>0, the trajectories of the system are dissipative and converges to the exponential form:
\[ \frac{dV}{dt} = e^{-(a-b+c+d)} \]  

(5)

When \( t \to \infty \), the system trajectories will eventually be limited to a zero volume, as detailed in reference [11].

C. Lyapunov Exponent Analysis

For the equation (3), The i-th Lyapunov exponent is the direction axle length of the i-th coordinate:

\[ \sigma_i = \lim_{t \to \infty} \frac{1}{t} \ln \left( \frac{\|x_i(t) - x_i(t)\|}{\|x_i(0) - x_i(0)\|} \right) \]

(6)

When \( a=10, b=55, c=8/3, d=0.6, e=-3, f=1, g=3, h=1, x=1, y=1, z=1, u=1, v=1, w=1 \) and \( dt=0.005 \), the Lyapunov exponents are 0.970, 3.108, -1.468, -0.461, -3.574 and -10.829, the system is in the hyperchaotic state.

D. The Structural of the Phase Space

The x-y-z phase space is shown in Fig.1.(a) and the others phase spaces are shown in Fig.1. (b)-(h).

E. Parameters Range

In the baseline parameters of \( a=10, b=55, c=8/3, d=0.6, e=-3, f=1, g=3, h=1, x=1, y=1, z=1, u=1, v=1, w=1 \) and \( dt=0.005 \), the Lyapunov exponent states of the equation (3) with the parameters change is shown in Table I.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>States</th>
<th>Steady</th>
<th>Chaos</th>
<th>Hyperchaos</th>
</tr>
</thead>
<tbody>
<tr>
<td>a [0,30]</td>
<td>[0,1], [25.9, 30]</td>
<td>(24.5, 25.9)</td>
<td>(1, 24.5)</td>
<td></td>
</tr>
<tr>
<td>b [0,70]</td>
<td>[0, 18.8]</td>
<td>(18.8, 22.8)</td>
<td>(22.8, 70)</td>
<td></td>
</tr>
<tr>
<td>c [0,10]</td>
<td>[0,0.2], [8.7,10]</td>
<td>(0.2,0.5)</td>
<td>(0.5, 8.7)</td>
<td></td>
</tr>
<tr>
<td>d [0,6]</td>
<td>no</td>
<td>no</td>
<td>[0, 6]</td>
<td></td>
</tr>
<tr>
<td>e [-20,20]</td>
<td>[-0.2,0.1]</td>
<td>[-20, -0.2), (0.1, 20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f [-10,20]</td>
<td>[-10, -9]</td>
<td>(-9, -6.2], (-4,-3.5], (16.5,20]</td>
<td>[-6.2, -4], (-3.5, 16.5]</td>
<td></td>
</tr>
<tr>
<td>g [-40,40]</td>
<td>no</td>
<td>no</td>
<td>[-40, 40]</td>
<td></td>
</tr>
<tr>
<td>h [-10,10]</td>
<td>[-10, -5.2]</td>
<td>(-5.2, -2.3], [7.3, 10)</td>
<td>(-2.3, 7.3]</td>
<td></td>
</tr>
</tbody>
</table>

III. NEW DUFFING-LORENZ CHAOTIC SYSTEM

A. The Form of the Lorenz Chaotic System is:

The Lorenz chaotic system is as the following:

\[
\begin{align*}
\dot{x} &= a(y - x) \\
\dot{y} &= bx - xz - y \\
\dot{z} &= xy - cz
\end{align*}
\]

(7)

The parameters of \( a \)–\( c \) in the system are real constants. The external excitation part of the original Duffing system is replaced by an autonomous part. Then, by the bridge of the autonomous part in the Duffing chaotic system, the Lorenz chaotic system of the equation (7) and the Duffing chaotic system of the equation (1) are overlaid into a new Duffing-Lorenz complex hyperchaotic system:

\[
\begin{align*}
-2 & \leq u \leq 2, \\
-2 & \leq v \leq 2, \\
-2 & \leq w \leq 2, \\
-2 & \leq x \leq 2, \\
-2 & \leq y \leq 2, \\
-2 & \leq z \leq 2
\end{align*}
\]

(8)

The equation (8) is a new six-dimensional complex chaotic system. The parameters of \( a \)–\( h \) in the system are real constants.
B. Dissipative Analysis

\[
\nabla V = \frac{\partial x}{x} + \frac{\partial y}{y} + \frac{\partial z}{z} + \frac{\partial u}{u} + \frac{\partial v}{v} + \frac{\partial w}{w} = -a - b - e
\]

(9)

When the parameters change meet the condition of \(a+1+b+c<0\), the system of the equation (8) is dissipative and converges to the exponential form:

\[
\frac{dV}{dt} = e^{-(a+1+b+c)}
\]

(10)

When \(t \to \infty\), the trajectories of the system will eventually be limited to a zero volume.

C. Lyapunov Exponent Analysis

For the equation (8), the initial conditions is taken with a six-dimensional ball in all directions on the contraction or the expansion. The \(i\)-th Lyapunov exponent of the equation (8) is the direction axle length of the \(i\)-th coordinate. The \(i\)-th Lyapunov exponent \((i = 1, 2, 3, 4, 5, 6)\) is defined as:

\[
\sigma_i = \lim_{t \to \infty} \frac{1}{t} \ln \left| \frac{\partial x_i(x_0, t)}{\partial x_i(x_0, 0)} \right|
\]

(11)

When the initial conditions are \(a=10, b=55, c=8/3, d=0.6, e=-3, f=1, g=3, x=1, y=1, z=1, u=1, v=1, w=1\) and \(dt=0.005\), the Lyapunov exponents are 1.399, 0.852, 0.039, -0.090, -1.450 and -14.980. Since there are three positive Lyapunov exponents, the system of equation (8) is in the hyperchaotic state.

D. The Structural of the Phase Space

The phase spaces of the equation (8) are shown in Fig.2, (a)-(e).

E. Parameters Range

Lyapunov exponent is a parameter which provides a quantitative description for chaotic system dynamics. When the parameters change, the system will be in a different state. In the baseline parameters of \(a=10, b=55, c=8/3, d=0.6, e=-3, f=1, g=3, x=1, y=1, z=1, u=1, v=1, w=1\) and \(dt=0.005\), the Lyapunov exponent states with the parameters change is shown in Table II.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>States</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steady</td>
<td>Chaos</td>
<td>Hyperchaos</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a ([0,40])</td>
<td>([0,3])</td>
<td>([3,4])</td>
<td>([4,40])</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>b ([0,70])</td>
<td>([0,21])</td>
<td>([21,27])</td>
<td>([27,70])</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c ([0,40])</td>
<td>([0.8,6.4,10])</td>
<td>([0,8.5,7,10,12])</td>
<td>([8.5,7,12,40])</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d ([0,6])</td>
<td>([5.5])</td>
<td>([5.5,5.6])</td>
<td>([5.5,5.6])</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e ([0,10])</td>
<td>([0,10])</td>
<td>([0,10])</td>
<td>([0,10])</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f ([-10,10])</td>
<td>([-10,7])</td>
<td>([-10,7])</td>
<td>([-7,8])</td>
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<td></td>
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</tr>
<tr>
<td>g ([0,40])</td>
<td>([0,40])</td>
<td>([0,40])</td>
<td>([0,40])</td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>
IV. THE OVERLAYING REGULARITY

By the analysis of the above experimental results of the Duffing-Chen hyperchaotic system and Duffing-Lorenz hyperchaotic system, three new overlaying regularities are found.

A new high-dimensional complex hyperchaotic system can be generated by overlaying a Duffing chaotic system with another low-dimensional chaotic system. The Chen chaotic system of the equation (2) and the Duffing chaotic system of the equation (1) are overlaid into a new the Duffing-Chen complex chaotic system of the equation (3). The first overlaying regularity is found. The external excitation part of the original Duffing chaotic system is replaced by an autonomous part. Then, by the bridge of the autonomous part in the Duffing system, the Duffing chaotic system and the other low-dimensional chaotic system are overlaid into a new high-dimensional complex hyperchaotic system. The first overlaying regularity of the Duffing chaotic system and the low-dimensional chaotic system is shown in Fig.3.

As shown in Fig.3, the new high-dimensional complex chaotic system is composed by three parts. The upper part is the Duffing chaotic system. The middle part is the low-dimensional chaotic system and the parameter feedback of F1. The lower parts is the external excitation part of the Duffing chaotic system. The external excitation part of the function F3 is the bridge of overlaying the low-dimensional chaotic system and the Duffing chaotic system. By selecting the different combination of parameters in function of the F2 and the F3, A new high-dimensional complex chaotic system can be generated based on the regularity of shown in Fig.3.

The Lorenz chaotic system of the equation (7) and the Duffing chaotic system of the equation (1) are overlaid into a new the Duffing-Lorenz complex hyperchaotic system of the equation (8). The second overlaying regularity is found. The external excitation part of the original Duffing chaotic system is replaced by an autonomous part. Then, by the bridge of the autonomous part in the Duffing system, the Duffing chaotic system and the other low-dimensional chaotic system are overlaid into a new high-dimensional complex hyperchaotic system. The second overlaying regularity is shown in Fig.4.

As shown in Fig.4, the new high-dimensional complex hyperchaotic system is composed by three parts. The upper part is the low-dimensional chaotic system and the parameters feedback of F2. The middle part is the Duffing chaotic system. The lower parts is the external excitation part of the Duffing chaotic system. The external excitation part of the F3 is the bridge of overlaying the low-dimensional chaotic system and the Duffing chaotic system. By selecting the different combination of parameters in function of the F2 and the F3, A new high-dimensional complex chaotic system can be generated based on the second regularity of shown in Fig.4.

By complexing the regularities of Fig.3 and Fig.4, the third regularity of generating a high-dimensional chaotic system is shown in Fig.5.
of the F1. The second part is the Duffing chaotic system. The third part is the external excitation of the Duffing chaotic system. The fourth part is the low-dimensional chaotic system and the parameters feedback. The fifth part is the external excitation part. The external excitation part of the F3 and the F4 are the bridge of overlaying the low-dimensional chaotic systems and the Duffing chaotic system. By selecting the different combination of parameters in function of the F1 and the F2, A new high-dimensional complex hyperchaotic system can be generated based on the third regularity in Fig.5.

V. THE FIRST VERIFY OF REGULARITY

A. The Duffing-Chebyshev Complex Chaotic System

The form of the Chebyshev system is:

$$ x_{n+1} = \cos(k \cos^{-1}(x_n)) - 1 \leq x_{n} \leq 1 $$

(12)

The parameters of $k$ in the system are real constants. To verify the validity of the first regularity indicated in Fig.3, a new high-dimensional Duffing-Chebyshev complex chaotic system is constructed based on the first overlaying regularity in Fig.3:

$$ \begin{align*}
  x &= y \\
  y &= -dy - x^3 + e \cos wt \\
  z &= \cos(k \cos^{-1}(\sin(w))) \\
  w &= F3
\end{align*} $$

(13)

Duffing chaotic system

Chebyshev system

External excitation part

By selecting the combination of the parameters of the external excitation part in the F3, the Duffing chaotic system and the Chebyshev chaotic system are overlayed into a new high-dimensional Duffing-Chebyshev complex hyperchaotic system.

$$ \begin{align*}
  x &= y \\
  y &= -dy - x^3 + b \cos(z) \\
  z &= k \cos(c \cos^{-1}(\sin(w))) \\
  w &= dx + ez
\end{align*} $$

(13)

B. Dissipative Analysis

$$ \begin{align*}
  \nabla V &= \frac{\partial \dot{x}}{x} + \frac{\partial \dot{y}}{y} + \frac{\partial \dot{z}}{z} + \frac{\partial \dot{w}}{w} \\
  &= -a
\end{align*} $$

(14)

When the parameter change meets the conditions of $a > 0$, the system of the equation (13) is dissipative and converges to the exponential form:

$$ \frac{dV}{dt} = e^{-a} $$

(15)

When $t \to \infty$, the trajectories of the system will eventually be limited to a zero volume.

C. Lyapunov Exponent Analysis

For equation (13), the initial conditions is taken with a four-dimensional ball in all directions on the contraction or expansion. The i-th Lyapunov exponent of equation (13) is the direction axle length of the i-th coordinate. The i-th Lyapunov exponent ($i = 1, 2, 3, 4$) is defined as:

$$ \sigma_i = \lim_{t \to \infty} \frac{1}{t} \ln \frac{||\dot{x}_i(x_0, t)||}{||\dot{x}_i(x_0, 0)||} $$

(16)

When the initial conditions are $a=0.09, b=25, c=4, d=3, e=15, k=1, x=0, y=0, z=0, w=1$ and $dt=0.005$, the picture of Lyapunov exponents are shown in Fig.7.

D. The Structural of the Phase Space

The x-y-z phase space of the equation (13) is shown in Fig.8.

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Figure 6. The relationship of overlaid
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Figure 7. Lyapunov exponents
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Figure 8. Phase space of x-y-z.
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VI. THE SECOND VERIFY OF REGULARITY

A. The Duffing-Lorenz-Sprott J Chaotic System

The form of the Sprott J chaotic system is:

\[
\begin{align*}
\dot{x} &= ax \\
\dot{y} &= -by + z \\
\dot{z} &= -x + y + y^3
\end{align*}
\]

(17)

The parameters of \(a\) and \(b\) are real constants. To verify the validity of the third regularity indicated in Fig.5, a new high-dimensional Duffing-Lorenz-Sprott J complex hyperchaotic system is constructed based on the third overlaying regularity:

\[
\begin{align*}
\dot{x} &= a(\dot{y} - x) + dz - w \\
\dot{y} &= c\dot{x} - xz - y \\
\dot{z} &= xy - bz \\
\dot{u} &= v \\
\dot{v} &= -ev - u^3 + f\cos(w) \\
\dot{w} &= gx \\
\dot{p} &= hr \\
\dot{q} &= -jq + r + kpr \\
\dot{r} &= -p + q + q^2 \\
\dot{s} &= -pr - s + p
\end{align*}
\]

(18)

The parameters of \(a\) to \(j\) in the system are real constants. The external excitation part of the original Duffing system is replaced by an autonomous part. By the bridge of the parameters \(d, z, s, w, r\), the Lorenz chaotic system of the equation (7), the Duffing chaotic system of the equation (1) and the Sprott J chaotic system of the equation (17) are overlaid into a new ten-dimensional Duffing-Lorenz-Sprott J complex hyperchaotic system.

B. Dissipative Analysis

\[
\nabla V = \frac{\partial \dot{x}}{x} + \frac{\partial \dot{y}}{y} + \frac{\partial \dot{z}}{z} + \frac{\partial \dot{u}}{u} + \frac{\partial \dot{v}}{v} + \frac{\partial \dot{w}}{w} \\
+ \frac{\partial \dot{p}}{p} + \frac{\partial \dot{q}}{q} + \frac{\partial \dot{r}}{r} + \frac{\partial \dot{s}}{s}
\]

(19)

When \(a+1+b+e+j+1>0\), equation (18) is dissipative and converges to the exponential form:

\[
\frac{dV}{dt} = e^{(a+1+b+e+j+1)}
\]

(20)

When \(t \to \infty\), the trajectories of the system will eventually be limited to a zero volume.

C. Lyapunov Exponent Analysis

For Equation (18), The i-th Lyapunov exponent is the direction axle length of the i-th coordinate:

\[
\sigma_i = \lim_{t \to \infty} \frac{1}{t} \ln \frac{\|\tilde{x}(t)\|}{\|\tilde{x}(0)\|} (i = 1, 2, \ldots, 10)
\]

(21)

When the parameters are \(a=10\), \(b=8/3\), \(c=28\), \(d=-2.5\), \(e=0.6\), \(f=-8\), \(g=28\), \(h=2\), \(j=2\), \(k=-2\), \(q=-2\), \(x=1\), \(y=1\), \(z=1\), \(u=1\), \(v=1\), \(w=1\) and \(dt=0.005\), the Lyapunov exponents are 0.622, 0.366, 0.116, 0.033, -0.364, -1.026,
-0.926, -1.846, -2.727 and -11.107, the system is in the hyperchaotic state.

D. The Structural of the Phase Space

The phase spaces are shown in Fig.10, (a)-(e).

VII. CONCLUSION

Now some five-dimensional complex chaotic systems have been built by adding a nonlinear controller to a three-dimensional chaotic systems. In this paper, there are two new six-dimensional complex chaotic systems and a new ten-dimensional complex chaotic system to be found. They are the six-dimensional Duffing-Chen complex chaotic system, the six-dimensional Duffing-lorenz complex chaotic system and the ten-dimensional Duffing-lorenz-Sprott J complex chaotic system. When overlaying low-dimensional chaotic systems and the Duffing chaotic system to generate a high-dimensional complex chaotic system, three new overlaying regularities are found. The results of theoretical analysis and experiment reveal the part of relationship between the low-dimensional chaotic system and high-dimensional hyperchaotic system. The characteristics of the new high-dimensional complex hyperchaotic systems are analyzed.

To study the chaotic phenomena and the regularity of producing high-dimensional chaotic system has important significance for the design of communication equipment and new computer chaotic encryption system.

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REFERENCES

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In 2000, he studied and researched computer communication security in university of northern virginia, U.S. His current job include: chaotic encryption algorithm, high-dimensional complex hyperchaotic model and cryptographic algorithm.

The published book:


He current interests are the ultra-high-dimensional chaotic model and the overlaying regularitys of high-dimensional complex hyperchaotic model.