Application of Curve Fitting Extrapolation in Measuring Transient Surface Temperature

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Abstract— The engine inner wall surface temperature was measured by the plug blind-hole extrapolation, and multiple thermocouples were installed at different depths in the substrate. The engine wall extrapolation model of transient high temperature was established according to the basic principles of heat transfer. The transient temperatures were measured by thermocouples buried at different depths of the engine wall and fitting curve was got. The transient temperature field which was generated by the three oxy-hydrogen flame guns was used to simulate the transient high temperature field inside the engine wall. The simulated inner wall surface temperature curves of the engine could be got by the curve fitting extrapolation of temperature sensor and the infrared thermometer respectively, which show good agreement in the overall trend and at the peak point, and verify the correctness of the extrapolation model and method.

Index Terms— temperature extrapolation, transient temperature, curve fitting, aircraft engine

I. INTRODUCTION

Testing is an important part of the whole process throughout the aero-engine test study. Depending on the accurate and reliable data, we can judge the aerodynamic and thermodynamic performances, the durability, the high reliability and the lifetime of the aero-engine throughout the flight. The transient high temperature generated by the components of the aero-engine propulsion which occurs very rapidly and under extreme conditions is one of the parameters of the engine performance. The structure of the aircraft components must be able to withstand the process of the aerodynamic heating and high overload during the high-speed flight. Therefore, it is great significance for the overall performance optimization and breakthroughs in key technologies by accurately measuring of the engine wall transient surface temperature which can help power train design and process engineer to understand the internal true thermodynamics state of the aero-engine combustion accurately [1, 2, 3, 4, 5, 6, 7].

The traditional measuring method is to use the thermocouples or other temperature sensors to measure the exhaust temperature and then indirectly calculating the gas temperature in the combustion chamber. The results of the method can not accurately reflect the temperature changes. It is important that the temperature measurements by an interior sensor of the thermocouple and the substrate. The traditional method for measuring the transient surface temperature is to establish the extrapolation model of a transient high-temperature field and install the thermocouples inside a cavity which is drilled in the substrate, and obtain the solution of the model by using the method of analysis, separation of variables or finite difference approximation.

Nanjing University of Science and Technology et al presented that the transient surface temperature measurements inside-wall of gun were performed by installing a single thermocouple inside a cavity which was drilled in the substrate to measure the transient temperature of one isothermal surface at a certain distance away from the inside-wall of gun with the help of extrapolation. In this paper, the plug blind-hole temperature sensor was designed by installing multiple thermocouples inside the cavities which were drilled at different depths in the substrate to measure the transient surface temperature inside-wall of aero-engine. By analyzing and studying the law of heat transfer among the different isothermal surface of the aero-engine wall according to the basic principles of heat transfer and the Law of Conservation of Energy, and then present a more suitable for engineering applications method for measuring the transient surface temperature of aero-engine inside-wall.

II. THE ESTABLISHMENT OF THE MEASUREMENT SYSTEM

The aero-engine transient surface temperature measurement system includes the special plug-sensor probe, signal conditioning circuits, data acquisition card and computer et al, as show in Fig. 1. In order to maintain the stability of the engine operation state, the probe is designed to drain plug formula. Pluralities of different depths of the blind holes were distributed evenly in the probe at a certain angle. The probe was connected with the engine wall as a single entity by thread.
III. THE PRINCIPLE OF THE MEASUREMENT SYSTEM

According to the characteristics of the aero-engine inside-wall surface, including the large value, transience and in a closed cavity, using instrument recorded the low magnitude and slow internal wall surface temperature values of one or a plurality of temperature layers from the test surface at a certain distance, and then using the principle of heat transfer calculated surface temperature.

A. Heat Transfer Extrapolation Model of the Engine Wall

In order to simplify the heat transfer of aero-engine wall and make it easy to solve, which combine the fundamental of heat transfer and the laws of thermodynamics, now we put forward the following 4 assumptions:

(1) Considering that the radial gradient of engine wall temperature distribution is much larger than the axial one, so the process of heat transfer can be simplified as only one-dimensional distribution along the radial of aero-engine wall;

(2) Compare with the thickness of aero-engine wall, the thermal diffusion layer is so thin that it can be simplified as semi-infinite flat plane model in heat transfer;

(3) Ignoring the temperature field distortion of the aero-engine caused by the screwed temperature sensor, and supposed that the thermal properties of the sensor components and the materials of aero-engine wall.

(4) Supposed that the engine wall thermal diffusivity \( \alpha \) is a constant.

As shown in Fig. 2, aero-engine wall temperature extrapolation model in Cartesian coordinate system.

The process of heat convective between fuel flow \( q_w(\tau) \) in combustion chamber and in-side wall can be simplified approximately as: at moment \( \tau \), the inner wall surface temperature of aero-engine has the same temperature value \( T(\tau) \). So the heat transfer mathematical model of the aero-engine wall can be expressed as:

\[
\begin{align*}
\frac{\partial T}{\partial \tau} &= \frac{\partial^2 T}{\partial x^2}, \quad 0 < x < \infty \\
T(X,0) &= T_0 \\
T(L_0,\tau) &= T_0 \\
T(X_p,\tau) &= T_p(\tau)
\end{align*}
\]

Where \( L_0 \) is the length of thermocouple sensor probe, \( T_0 \) is the initial temperature value; \( T_p(\tau) \) is the true temperature value at \( X_p \), temperature layer of the engine wall and \( \alpha \) is the thermal diffusivity of aero-engine.

B. The Implementation Process of Extrapolation of the Curve Fitting

When the partial differential equations and single-valued conditions of heat transfer of engine wall are known, the temperature field distribution is got simply by the appropriate method [8]. An approximate solution was presented to solve the problem of heat transfer in accordance with limitations of analysis [9], separation of variables [10] and finite difference method [11], which failed to play a practical role in applications of engineering, especially involving nonlinear or other complex boundary conditions. The integral equation method is an approximation method to solve partial differential equations, which provides a systematic and direct way to solve this kind of problem. We need to seek an appropriate explicit expression of temperature distribution in order to calculate the integral and the derivative easily when integral equation method is used to solve partial differential equations.

The Aero-engine wall can be simplified as semi-infinite flat plane model in heat transfer. The initial temperature \( t_0 \) is uniform, and there is instantaneous constant heat flow \( q_w(\tau) \) in the boundary. The surplus temperature \( \theta = t - t_0 \) is Introduced. So the heat transfer mathematical model of the aero-engine wall can be expressed as:

\[
\begin{align*}
\frac{\partial \theta}{\partial \tau} &= \alpha \frac{\partial^2 \theta}{\partial x^2}, \quad 0 < x < \infty \\
\theta &= 0, \quad x \geq 0, \tau = 0 \\
\frac{\partial \theta}{\partial x} &= -\frac{q_w(\tau)}{\lambda}, \quad x = 0, \tau > 0
\end{align*}
\]

According to the integral equation method and Fourier's law, the solution of equations is:

\[
\theta = \frac{2q_w}{\lambda} \sqrt{\alpha \tau} \cdot \text{erfc} \left( \frac{x}{2\sqrt{\alpha \tau}} \right) + C
\]
In the solution of equations,
\[
\text{ierfc}(u) = \frac{e^{-u^2}}{\sqrt{\pi}} - \text{uerfc}(u)
\]
\[
\text{ierfc}(0) = \frac{1}{\sqrt{\pi}}, \quad \text{ierfc}(\infty) = 0
\]
When \( x \to \infty, \ \theta \to 0 \), then \( C=0 \).
\[
\theta = \frac{2q_w}{\lambda} \sqrt{\alpha \tau} \cdot \text{ierfc}\left(\frac{x}{2\sqrt{\alpha \tau}}\right)
\]
(4)
When \( x=0 \), the aero-engine wall temperature is got as follows:
\[
\theta(0, \tau) = \frac{2q_w}{\lambda} \sqrt{\alpha \tau}
\]
(5)
In semi-infinite objects, the approximate solution of temperature response can be got by integral equation method. The approximate equation is:
\[
\int_0^\infty q_w d\tau = \rho c \int_0^\infty \theta(x, \tau) dx
\]
(6)
Where \( \rho \) and \( c \) are physical parameters.
The thickness of heat penetration layer is \( \delta \). The temperature rise of the region \((x > \delta)\) is considered as 0. The Integration interval is changed to 0 ~ \( \delta \). Assuming that the temperature distribution is a quadratic polynomial approximatively as follows:
\[
\theta = A + Bx + Cx^2
\]
(7)
According to boundary conditions and the hypothesis of heat penetration layer hypothesis, we can know:
\[
\begin{align*}
\left\{ \begin{align*}
x = \delta, \ \theta &= 0, \frac{\partial \theta}{\partial x} = 0 \\
x = 0, \ \frac{\partial \theta}{\partial x} &= -\frac{q_w(\tau)}{\lambda}
\end{align*} \right.
\]
(8)
The polynomial coefficients of the temperature distribution are determined by above conditions.
\[
\theta = \frac{q_w \delta}{2\lambda} \left( 1 - \frac{x}{\delta} \right)^2
\]
(9)
According to (6) and (9), the thickness of heat penetration layer \( \delta \) and the temperature rise of the wall \( \theta_w \) can be got as follows:
\[
\delta = \sqrt{6\alpha \tau}
\]
\[
\theta_w = \frac{3}{2} \frac{q_w}{\lambda} \sqrt{\alpha \tau}
\]
(10)
In Reference [12], assuming separately that the temperature distribution is a quadratic polynomial, cubic polynomial, quartic polynomial or quintic polynomial, we can get the temperature distributions and compare them with exact solution, as shown in TABLE 1. Although the choice of explicit expression is random and the solution of equation is inaccurate in local for the certain temperature distribution and heat flow, the integral equation can accurately meet the energy balance on the whole. It is found that the quartic polynomial approximation solution is closest to the exact solution only 0.92% in error, which can meet the accuracy requirements for temperature measurement in most cases.
Simplifying the boundary conditions of heat conduction equation, take the values of the different temperature levels in the wall as the boundary conditions of the heat conduction equation. Thus, the mathematical model of the thermal conductivity of the wall surface can be expressed as:
\[
\begin{align*}
\frac{\partial \theta}{\partial \tau} &= \alpha \frac{\partial^2 \theta}{\partial x^2}, \quad 0 < x < l, \ \tau \geq 0 \\
\theta_i &= \theta(x_i, \tau), \quad x = x_i, \ \tau \geq 0 \\
\theta_2 &= \theta(x_2, \tau), \quad x = x_2, \ \tau \geq 0 \\
\theta_3 &= \theta(x_3, \tau), \quad x = x_3, \ \tau \geq 0
\end{align*}
\]
(11)
Where \( l \) is the thickness of the engine wall, \( \theta_i, \ \theta_2, \ \theta_3 \) is the temperature curve at three different temperature layers of the engine wall, \( \alpha \) is the thermal diffusivity of the wall, \( \tau \) is time.
Assuming that the temperature distribution of the aero-engine wall is a quadratic polynomial approximatively: \( \theta = a + bx + cx^2 \), where \( a, b \) and \( c \) are time-variable functions. According to (11) and \( x_i \neq x_j \neq x_k \neq 0 \), we obtain:
\[
\begin{align*}
\left\{ \begin{align*}
a + bx_1 + cx_1^2 &= \theta_1 \\
a + bx_2 + cx_2^2 &= \theta_2 \\
a + bx_3 + cx_3^2 &= \theta_3
\end{align*} \right.
\]
(12)
If \( \Delta=0.1 \text{mm} \), then \( x_1 = \Delta, \ x_2 = 2 \Delta, \ x_3 = 3 \Delta \). We obtain:
\[
\begin{align*}
a &= 2\theta_1 - 3\theta_2 + \theta_3 \\
b &= \frac{-2\theta_3 + 5\theta_2 - 3\theta_1}{2\Delta} \\
c &= \frac{\theta_3 - 2\theta_2 + \theta_1}{2\Delta^2}
\end{align*}
\]
(13)
The temperature distribution of the aero-engine wall is:
\[
\theta(x, \tau) = a + bx + cx^2
\]
\[
= \left[ 2\theta(x_1, \tau) - 3\theta(x_2, \tau) + \theta(x_3, \tau) \right] + \frac{-2\theta(x_3, \tau) + 5\theta(x_2, \tau) - 3\theta(x_1, \tau)}{2\Delta} \cdot x
\]
TABLE I.
THE TEMPERATURE RESPONSE OF SEMI-INFINITE OBJECTS

<table>
<thead>
<tr>
<th>Temperature distribution function</th>
<th>The thickness of heat penetration layer $\delta$</th>
<th>The temperature rise of the wall $\theta_w$</th>
<th>The relative error between $\theta_w$ and the exact solution $\frac{\theta_w - \theta_{exact}}{\theta_{exact}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = \frac{q_w}{2\lambda} \sqrt{\alpha \tau} \cdot \text{erfc} \left( \frac{x}{2\sqrt{\alpha \tau}} \right)$</td>
<td>$\delta = \sqrt{6\alpha \tau}$</td>
<td>$\theta_w = \frac{3}{2} \frac{q_w}{\sqrt{\lambda}} \sqrt{\alpha \tau}$</td>
<td>$+8.54%$</td>
</tr>
<tr>
<td>$\theta = \frac{q_w \delta}{3\lambda} \left( 1 - \frac{x}{\delta} \right)^2$</td>
<td>$\delta = \sqrt{12\alpha \tau}$</td>
<td>$\theta_w = \frac{3}{2} \frac{q_w}{\sqrt{\lambda}} \sqrt{\alpha \tau}$</td>
<td>$+2.33%$</td>
</tr>
<tr>
<td>$\theta = \frac{q_w \delta}{4\lambda} \left( 1 - \frac{x}{\delta} \right)^4$</td>
<td>$\delta = \sqrt{20\alpha \tau}$</td>
<td>$\theta_w = \frac{3}{2} \frac{q_w}{\sqrt{\lambda}} \sqrt{\alpha \tau}$</td>
<td>$-0.92%$</td>
</tr>
<tr>
<td>$\theta = \frac{q_w \delta}{5\lambda} \left( 1 - \frac{x}{\delta} \right)^5$</td>
<td>$\delta = \sqrt{30\alpha \tau}$</td>
<td>$\theta_w = \frac{3}{2} \frac{q_w}{\sqrt{\lambda}} \sqrt{\alpha \tau}$</td>
<td>$-2.92%$</td>
</tr>
</tbody>
</table>

\[ \sum_{i=1}^{n} \left[ \theta_i(x, \tau) - \theta(x, \tau) \right]^2 \] (16)

So, we can get the temperature variations of the various temperature layers of the entire engine at any time.

When $x = 0$, the temperature distribution of aero-engine inner wall is:

\[ \theta(0, \tau) = a = 2\theta(x_1, \tau) - 3\theta(x_2, \tau) + \theta(x_3, \tau) \] (15)

The substance of approximate solution method of integral equations is using a fitting function $\theta_i(x_i, \tau)$ to approximate the actual temperature distribution function $\theta(x, \tau)$. In order to make the fitting function as much as possible to approximate in accordance with the trend of the experimental data points[13], the least squares method was used to minimize the summation of all the squares of residuals, that is minimize the below expression:

\[ \sum_{i=1}^{n} \left| \theta_i(x, \tau) - \theta(x, \tau) \right|^2 \] (16)

Several measured values $\theta(x_1, \tau), \theta(x_2, \tau), \ldots, \theta(x_n, \tau), x_1 \neq x_2 \neq \ldots \neq x_n \neq 0$ were obtained by thermocouples buried at different depths of the sensor at the moment $\tau$. Supposed that the temperature distribution of the aero-engine wall can be approximated to a polynomial fitting function: $\theta(x, \tau) = a_1 + a_2 x + \cdots + a_{n-1} x^{n-1}$, where $a_1, a_2, \ldots, a_{n-1}$ are the time function. The mathematical model of heat transfer for the aero-engine wall can be expressed as:

\[ \begin{align*}
\theta_1(x, \tau) & = a_1 \quad 0 < x < \tau, \quad \tau \geq 0 \\
\theta_2(x_1, \tau) & = a_1 + a_2 x_1 \quad x = x_1, \tau \geq 0 \\
\theta_2(x_2, \tau) & = a_1 + a_2 x_2 + a_3 x_2^2 \quad x = x_2, \tau \geq 0 \\
\vdots & \\
\theta_n(x_n, \tau) & = a_1 + a_2 x_n + \cdots + a_{n-1} x_n^{n-1} \quad x = x_n, \tau \geq 0
\end{align*} \] (17)

The distance between thermocouples 1, 2, 3, ..., $n$ and the bottom of the probe have been given, the measured values of thermocouples $t_1, t_2, \ldots, t_n$ to fit a curve according to certain rules, as show in Fig. 3, which can calculate the temperature at $d=0$, that is the transient surface temperature $t_0$ of aero-engine wall to be measured.

![Fig. 3 Schematic view of curve fitting extrapolation](image)
IV. EXPERIMENTAL VALIDATION SCHEME AND DEVICES

A. The Experimental Validation Scheme

To understand how to assess the reliability and accuracy of the model is the matter of first importance question after the implementation of heat transfer extrapolation model for aero-engine wall. The validation scheme was shown in Fig. 4.

Several thermocouples measured temperature-time curve (maximum value is $T_1$), curve 2 (maximum value is $T_2$), ..., curve n (maximum value is $T_n$) and standard infrared thermometer measured temperature curve $a$ (maximum value is $T_0$) can be obtained by heating the thermocouple probes using oxy-hydrogen flame. Curves 1, 2, 3, ... , n can be used as a temperature source, which obtained temperature-time curve by extrapolation of the curve fitting according to certain rules. If the curve $b$ is consistent with the curve $a$ within the uncertainty range, the extrapolation model used is reasonable. When the two curves are inconsistent, we need to keep on trying to change with the new model for curve fitting until meeting the requirements of uncertainty. On this basis, the error analysis and model improvement were performed.

B. The Experimental Validation Devices

Experimental verification apparatus is shown in Fig. 5, including the special plug-sensor probe, transmission line, signal conditioning circuits, infrared thermometer Modline 5, oxy-hydrogen flame machine, multi-channel data acquisition card and computer et al. The transient temperature field generated by the three oxy-hydrogen flame guns ejecting oxy-hydrogen flame was used to simulate the transient high temperature field inside the engine wall. By the heating process of the bottom surface of the sensor probe, the inner wall surface temperature curves of the engine could be measured by the curve fitting extrapolation of temperature sensor and the infrared thermometer respectively.

Fig. 5 Schematic view of the experimental validation devices

(1) Temperature sensor probe
The diameter of K-type thermocouples is 0.5mm, and the length is 0.3m.
Three different depths of the blind holes were distributed evenly in the probe at a certain angle. The engineering drawing of the structure of the sensor probe is shown in Fig. 6.

(2) Data acquisition card
The data acquisition card produced by Top Measurement & Control Technology Co., Ltd is four-channel data acquisition card PCI-20612 based on 32-bit PCI bus. Its main performance parameters are as follows:
- Maximum sampling rate: 50Msps/CH
- Input range: $(\pm 100, \pm 200, \pm 500) \text{ mV}, (\pm 1, \pm 2, \pm 4, \pm 10, \pm 20) \text{ V}$
- A/D resolution: 12 bit
- SNR: 52dB
- Input signal bandwidth: 0–7 MHz
- Trigger modes: manual trigger, inner trigger and external trigger

(3) Oxy-hydrogen flame machine
The oxy-hydrogen flame machine JD180-type produced by JINDIAN Oxy-hydrogen Equipment Manufacturer provides transient high temperature heat source. The gas production is 180 L/h, and the flame temperature can reach 2800°C.

(4) Infrared thermometer Modline 5
The infrared thermometer Modline 5 is produced by IRCON Corporation, including 52, 5R and 5G series. In this experiment, we used 5R-3015 type. Its performance indicators are as follows:
- Combination lenses type: RA
- Spectral range: 0.7–1.05μm, 1.0–1.10μm (colorimetric)
- Temperature range: 600~1400°C
- Responding time: 0.1~60s
- Optical resolution $F$: 100
Focusing range: $330\text{mm} \sim \infty$ (The minimum diameter of the target is $3.3\text{mm}$)

C. The Data Processing

In the experiment, the output voltage curves were obtained by three thermocouples which away from the probe followed by 0.34mm, 0.40mm, 0.50mm respectively and the infrared thermometer, as shown in Fig. 7, and two extrapolations and the infrared thermometer measurement curves are shown in Fig. 8.

![Fig. 7 The output voltage curves of three thermocouples and infrared thermometer](image)

The output voltage curves were obtained by four thermocouples which away from the probe followed by 0.14mm, 0.30mm, 0.64mm and 0.86mm respectively, as shown in Fig. 9. The curves were fitted by quadratic and cubic polynomial with the help of four thermocouples in Fig. 10.

![Fig. 8 Comparison between two extrapolations and the infrared thermometer measurement curves](image)
As shown in Fig. 8, compared to the extrapolated curve with the finite difference approximation, the error of curve fitting extrapolation is less. As shown in Fig. 10, the maximum measured curve of infrared thermometer is 1065°C which appeared at 6.58s, and the maximum extrapolated value of curve fitting is 1094°C which appeared at 6.70s with error of 2.72%. Compared with Fig. 8, the result of curve fitting with four thermocouples is more precise, and the error is less. In addition, the result of quadratic polynomial fitting extrapolation is more accurate than three ones with the help of four thermocouples.

V. CONCLUSION

The experimental results show that the measuring accuracy of the curve fitting extrapolation was affected by the curve fitting model, fit exponent and one-dimensional heat conduction model idealization et al. But the extrapolation temperatures obtained by curve fitting are consistent with the temperatures detected by
infrared thermometer in the overall trend, especially in the peak value, which proves the feasibility of this extrapolation model and method. Compared with traditional methods, the solution of this method is simple in form and applicable in the applications of engineering.

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