Optimal Selection of Addendum Modification Coefficients of Involute Cylindrical Gears

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Abstract—**Based on the principles of equalized sliding coefficients and equalized teeth bending strength, the optimization mathematical model for calculating and allocating the addendum modification coefficients of involute cylindrical gear is firstly established. Then the measures to calculate the tooth parameters in real-time during the optimization steps are achieved. Finally, the optimization program for calculating and allocating the modification coefficients is developed using the Optimization Toolbox in MATLAB combined with VC++. Compared with the traditional methods such as enclosed chart and graph method, the proposed optimization method in this paper can update the constraints during the optimization process and obtain the accurate modification coefficients. The examples show that the proposed optimization method is more rational and accurate.**

Index Terms—**optimization design, addendum modification coefficients, cylindrical gears, sliding coefficients, tooth bending strength**

I. INTRODUCTION

Gears with addendum modifications have the advantages of improving the meshing performances and loading capacities. A rational selection of addendum modification coefficients can efficiently help improve the fatigue strength of gears, reduce vibrations, suppress the noises and extend service life [1-2]. Hence, gears with addendum modifications are widely used in the fields of machinery industries, and the selection of addendum modification coefficients is one of the most important research orientations in the gear design area.

Among the numerous selection methods, the enclosed chart method introduced by Niemann and Winter [3] is most frequently used by the designers. As both the limitations of selections and principles of allocations are well balanced in this method, the design process is usually directive and fast. But this method consists of hundreds of graphs with different teeth numbers and pressure angles, which results in an inconvenient usage. The graph method proposed by Wang [4] is widely applied in the civil gear manufactures due to the brief procedure and well-balanced limitations. But when it comes to the allocation of the modification coefficients, only one principle based on equalized sliding coefficients is provided in this method without other principles, e.g., equalized bending strength. Therefore, the limitation of the graph method is obvious.

With the developments of computing technology, numerical methods and optimization theories have been applied in the design of engineering areas [5-7]. Based on the allocating principles of equalized sliding coefficients, GA is used by Antal in the design of helical gears [8]. With the improved constraint conditions and non-dimensional gear tooth modeling, the Complex optimization algorithm is applied in the process of optimizing involute gear design by Spitas etc. [9]. And four different allocating methods of modification coefficients are compared by Baglioni et al. [10], and based on the comparison result the influence of the addendum modification coefficients on gear efficiency is researched. And GA is also used by Zhang et al. to carry out an optimization for bevel gear drive [11]. Different optimization algorithms for the optimal design of the modification coefficients are applied in the above practices. But in the optimization steps, the constraints fail to update with the design variables due to the complex and time-consuming calculation process of gear parameters [12]. In order to simplify the calculation, approximated curves are usually used to simulate the constraint parameter, which result in the inaccurate optimization results.

In this paper, the dynamic optimization method is used to obtain the optimal selection of modification coefficients in order to overcome the limitations in the above methods. Firstly, the mathematical model is established based on the principles of equalized sliding coefficients and equalized bending strength. Then, the optimization process is achieved by the Optimization Toolbox in MATLAB and the optimization program for gear design is implemented by VC++. In the end, two optimization examples using different methods are

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carried out for a pair of gears. The optimization results show that combined with the traditional methods, the proposed method in this paper has a faster solution and a more accurate allocating result, which can improve the design efficiency.

II. METHODOLOGY

With the establishment of the mathematical model and the dynamic constraints, the optimization process is achieved using the mature optimization program.

A. Limitations on Selecting Addendum Modification Coefficients

Before the optimization process, the following constraints need to be considered [13]:

(1) Make sure there is no undercut and excessively-thinned tooth;

(2) Make sure the sufficient tooth thickness in order to guarantee the bending strength;

(3) Make sure there is no interference between the addendum and the corresponding dedendum fillet curve;

(4) Make sure the contact ratio is greater than 1.0 in order to satisfy the gears' continuous transmission condition;

(5) Under some circumstances, the transmission of no flank clearance is demanded;

(6) Make sure the strength conditions after the modification.

B. Mathematical Model

The optimization design of modification coefficients is engaged under many constraints and based on the specified allocating principles. And its nature is a nonlinear constraint programming problem. According to the three elements of optimization method, the design variables [14], optimization goal and constraint conditions are respectively determined.

1) Design Variables

Before the optimization, we assume that the gear structure parameters such teeth numbers, module, transmission ratio, etc. are designed and obtained. So the design variables are actually the modification coefficients x_{n1},x_{n2} of two gears [15], as shown in (1).

$$X = \left[x_1, x_2\right]^T = \left[x_{n1}, x_{n2}\right]^T \tag{1}$$

2) Optimization Goal

In order to overcome the limitation of the enclosed chart method and graph method, both the principle of equalized sliding coefficients and the principle of equalized of bending strength are taken into considerations when allocating the sum of modification coefficient. The principle of equalized sliding coefficients can make sure the two gears have equal sliding, which can reduce the abrasion and extend the service life. And the principle of equalized bending strength can guarantee both the pinion and the wheel have equal bending strength in order to avoid the broken failure. So two types of optimization goals are provided in this paper and different principle leads to different objective function.

a) Objective function based on the principle of equalized bending strength

According to ISO 6336-1996 standard, the calculation equation of bending stress is defined as in (2).

$$\sigma_F = \frac{F_t}{bm_n} Y_F Y_S Y_\beta K_A K_v K_{F\beta} K_{F\alpha} \tag{2}$$

Where, the factors can be referred by the standard ISO 6336-1996.

For a pair of gears, all the factors except for the tooth form factor Y_F and stress correction factor Y_S are equal. So the objective function can be established as in (3).

$$min\ f(X) = \left|Y_{F1} \cdot Y_{S1} - Y_{F2} \cdot Y_{S2}\right| \tag{3}$$

b) Objective function based on the principle of equalized sliding coefficients

Among many geometry parameters that influence the meshing performance, the relative sliding velocity is the most important factor [3]. The largest sliding coefficients occur at the position when the tooth addendum is meshing with the corresponding tooth. Under the circumstances of high-speed and heavy-load, the sliding coefficient will severely influence the meshing performance. Hence, in order to allocate the modification coefficients based on the principle of equalized sliding coefficients, the equation of sliding coefficient should firstly be derived.

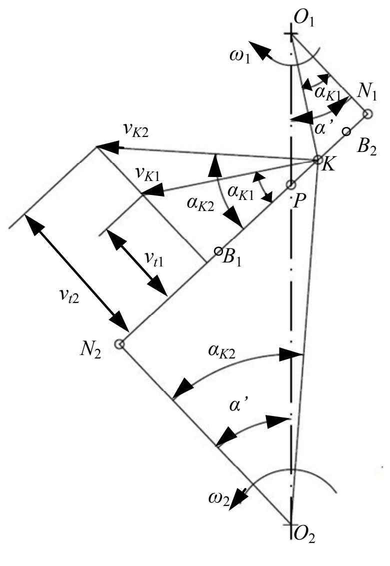

Figure 1. Sketch for the calculation of sliding coefficients.

Fig. 1 shows the meshing sketch of a pair of involute cylindrical gears, and the pinion and wheel are meshing at point K. The linear velocities of pinion and wheel are correspondingly $\boldsymbol{v_{K1}}$ and $\boldsymbol{v_{K2}}$, which are not equal. In order to achieve the continuous transmission, the velocities of two gears should be equal along the direction of common normal line at point K. Since the velocities along the tangential direction $\boldsymbol{v_{t1}}$ and $\boldsymbol{v_{t2}}$ of two gears are not equal, sliding phenomenon happens at the meshing points along the meshing line N_1N_2 except for the pitch point P, and the relative sliding velocity $\boldsymbol{v}_{21}$ can be expressed as in (4) shown.

$$v_{21} = v_{t2} - v_{t1} \tag{4}$$

The sliding coefficient η can be used to express the relative sliding degree of two gears, and it's defined as the ratio of the relative sliding velocity and the tangential velocity. Therefore, the sliding coefficients η_1 and η_2 of pinion and wheel can be obtained as in (5) and (6) shown [3].

$$\eta_1 = \frac{v_{t2} - v_{t1}}{v_{t1}} \tag{5}$$

$$\eta_2 = \frac{v_{t1} - v_{t2}}{v_{t2}} \tag{6}$$

From the geometric relationship, the sliding coefficients can be further derived as in (7) and (8) shown.

$$\eta_1 = \frac{v_{t2} - v_{t1}}{v_{t1}} = \frac{\overline{PK}}{\overline{\overline{N_1 K}}}\left(\frac{u + 1}{u}\right) \tag{7}$$

$$\eta_2 = \frac{v_{t1} - v_{t2}}{v_{t2}} = \frac{\overline{PK}}{\overline{\overline{N_2 K}}}\left(u + 1\right) \tag{8}$$

Where, ω1 and ω2 stand for the angular velocities of pinion and wheel. u denotes the transmission ration of two gears.

Hence, the sliding coefficient is the function of the position of meshing point K. At point N_1, $\eta_1=\infty$,$\eta_2=1$ and at the pitch point P, $\eta_1=\eta_2=0$, while at point N_2, $\eta_1=1$,$\eta_2=\infty$. In fact, the two gears can only mesh along the actual meshing line B_1B_2. At point B_2, the sliding coefficient of pinion gets its maximum $\eta_{1\max}$ and for B_2 is $\eta_{2\max}$. The calculation equations of $\eta_{1\max}$ and $\eta_{2\max}$ can be derived as (9) and (10) shown.

$$\eta_{1\,max} = \frac{tan\ \alpha_{a2} - tan\ \alpha'}{\left(1 + \frac{z_1}{z_2}\right)tan\ \alpha' - tan\ \alpha_{a2}}\left(\frac{u + 1}{u}\right) \tag{9}$$

$$\eta_{2\,max} = \frac{tan\ \alpha_{a1} - tan\ \alpha'}{\left(1 + \frac{z_2}{z_1}\right)tan\ \alpha' - tan\ \alpha_{a1}}\left(u + 1\right) \tag{10}$$

In order to reduce the wear of two gears and extend lifetime, the maximum of sliding coefficients should be equal as much as possible. And the objective function based on the principle of equalized sliding coefficients can be defined as (11) shown.

$$min\ f\left(X\right) = \left|\eta_{1\,max} - \eta_{2\,max}\right| \tag{11}$$

3) Constraint Conditions

According to the limitations on selecting the addendum modification coefficients, the equality and inequality constraints can be formed [10].

a) Undercut constraint

The constraint conditions to avoid undercut for pinion and wheel can be expressed as in (12) and (13).

$$x_1 \geq \frac{z_{min} - z_1}{z_{min}} \cdot h_{an}^{\ *} \tag{12}$$

$$x_2 \geq \frac{z_{min} - z_2}{z_{min}} \cdot h_{an}^{\ *} \tag{13}$$

Where, *Zmin* is the minimum teeth number to avoid undercut and h_{an}* stands for the normal addendum coefficient.

b) Essential tooth thickness constraint

In order to maintain the necessary and expected contact and bending strength after the modification, the tooth addendum should be greater than essential thickness, as (14) and (15) shown.

$$d_a\left(\frac{\pi + 4x_1\ tan\ \alpha_n}{2z_1} + inv\alpha_t - inv\alpha_t'\right) - S_{a1} \geq 0 \tag{14}$$

$$d_a\left(\frac{\pi + 4x_2\ tan\ \alpha_n}{2z_2} + inv\alpha_t - inv\alpha_t'\right) - S_{a2} \geq 0 \tag{15}$$

Where, S_{a1} and S_{a2} respectively stand for the minimum addendum thickness. For gears with soft surface, S_a=0.25m, and for gears with hard surface, S_a=0.4m (m denotes the module).

c) Interference constraint

During the meshing process, to avoid the addendum interferes with the fillet curve of the respective dedendum, the interference constraint condition should be defined, as (16) and (17) shown.

$$tan\ \alpha_t' - \frac{z_2}{z_1}\left(tan\ \alpha_{a2} - tan\ \alpha_t'\right) \geq \left(tan\ \alpha_n - \frac{4\left(h_{an}^{\ *} - x_1\right)}{z_1\ sin\ 2\alpha_n}\right) \tag{16}$$

$$tan\ \alpha_t' - \frac{z_1}{z_2}\left(tan\ \alpha_{a1} - tan\ \alpha_t'\right) \geq \left(tan\ \alpha_n - \frac{4\left(h_{an}^{\ *} - x_2\right)}{z_2\ sin\ 2\alpha_n}\right) \tag{17}$$

d) Contact ratio constraint

According to the conditions of continuous transmission, the contact ratio ε_α should be greater than 1.0. In actual situation, in order to obtain a stable transmission and reduce vibration, the contact ratio need to be greater than 1.2, as in (18).

$$\frac{1}{2\pi}\left[z_1\left(tan\ \alpha_{at1} - tan\ \alpha_t'\right) + z_2\left(tan\ \alpha_{at2} - tan\ \alpha_t'\right)\right] \geq 1.2 \tag{18}$$

e) No flank meshing constraint

To meet the condition of no flank meshing, the sum of modification coefficients should satisfy the equality constraint as (19) shown.

$$x_1 + x_2 = \frac{z_1 + z_2}{2\ tan\ \alpha_n}\left(inv\alpha_t' - inv\alpha_t\right) \tag{19}$$

f) Strength constraints

After the modification, the calculated tooth surface contact and bending stressed of pinion and wheel should be less than the allowed contact and bending stresses. These constraints can be expressed as (20) and (21) shown.

$$\sigma_{Hi} \leq [\sigma_{Hi}] \tag{20}$$

$$\sigma_{Fi} \leq [\sigma_{Fi}] \tag{21}$$

Where, σ_{Hi}, σ_{Fi} (i=1,2) stand for the calculated contact and bending stresses and $[\sigma_{Hi}]$, $[\sigma_{Fi}]$ (i=1,2) denote the allowed contact and bending stresses.

C. Implementation of the Optimization Program

During the iterative process, the modification coefficients may change at each iterative step. And the changes of the coefficients will influence some tooth parameters such as the pressure angle at addendum circle and the diameter of addendum circle. Since these tooth parameters can directly determine the constraints, the constraints need synchronism with the design variables at each iterative step. Based on the programming technology, the optimization based on dynamic constraints is achieved. In this method, after each iterative step, the design variables are saved to local files and then used to re-calculate the tooth parameters. And the new constraints are formed using these parameters in order to carry out a second time iterative step. These processes will loop until the accurate results are obtained. Since MATLAB has the functions including solving numerous equations and computing mathematical expressions [16], to implement this process, the fmincon() function in MATLAB Optimization Toolbox is used and optimization program for modification coefficients optimization is developed by VC++. Fig. 2 shows the interface of this program. Using the program, the modification coefficients can be calculated fast and directly.

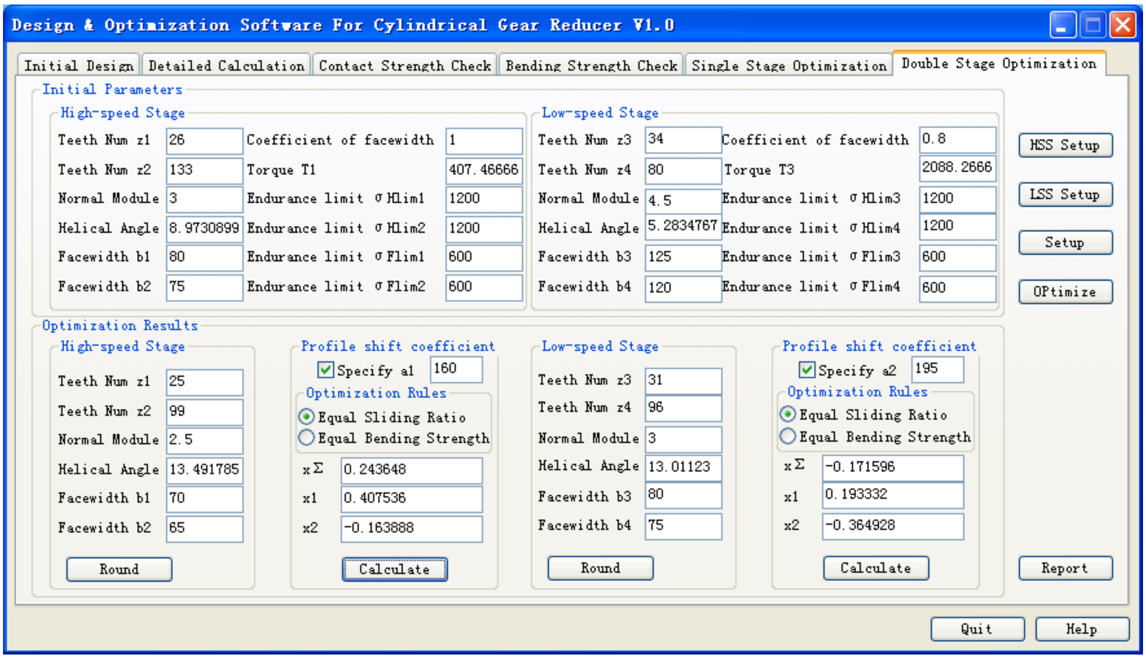

Figure 2. Interface of the optimization program.

III. Case Study and Analysis

A pair of cylindrical spur gears in the gearbox in a certain machine tool is used to carry out an optimization for modification coefficients respectively based on the principles of equalized bending strength and sliding coefficients. The initial parameters are known as: teeth number of pinion z_1=21, teeth number of wheel z_2=33, module m=2.5mm, the actual center distance a'=70mm, pressure angle α=20° and addendum coefficient h_a^*=1.0.

A. Based on the Principle of Equalized Bending Strength

Based on the principle of equalized bending strength, the traditional enclosed chart method (TECM) and dynamic constraint method (DCM) are correspondingly used to calculate and allocate the modification coefficients. And the modification coefficients x_1, x_2 and the re-calculated bending strength σ_{F1}, σ_{F2} are tabulated as Table 1 shown.

TABLE I.
COMPARISON BETWEEN TM AND DCM

Method	x_1	x_2	σ_{F1}(MPa)	σ_{F2}(MPa)
TECM	0.8200	0.3046	375.9076	410.7910
DCM	0.0649	1.0597	359.2055	359.2057

From the data in Table 1, when the enclosed chart method is used, there is combination for teeth number 21/33. So the enclosed chart for teeth number 20/33 is instead for allocating the coefficients, which results in the inaccurate results. And after the allocation, bending strength of two gears are not equal due to the allocation error. Using the method in this paper, the accurate modification coefficients are obtained after the optimization and equalized bending strength is guaranteed.

B. Based on the Principle of Equalized Sliding Coefficients

Based on the principle of equalized sliding coefficients, the allocation for modification coefficients using graph method (GM) and DCM are respectively carried out. And the coefficients x_1,x_2 and sliding coefficients η_1、η_2 after the optimization are shown in Table 2.

TABLE II.
COMPARISON BETWEEN GM AND DCM

Method	x_1	x_2	η_1	η_2
GM	0.5500	0.5746	0.9533	0.9702
DCM	0.6999	0.4247	0.9648	0.9648

From Table 2, it's obvious that results obtained by two methods are different to some extent. The sliding coefficients η_1 and η_2 are not actually equal using GM because the results are approximated in the diagram, which leads to certain error. But using DCM, two modification coefficients are obtained after the optimization, which can make sure the two gears have the same sliding coefficients. Hence, DCM is more accurate than the traditional methods.

It can be seen that the modification coefficients obtained by the traditional methods change greatly with those obtained by the method in this paper. This is because a manual selection is used on the diagram which causes great error, and cannot guarantee the essential equalized sliding coefficients or bending strength. While using the optimization method combined with computer technology, the coefficients can be allocated fast and accurately.

IV CONCLUSION

In this paper, using dynamic constraint optimization, the mathematical model for the allocation of modification coefficients is established based on the principles of equalized sliding coefficients and bending strength. And the process is achieved using the Optimization Toolbox in MATLAB combined with VC++. Compared with traditional manual methods such as enclosed chart method or graph method, the dynamic optimization method can allocate the coefficients more accurately, also reduce the calculation time. This method can improve the design efficiency and has an important practical value.

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