Compressed Sensing Based on Best Wavelet Packet Basis for Image Processing

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***Abstract*—In this paper, an algorithm named best wavelet packet tree decomposition (BWPTD) is proposed for image compression. In order to obtain better sparse representation of image, best wavelet packet basis is introduced to decompose image signal in the algorithm. Experimental results show that BWPTD is better than single layer wavelet decompression (SLWD) and original compressed sensing (OCS) in peak signal to noise ratio (PSNR) by 2db and 8db, respectively. In addition, the reconstruction time of BWPTD is only half as that of SLWD and OCS.**

***Index Terms*—compressed sensing, image processing, wavelet packet, matching pursuit algorithms**

I. Introduction

Compressed sensing (CS) proposed by Donoho [1] in 2006 has aroused widespread concerns in signal processing. It has been widely used in the area of high-resolution radar imaging [2], medical magnetic resonance imaging (MRI) [3] and astronomy [4]. In image compression, CS applies the fact that images are highly compressible and certain transforms can make them sparse, that is, they can be accurately reconstructed by only a certain number of non-zero elements. These techniques take advantage of convex optimization based on l_0 or l_1 norm which relies on the sparsity hypothesis [5].

Although many current CS methods can reconstruct the images well, they have some disadvantages. The reconstruction time of matching pursuit (MP) algorithm [6] is short, but the reconstruction quality is bad in the algorithm. Moreover, the fact that the MP algorithm requires a number of measured samples hinders the effectiveness of image compression. In order to make up the deficiency of MP, researchers have proposed some improved algorithms based on MP, such as orthogonal matching pursuit [7], regularized orthogonal matching pursuit [8] and sparsity adaptive MP [9].

The basis pursuit (BP) algorithm [10] is a typical representative of linear programming algorithms.And its reconstruction quality is better than that obtained from the MP algorithm used lesser samples, but its reconstruction speed is slower than that of MP. So it can be concluded that interior-point methods [11] 、basis-pursuit denoising [12] and improved algorithm-basis pursuit [13] are improved algorithms based on BP.

The orthogonal wavelet basis has been applied in solving the problem of Doppler parameters of synthetic aperture radar [14]. And its advantages are as follows: 1) the scaling function and generating function are tightly supported, 2) the numerical calculation is simple and convenient, 3) the remnant components would rapidly decay with the decomposition processing and can be astringed after finite iterations when the selected basis meet certain conditions. Thus, the original signal can be expressed well with a small amount of basis selected in best wavelet packet tree decomposition algorithm (BWPTD).

II. CS With Best Wavelet Packet Tree Decomposition

Consider a signal $x=\{x_1,x_2...x_n\}$, the best wavelet packet tree is selected by the Shannon entropy which is defined by the following formula:

$$M(x)=-\sum_i P_i\log P_i \qquad (1)$$

Where $P_i = |x_i|^2 / \|x\|^2$,if $P=0$, then we set $P\log P=0$.

Since $M(x)$ is semi-additive, an additive function is introduced in this paper as:

$$\lambda(x)=-\sum_i |x_i|^2\log|x_i|^2 \ . \qquad (2)$$

With the substitution of Equation (2) into Equation (1),

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we get:

$$M(x) = \|x\|^2 \lambda(x) + \log \|x\|^2 . \qquad (3)$$

One can see that $\lambda(x)$ and $M(x)$ reaching their minimum values at same time. $M(x)$ is used to select optimal basis in BWPTD algorithm. BWPTD obtains better sparse representation with the introduction of best wavelet packet basis in the process of decomposition. The main steps of BWPTD are as follows:

Step 1: Set the decomposition level k and the wavelet function $\psi = \{\psi_1, \psi_2, ..., \psi_N\}$. If $k \geq 2$, decompose the input $N \times N$ signals, else stop.

Step 2: Calculate M of all quad-tree matrix, set true flag for children-matrixes in k level.

Step 3: For the level $(k-1)$ to 1, if parent-matrix's M is less than the sum of children-matrix's M ,then set a true flag for the parent matrix and a false flag for children-matrixes, otherwise set parent matrix's M with the sum of children-matrix's M .

Step 4: Save approximate children-matrix in k level to $Coef(1)$.

Step 5: Copy the matrix with true flag to $CFS(i)$, i=1,2,3,... .

Step 6: Save $Coef(i+1)$ with the product of Gaussian random measurement matrix Φ and $CFS(i)$, i=1,2,3,... .

Step 7: Reconstruct the image with $Coef(l)$ by orthogonal matching pursuit (OMP), l=1,2,3,... .

III. Simulation and Results

The two 256 × 256 standard images (Lena and Cameraman) are used to illustrate the performance of quality of reconstructed image, decomposition time, reconstruction time. Our simulation experiments were performed in the MATLAB2010b environment using an AMD Athlon II X2 245 processor with 2GB of memory. The Gaussian random matrix was applied to measure the coefficient of BWPTD, single layer wavelet decomposition (SLWD) [15] and original compressed sensing (OCS) [12]. The reconstructed process of these algorithms is a typical improved algorithm based upon MP.

Figure 1 is the comparison of reconstructed image of Lena and Cameraman in the similar restore data (RD). It shows that the quality of reconstructed images by BWPTD is better than that of those by SLWD and OCS algorithms for both images. However the reconstructed quality of Cameraman is better than that of Lena. The reason is that there are more high frequency components in Lena than that in Cameraman, and high frequency sub-band coefficients are easy to be ignored by measuring random matrix. In other words, the more low frequency components appearing, the better reconstructed quality can be obtained.

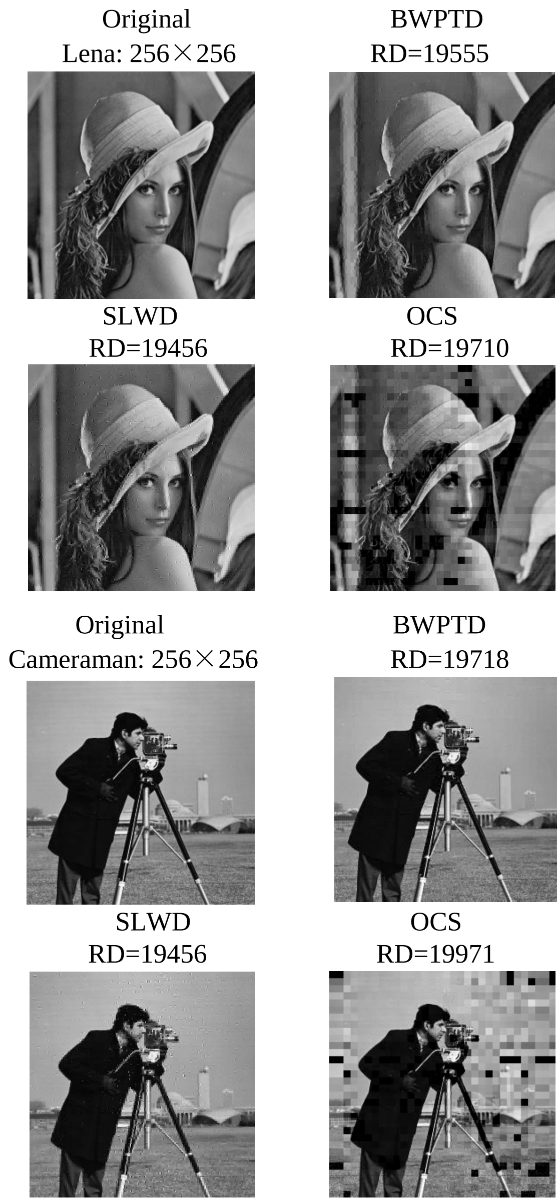
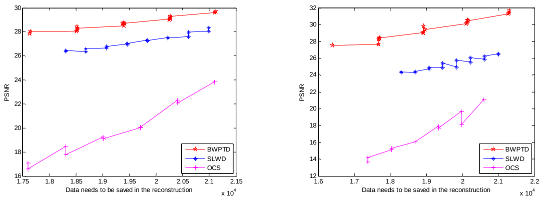


Fig.1 Comparison of reconstructed image

The peak signal to noise ratio (PSNR) comparison of Lena and Cameraman are given in Figures 2 (a) and 2 (b) respectively. From Figure 2, one can see that BWPTD is better than the SLWD and OCS algorithms in PSNR at least by 2db、8db respectively. SLWD applies 1-level of discrete wavelet transform. Thus, low-pass sub-band has 16384(128×128) coefficients. If the saved data is less than 16384 coefficients (called a threshold), the reconstructed image will be sharply deteriorated. Although OCS does not have threshold effect, the quality of reconstructed image declines in spite of the decrease of recovery data.

The decomposition and reconstruction time of BWPTD、SLWD and OCS with noise images are given in Table 1 and Table 2. The Gaussian noise $(0, 0.0005)$ was added into the original image in these experiments.



(a) (b)

Fig. 2 Comparison of PSNR ((a) Lena (b) Cameraman)

Table 1 show that the decompression times of BWPTD is longer than those of SLWD and OCS. In order to depict the image with the optimal sparse matrix, BWPTD spends longer time to find the best basis. However, BWPTD has a better adaptability because it decomposes different images with different best matching basis. Furthermore, the reconstruction time benefits from the selection of best basis.

TABLE I
COMPARISON OF DECOMPOSITION TIME (UNIT: SECOND)

	BWPTD	SLWD	OCS
Lena	0.0325	0.0111	0.0180
Cameraman	0.0281	0.0142	0.0209

Table II Comparison of reconstruction time (Unit: second)

	BWPTD	SLWD	OCS
Lena	0.3682	0.7205	0.6928
Cameraman	0.3731	0.7399	0.7205

The times given in Table 2 are ten times longer than those in Table 1. It indicates that reconstruction time is the key performance index forto determining the effectiveness of an algorithm. Table 2 shows that the reconstruction times of BWPTD is half as those of SLWD and OCS. The main reason is that the selected basis is the best expression of original signal in BWPTD. Therefore, the reconstruction time can be reduced significantly.

The total time was defined as the sum of decomposition time and reconstruction time. For the image of Lena, the total time of BWPTD, SLWD and OCS are 0.4007s, 0.7316s and 0.7108s, respectively. For the image of Cameraman, the total time of BWPTD, SLWD and OCS are 0.4012s, 0.7541s and 0.7414s, respectively. It is observed that the total time of SLWD cost as much as that of OCS, but more than that of BWPTD. Thus it is concluded that BWPTD is more effective than SLWD in image compression.

IV.CONCLUSION

The BWPTD proposed in this paper is efficient, and easy to implement image compression. As best basis is applied to render original signals becoming most sparse, the signals can be reconstructed well with fewer samples. Experimental results show that BWPTD is better than the SLWD and OCS algorithms in PSNR by 2db、8db, respectively. For images with noise, the reconstruction times of BWPTD are half as those of SLWD and OCS. However, it is sensitive to big noise, too. The development of more robust algorithms for images with noise is our future work.

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