Research on the Properties of DST Combination Rule and its Parallel Algorithm

Jinguang Chen
School of Computer Science, Xi'an Polytechnic University, Xi'an 710048, China
School of Electronic Engineering, Xidian University, Xi'an 710071, China
Email: xacjg@163.com

Fen Zhang, Huiran Wang
School of Computer Science, Xi'an Polytechnic University, Xi'an 710048, China
Email: zhangfen1214@126.com

Abstract—**The DST combination rule is studied and corresponding rigorous proofs of its properties are given, including two properties, i.e., sequential DST combination rule meets the commutative law and associative law; the sequential DST combination rule and the centralized DST combination rule are equivalent. The two properties show that the fusion result of multiple evidence sources is irrelevant to fusion order, and the results of sequential fusion and centralized fusion are the same. In addition, employing the advantage of parallel computing feature in the sequential DST fusion process, the parallel DST algorithm is given, and the computing time complexity of parallel DST algorithm is reduced further. The results of example are consistent with the conclusions above. The properties of the DST combination rule and the parallel DST algorithm are of benefit to the engineering practice.**

Index Terms—**DST combination rule; sequential fusion; centralized fusion; information fusion; parallel computing**

I. INTRODUCTION

Evidence theory originated the up probability and the below probability which derived from multiple value mapping, which was proposed by Dempster in 1976. Then Shafer gave some further improvements, established the relationship between propositions and sets of one to one. In uncertainty reasoning, its usage is more convenient and flexible, and then the reasoning mechanism is more concise. Evidence theory is also called Dempster-Shafer evidence theory (DST evidence theory) [1]. When the conflict is lower among evidence sources, DST (Dempster-Shafer Theory, DST) combination rule has a good fusion effect, and DST combination rule's time complexity is lower than the other information fusion algorithms. Due to the benefits of DST, its important role is clearly and easily accepted in the field of information fusion [2, 3]. DST has wide applications in artificial intelligence [4, 5], detection and diagnosis [6] etc.

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Corresponding author: Jinguang Chen.

It is important to study the properties of DST combination rule. We aim at two properties of DST in the paper, i.e., 1) DST combination rule meets commutative law and associative law; 2) sequential DST fusion and centralized DST fusion are equivalent. In some early literatures and books, some properties of DST have been mentioned. Reference [1] pointed out that DST combination rule's order is irrelevant to the calculation for orthogonal of the belief function, and its proof process are the same as reference [7]. Reference [7] gives some properties of DST combination rule and proof process of the properties, and the properties are similar to this paper, but they are not all the same, and their proof process are different from our work. In contrast, this paper's proof process is more strict than [7], and gives the physical meaning of the properties of DST in the application. Article [8] pointed out that whether the evidence synthesis rules or improved synthetic rules do not satisfy the associative law. It should be pointed out that the synthesis rule in [8] is different from this paper. In [8], it is Dempster synthesis rule. However, our research object is Dempster-Shafer combination rule, so the contribution of them is not conflict with us. References [9, 10] studied on properties of the combination rule, but their research concentrate on the improved DST combination rule, And essentially, its research emphasis is different from our work.

Our contributions include two aspects. On the one hand, two properties of DST are validated and studied, and their physical meanings are given. On the other hand, in order to reduce time complexity of algorithm, parallel DST algorithm is presented. Conclusions and numerical results show that DST combination rule meets commutative law and associative law. The results of the sequential DST fusion and the centralized DST fusion are the same. Considering the sequential DST fusion and centralized DST fusion are equivalent, in order to further reduce the time complexity of algorithm, we employ the properties of DST to conduct the parallel computing [11, 12]. At the same time, the time complexity of algorithm of sequential DST fusion, centralized DST fusion and parallel DST algorithm are analyzed, respectively. The results show

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that the computing complexity of the parallel DST algorithm is the lowest.

II. DST Combination Rule

Evidence theory is based on the merger of the evidence sources and the update of the belief functions, which has the following basic concepts: probability distribution function, belief function and plausibility function, etc. They can be used to deal with the uncertainty of the proposition. Probability distribution function expresses the exact confidence for the corresponding proposition. Belief function is also known as lower limit function, which expresses the proposition on the level of trust. Plausibility function is also known as irrefutable function or upper limit function, which expresses the proposition non-false on the level of trust.

Definition 1 DST combination rule

For two evidence sources, the combination rule is given as:

$$m(A)=\begin{cases}0 & A=\varnothing\\ \dfrac{\sum_{A_i\cap B_j=A} m_1(A_i)m_2(B_j)}{1-k} & A\neq\varnothing\end{cases} \tag{1}$$

Assume that two evidence sources E_1, E_2 are under the recognition framework U, their corresponding basic trust distribution functions are m_1 and m_2, respectively. The focal elements are A_i and B_j, $k<1$, which is a conflict factor among the evidence sources.

Conflict factor objectively reflects the level of conflict among the evidence sources in the fusion process. Conflict factor of two evidence sources can be calculated by

$$k=\sum_{A_i\cap B_j=\varnothing} m_1(A_i)m_2(B_j) \tag{2}$$

In the case of multiple evidence sources, assume that the number of evidence sources is N ($N>2$), and the combination rule is given as:

$$m(A)=\begin{cases}0 & A=\varnothing\\ \dfrac{\sum_{\cap A_i=A}\prod_{1\le i\le N} m_i(A_i)}{1-k} & A\neq\varnothing\end{cases} \tag{3}$$

$$k=\sum_{\cap A_i=\varnothing}\prod_{1\le i\le N} m_i(A_i) \tag{4}$$

Assume that $m_1, m_2,\cdots, m_n$ are n basic trust distribution functions under the same recognition framework U, focal elements are $A_i (i=1,2,\cdots,N)$.

Formula (1) is regarded as the sequential fusion method. Sequential fusion [13, 14] is refers to use formula (1) to fuse multiple evidence sources in the manner of two by two. Through the combination of two by two, we can get the final fusion results. Process is described as follows. Assume that there are evidence sources $m_1, m_2,\cdots, m_n$, first of all, m_1 and m_2 are fused using formula (1). Then, the fusion results are used to fuse with m_3, and so on. Formula (3) is the calculation formula of centralized fusion. Centralized fusion is refers to fuse all the evidence sources $m_1, m_2,\cdots, m_n$ at one-time. When we use formula (1) to fuse, the fusion order of the evidence sources will not affect fusion results. Property 1 described in section 3 will prove it. In addition, both formula (1) and (3) can fuse multiple evidence sources, for the same set of evidence sources, fusion results are the same. Property 2 in section 4 makes strict proof for it. These are two problems that we need to study in the following sections. In most information fusion process, sequence fusion style is often used, the reason is that sequential fusion and centralized fusion have the same results for the same set of evidence sources, and the algorithm's time complexity of sequential fusion is much lower than that of centralized fusion, so the sequential fusion method has become the first choice.

III Property 1: DST Combination Rule Meets Commutative Law and Associative Law

Proof:

Assume that there are two evidence sources E_1, E_2: $m_1(A), m_1(B), m_1(C)$; $m_2(A), m_2(B), m_2(C)$. If the fusion of m_1 and m_2 is recorded as $m_1\oplus m_2$, and we will prove that DST combination rule meets the commutative law, i.e. to prove $m_1\oplus m_2 = m_2\oplus m_1$.

Step 1: fuse E_1 and E_2 using formula (1), i.e., calculate $m_{12}=m_1\oplus m_2$.

$$k_{12}=m_1(A)m_2(B)+m_1(A)m_2(C)+m_1(B)m_2(A)+m_1(C)m_2(A)+m_1(B)m_2(C)+m_1(C)m_2(B)$$

$$m_{12}(A)=m_1(A)m_2(A)/(1-k_{12})$$

$$m_{12}(B)=m_1(B)m_2(B)/(1-k_{12})$$

$$m_{12}(C)=m_1(C)m_2(C)/(1-k_{12})$$

Step 2: fuse E_2 and E_1 using formula (1), i.e., calculate $m_{21}=m_2\oplus m_1$.

$$k_{21}=m_2(A)m_1(B)+m_2(A)m_1(C)+m_2(B)m_1(A)+m_2(C)m_1(A)+m_2(B)m_1(C)+m_2(C)m_1(B)$$

$$m_{21}(A)=m_2(A)m_1(A)/(1-k_{21})$$

$$m_{21}(B)=m_2(B)m_1(B)/(1-k_{21})$$

$$m_{21}(C)=m_2(C)m_1(C)/(1-k_{21})$$

Step 3: compare the fusion results m_{12} of the first step and the fusion results m_{21} of the second step.

$$\begin{aligned}m_{12}(A)-m_{21}(A)=&m_1(A)m_2(A)/[1-(m_1(A)m_2(B)+m_1(A)\\&m_2(C)+m_1(B)m_2(A)+m_1(C)m_2(A)+\\&m_1(B)m_2(C)+m_1(C)m_2(B))]-m_2(A)\\&m_1(A)/[1-(m_2(A)m_1(B)+m_2(A)m_1(C)\\&+m_2(B)m_1(A)+m_2(C)m_1(A)+m_2(B)\\&m_1(C)+m_2(C)m_1(B))]\\=&0\end{aligned}$$

So, $m_{12}(A)=m_{21}(A)$. In the same way, we will get $m_{12}(B)=m_{21}(B)$ and $m_{12}(C)=m_{21}(C)$. Finally, we get

$m_1 \oplus m_2 = m_2 \oplus m_1$, i.e.: DST combination rule meets commutative law.

Now we will prove DST combination rule meets the associative law. Assume that there are three evidence sources E_1, E_2, E_3. They are $m_1(A), m_1(B), m_1(C)$, $m_2(A), m_2(B), m_2(C)$ and $m_3(A), m_3(B), m_3(C)$. If we want to prove DST combination rule meets the associative law, i.e., to prove $(m_1 \oplus m_2) \oplus m_3 = m_1 \oplus (m_2 \oplus m_3)$. According to formula (1) and (2), we have,

Step 1: calculate the fusion results of the evidence sources E_1 and E_2, i.e. calculate $m_{12} = m_1 \oplus m_2$.

$$k_{12} = m_1(A)m_2(B) + m_1(A)m_2(C) + m_1(B)m_2(A) + m_1(C)m_2(A) + m_1(B)m_2(C) + m_1(C)m_2(B)$$

$$m_{12}(A) = m_1(A)m_2(A)/(1-k_{12})$$
$$m_{12}(B) = m_1(B)m_2(B)/(1-k_{12})$$
$$m_{12}(C) = m_1(C)m_2(C)/(1-k_{12})$$

Step 2: use the results above to fuse with E_3, i.e., calculate $m_{12} \oplus m_3 = (m_1 \oplus m_2) \oplus m_3$.

$$\begin{aligned} k_{(12)3} &= m_{12}(A)m_3(B) + m_{12}(A)m_3(C) + m_{12}(B)m_3(A) + m_{12}(C)m_3(A) + m_{12}(B)m_3(C) + m_{12}(C)m_3(B) \\ &= m_1(A)m_2(A)m_3(B)/(1-k_{12}) + m_1(A)m_2(A)m_3(C)/(1-k_{12}) + m_1(B)m_2(B)m_3(A)/(1-k_{12}) \\ &\quad + m_1(C)m_2(C)m_3(A)/(1-k_{12}) + m_1(B)m_2(B)m_3(C)/(1-k_{12}) + m_1(C)m_2(C)m_3(B)/(1-k_{12}) \end{aligned}$$

$$m_{(12)3}(A) = m_{12}(A)m_3(A)/(1-k_{(12)3})$$
$$m_{(12)3}(B) = m_{12}(B)m_3(B)/(1-k_{(12)3})$$
$$m_{(12)3}(C) = m_{12}(C)m_3(C)/(1-k_{(12)3})$$

Step 3: calculate the fusion results of E_2 and E_3, i.e., calculate $m_{23} = m_2 \oplus m_3$.

$$k_{23} = m_2(A)m_3(B) + m_2(A)m_3(C) + m_2(B)m_3(A) + m_2(C)m_3(A) + m_2(B)m_3(C) + m_2(C)m_3(B)$$
$$m_{23}(A) = m_2(A)m_3(A)/(1-k_{23})$$
$$m_{23}(B) = m_2(B)m_3(B)/(1-k_{23})$$
$$m_{23}(C) = m_2(C)m_3(C)/(1-k_{23})$$

Step 4: use evidence source E_1 and results m_{23} of the third step to fuse, i.e., calculate $m_1 \oplus m_{23} = m_1 \oplus (m_2 \oplus m_3)$.

$$\begin{aligned} k_{1(23)} &= m_1(A)m_{23}(B) + m_1(A)m_{23}(C) + m_1(B)m_{23}(A) + m_1(C)m_{23}(A) + m_1(B)m_{23}(C) + m_1(C)m_{23}(B) \\ &= m_1(A)m_2(B)m_3(B)/(1-k_{23}) + m_1(A)m_2(C)m_3(C)/(1-k_{23}) + m_1(B)m_2(A)m_3(A)/(1-k_{23}) \\ &\quad + m_1(C)m_2(A)m_3(A)/(1-k_{23}) + m_1(B)m_2(C)m_3(C)/(1-k_{23}) + m_1(C)m_2(B)m_3(B)/(1-k_{23}) \end{aligned}$$

$$m_{1(23)}(A) = m_1(A)m_{23}(A)/(1-k_{1(23)})$$
$$m_{1(23)}(B) = m_1(B)m_{23}(B)/(1-k_{1(23)})$$
$$m_{1(23)}(C) = m_1(C)m_{23}(C)/(1-k_{1(23)})$$

Step 5: compare the fusion results $m_{(12)3}$ of the second step and the fusion results $m_{1(23)}$ of the forth step.

$$\begin{aligned} & m_{(12)3}(A) - m_{1(23)}(A) \\ &= m_{12}(A)m_3(A)/(1-k_{(12)3}) - m_1(A)m_{23}(A)/(1-k_{1(23)}) \\ &= m_1(A)m_2(A)m_3(A)/(1-k_{12})(1-k_{(12)3}) - m_1(A)m_2(A)m_3(A)/(1-k_{1(23)})(1-k_{23}) \\ &= m_1(A)m_2(A)m_3(A)/[1-k_{12}-(1-k_{12})k_{(12)3}] - m_1(A)m_2(A)m_3(A)/(1-k_{23}-(1-k_{23})k_{1(23)}) \\ &= m_1(A)m_2(A)m_3(A)/[1-(m_1(A)m_2(B)+m_1(A)m_2(C)+m_1(B)m_2(A)+m_1(C)m_2(A)+m_1(B)m_2(C)+m_1(C)m_2(B)) \\ &\quad -(m_1(A)m_2(A)m_3(B)+m_1(A)m_2(A)m_3(C)+m_1(B)m_2(B)m_3(A)+m_1(C)m_2(C)m_3(A)+m_1(B)m_2(B)m_3(C) \\ &\quad +m_1(C)m_2(C)m_3(B))] - m_1(A)m_2(A)m_3(A)/[1-(m_2(A)m_3(B)+m_2(A)m_3(C)+m_2(B)m_3(A)+m_2(C)m_3(A) \\ &\quad +m_2(B)m_3(C)+m_2(C)m_3(B))-(m_1(A)m_2(B)m_3(B)+m_1(A)m_2(C)m_3(C)+m_1(B)m_2(A)m_3(A) \\ &\quad +m_1(C)m_2(A)m_3(A)+m_1(B)m_2(C)m_3(C)+m_1(C)m_2(B)m_3(B)] \\ &= m_1(A)m_2(A)m_3(A)/[1-(m_1(A)m_2(B)+m_1(A)m_2(C)+m_1(B)m_2(A)+m_1(C)m_2(A)+m_1(B)m_2(C)+m_1(C)m_2(B)) \\ &\quad -((1-m_1(B)-m_1(C))(m_2(A)m_3(B)+m_2(A)m_3(C))+(1-m_1(A)-m_1(C))(m_2(B)m_3(A)+m_2(B)m_3(C)) \\ &\quad +(1-m_1(A)-m_1(B))(m_2(C)m_3(A)+m_2(C)m_3(B)))] - m_1(A)m_2(A)m_3(A)/[1-(m_2(A)m_3(B) \\ &\quad +m_2(A)m_3(C)+m_2(B)m_3(A)+m_2(C)m_3(A)+m_2(B)m_3(C)+m_2(C)m_3(B))-((1-m_3(B)-m_3(C))(m_1(B)m_2(A) \\ &\quad +(1-m_3(A)-m_3(C))(m_1(A)m_2(B)+m_1(C)m_2(B))+(1-m_3(A)-m_3(B))(m_1(A)m_2(C)+m_1(B)m_2(C))+m_1(C)m_2(A))] \\ &= 0 \end{aligned}$$

So, $m_{(12)3}(A) = m_{1(23)}(A)$. In the same way, we can obtain equations $m_{(12)3}(B) = m_{1(23)}(B)$ and $m_{(12)3}(C) = m_{1(23)}(C)$. That is to say, DST combination rule meets the associative law.

DST's commutative law and associative law are both relative to formula (1) which fuses evidence sources in the manner of two by two. DST's commutative law means that if we inverse the order of two evidence sources in the process of fusion, the results are unchanged. DST's associative law means that we can disturb the order randomly. That is to say, if we fuse any two evidence sources, and then use the fusion results with other evidence sources to fuse, the results are unchanged. For the same set of evidence sources, DST combination rule meets commutative law and associative law. That is to say, the fusion results of DST combination rule and fusion order have no effect on the results. In the process when we use formula (1) to fuse, whether a certain evidence source is placed in whatever stage will not affect fusion results.

IV Property 2: Sequential DST Fusion Rule and Centralized DST Fusion Rule are Equivalent

Proof:

Assume that there are three evidence sources, $m_1(A), m_1(B), m_1(C)$, $m_2(A), m_2(B), m_2(C)$, and $m_3(A), m_3(B), m_3(C)$.

Step 1: calculate the results of sequential fusion, i.e., $(m_1 \oplus m_2) \oplus m_3$. According to the results of section 3, we can get the conflict value and belief function values of sequential fusion. They can be represented as k_{123}^{S} , $m_{123}^{\mathrm{S}}(A)$, $m_{123}^{\mathrm{S}}(B)$ and $m_{123}^{\mathrm{S}}(C)$.

$$
\begin{aligned}
k_{123}^{\mathrm{S}} &= m_{12}(A)m_3(B) + m_{12}(A)m_3(C) + m_{12}(B)m_3(A) + \\
&\quad m_{12}(C)m_3(A) + m_{12}(B)m_3(C) + m_{12}(C)m_3(B) \\
&= m_1(A)m_2(A)m_3(B)/(1-k_{12}) + m_1(A)m_2(A) \\
&\quad m_3(C)/(1-k_{12}) + m_1(B)m_2(B)m_3(A)/(1-k_{12}) \\
&\quad + m_1(C)m_2(C)m_3(A)/(1-k_{12}) + m_1(B)m_2(B) \\
&\quad m_3(C)/(1-k_{12}) + m_1(C)m_2(C)m_3(B)/(1-k_{12})
\end{aligned}
$$

$$
\begin{aligned}
m_{123}^{\mathrm{S}}(A) &= m_{12}(A)m_3(A)/(1-k_{123}^{\mathrm{S}}) \\
m_{123}^{\mathrm{S}}(B) &= m_{12}(B)m_3(B)/(1-k_{123}^{\mathrm{S}}) \\
m_{123}^{\mathrm{S}}(C) &= m_{12}(C)m_3(C)/(1-k_{123}^{\mathrm{S}})
\end{aligned}
$$

Step 2: calculation the results of centralized fusion, i.e., $m_1 \oplus m_2 \oplus m_3$. According to the formula (3) and (4), k_{123}^{C} , $m_{123}^{\mathrm{C}}(A)$, $m_{123}^{\mathrm{C}}(B)$ and $m_{123}^{\mathrm{C}}(C)$ express the conflict value and belief function values of centralized fusion.

$$
\begin{aligned}
k_{123}^{\mathrm{C}} &= m_1(A)m_2(A)(m_3(B)+m_3(C)) + m_1(A)m_2(B) \\
&\quad (m_3(A)+m_3(B)+m_3(C)) + m_1(A)m_2(C)(m_3(A) \\
&\quad + m_3(B)+m_3(C)) + m_1(B)m_2(A)(m_3(A)+m_3(B) \\
&\quad + m_3(C)) + m_1(B)m_2(B)(m_3(A)+m_3(C)) + m_1(B) \\
&\quad m_2(C)(m_3(A)+m_3(B)+m_3(C)) + m_1(C)m_2(A) \\
&\quad (m_3(A)+m_3(B)+m_3(C)) + m_1(C)m_2(B)(m_3(A) \\
&\quad + m_3(B)+m_3(C)) + m_1(C)m_2(C)(m_3(A)+m_3(B))
\end{aligned}
$$

Due to $m_3(A)+m_3(B)+m_3(C)=1$

So,

$$
\begin{aligned}
k_{123}^{\mathrm{C}} &= m_1(A)m_2(A)(m_3(B)+m_3(C)) + m_1(A)m_2(B) + \\
&\quad m_1(A)m_2(C) + m_1(B)m_2(A) + m_1(B)m_2(B)(m_3(A) \\
&\quad + m_3(C)) + m_1(B)m_2(C) + m_1(C)m_2(A) + m_1(C) \\
&\quad m_2(B) + m_1(C)m_2(C)(m_3(A)+m_3(B))
\end{aligned}
$$

$$
\begin{aligned}
m_{123}^{\mathrm{C}}(A) &= m_1(A)m_2(A)m_3(A)/(1-k_{123}^{\mathrm{C}}) \\
m_{123}^{\mathrm{C}}(B) &= m_1(B)m_2(B)m_3(B)/(1-k_{123}^{\mathrm{C}}) \\
m_{123}^{\mathrm{C}}(C) &= m_1(C)m_2(C)m_3(C)/(1-k_{123}^{\mathrm{C}})
\end{aligned}
$$

Step 3: compare the results m_{123}^{S} of sequential fusion in the first step and the results m_{123}^{C} of centralized fusion in the second step.

$$
\begin{aligned}
& m_{123}^{\mathrm{S}}(A) - m_{123}^{\mathrm{C}}(A) \\
&= m_{12}(A)m_3(A)/(1-k_{123}^{S}) - m_1(A)m_2(A)m_3(A)/(1-k_{123}^{C}) \\
&= m_1(A)m_2(A)m_3(A)/(1-k_{12})(1-k_{123}^{S}) - m_1(A)m_2(A) \\
&\quad m_3(A)/(1-k_{123}^{C}) \\
&= m_1(A)m_2(A)m_3(A)/(1-k_{12} - (m_1(A)m_2(A)m_3(B) + \\
&\quad m_1(A)m_2(A)m_3(C) + m_1(B)m_2(B)m_3(A) + m_1(C)m_2(C) \\
&\quad m_3(A) + m_1(B)m_2(B)m_3(C) + m_1(C)m_2(C)m_3(B))) - \\
&\quad m_1(A)m_2(A)m_3(A)/[1-(m_1(A)m_2(A)(m_3(B)+m_3(C)) \\
&\quad + m_1(A)m_2(B)(m_3(A)+m_3(B)+m_3(C)) + m_1(A)m_2(C) \\
&\quad (m_3(A)+m_3(B)+m_3(C)) + m_1(B)m_2(A)(m_3(A)+m_3(B) \\
&\quad + m_3(C)) + m_1(B)m_2(B)(m_3(A)+m_3(C)) + m_1(B)m_2(C) \\
&\quad (m_3(A)+m_3(B)+m_3(C)) + m_1(C)m_2(A)(m_3(A)+m_3(B) \\
&\quad + m_3(C)) + m_1(C)m_2(B)(m_3(A)+m_3(B)+m_3(C)) + m_1(C) \\
&\quad m_2(C)(m_3(A)+m_3(B)))] \\
&= m_1(A)m_2(A)m_3(A)/(1-(m_1(A)m_2(B) + m_1(A)m_2(C) + \\
&\quad m_1(B)m_2(A) + m_1(C)m_2(A) + m_1(B)m_2(C) + m_1(C)m_2(B)) \\
&\quad -(m_1(A)m_2(A)m_3(B) + m_1(A)m_2(A)m_3(C) + m_1(B)m_2(B) \\
&\quad m_3(A) + m_1(C)m_2(C)m_3(A) + m_1(B)m_2(B)m_3(C) + m_1(C) \\
&\quad m_2(C)m_3(B))) - m_1(A)m_2(A)m_3(A)/[1-(m_1(A)m_2(A) \\
&\quad (m_3(B)+m_3(C)) + m_1(A)m_2(B) + m_1(A)m_2(C) + m_1(B) \\
&\quad m_2(A) + m_1(B)m_2(B)(m_3(A)+m_3(C)) + m_1(B)m_2(C) + \\
&\quad m_1(C)m_2(A) + m_1(C)m_2(B) + m_1(C)m_2(C)(m_3(A) + \\
&\quad m_3(B)))] \\
&= 0
\end{aligned}
$$

So, $m_{123}^{\mathrm{S}}(A) = m_{123}^{\mathrm{C}}(A)$. In the same way, we will get $m_{123}^{\mathrm{S}}(B) = m_{123}^{\mathrm{C}}(B)$ and $m_{123}^{\mathrm{S}}(C) = m_{123}^{\mathrm{C}}(C)$. At this time, the two properties are proved completely.

Now we have proved that sequential DST fusion and centralized DST fusion can obtain the same results. Because algorithm's time complexity of centralized fusion is higher than that of sequential fusion, sequential DST fusion has become the prefer method for the fusion of multiple evidence sources. Considering the sequential DST fusion method has good parallel computing feature, in order to improve time complexity of the algorithm further, the parallel DST algorithm is presented in the following section.

V Parallel DST Algorithm

Parallel computing indicates that the computing tasks are divided into subtasks which can be executed at the same time, and these subtasks are executed in parallel. In this manner, the entire computing tasks are completed [15, 16]. Because the fusion order has no effect on results in sequential DST fusion, and the results of sequential fusion and centralized fusion are the same, therefore evidence sources may be distributed to different computers for parallel computing.

Fig. 1 is a schematic diagram of the parallel DST algorithm in ideal circumstance. For simplicity, we assume that the number of computers available is unlimited. For the number of evidence sources is N, we need $N/2$ computers to fuse the evidence sources. First of all, the evidence sources are arbitrarily divided into some groups. Each group has two evidence sources, which is assigned to one computer to fuse using sequential DST combination rule separately. Secondly, the results are calculated two by two by using different computers in the first layer. Further, using two fusion results above to fuse until all the fusion results of the previous layer are completed. This step may be repeated many times. Finally, only one fusion result is obtained, which is also the final result. Here it should be pointed out that when one layer's calculations are finished, there might be only one result left without calculation, putting it into the next layer to fuse.

Due to the algorithm's time complexity of the sequential fusion is much lower than that of the centralized fusion, using parallel computing feature of the sequential fusion to do parallel computing will reduce the time complexity of algorithm further. We analyze the algorithm's time complexity of sequential DST fusion, centralized DST fusion and parallel DST algorithm. Because the time complexity of addition and subtraction is much smaller than that of multiplication and division in the computing process, we regard the number of multiplication and division as the algorithm's basic operations. As we all know, the number of these basic operations can be regarded as time complexity measure.

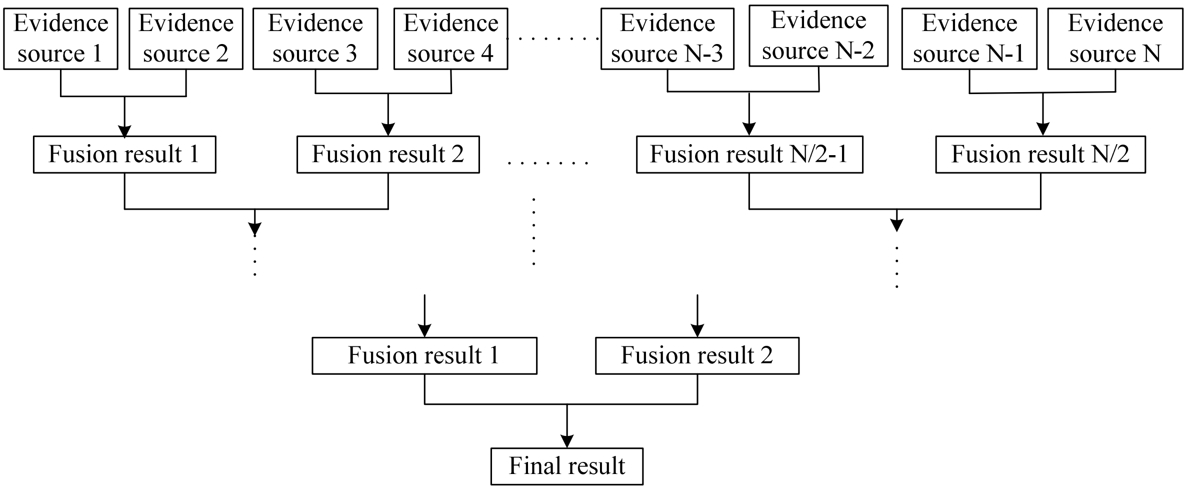

Fig. 1 Schematic diagram of parallel DST algorithm

TABLE 1
TIME COMPLEXITY OF ALGORITHMS

	Time complexity
sequential DST fusion	$8*(N-1)$
centralized DST fusion	$[(N-1)*2*(3^0+3^1+3^2+\cdots+3^Q)]*3+N$
parallel DST algorithm	$8*(N-1)/P$

In order to measure the time complexity of DST fusion algorithm, we regard multiplication and division as basic operation to calculate the number of basic operations in the worst case, and make it as a measurement criteria, the results are shown in Table 1. Table 1 assume that the number of evidence sources is N, so algorithm's time complexity of the sequential fusion is $8*(N-1)$; algorithm's time complexity of centralized fusion is $\left[(N-1)*2*(3^0+3^1+3^2+\cdots+3^Q)\right]*3+N$, where $Q=N-2$; algorithm's time complexity of parallel computing is $8*(N-1)/P$, where P is the number of computers used in the parallel DST algorithm. It should be pointed out that, for easily, the time complexity of the parallel DST algorithm ignores the time consuming of communication and synchronization among multiple CPUs.

As shown in Table 2, we assume that the number of computers is 4 in parallel DST fusion, and the number of evidence sources is 3, 4 and 5. The algorithm's time complexity of centralized DST fusion is much larger than that of sequential DST fusion and parallel DST algorithm. And time complexity of parallel DST algorithm is clearly smaller than the others, especially when the number of evidence sources becomes more and more. From Table 2, there are some facts, such as, the difference of the algorithm's time complexity of the sequential DST fusion is 4, while the parallel DST algorithm's is 1 when the number of evidence sources is 4 to 5.

TABLE 2
INSTANCE FOR TIME COMPLEXITY OF ALGORITHMS

	time complexity		
	3 evidence sources	4 evidence sources	5 evidence sources
sequential DST fusion	16	28	32
centralized DST fusion	51	238	965
parallel DST algorithm	4	7	8

The fusion results of parallel DST algorithm, sequential DST fusion and centralized DST fusion are the same, and algorithm's time complexity of parallel DST algorithm is much lower. When there are a large number of evidence sources in some actual applications, the fusion efficiency of parallel DST algorithm is much higher than the others.

VI EXAMPLE

Assume there is a group of evidence sources [17], as shown in Table 3. In order to study whether the fusion order of multiple evidence sources influences on the results of sequential DST fusion or not, we sort the evidence sources arrangement as m_1, m_2, m_3, m_4, m_5 and m_5, m_4, m_3, m_2, m_1 , the former is called positive sequential sort and the latter is called the inverse sequential sort.

TABLE 3
EVIDENCE SOURCES

target type	evidence sources				
	m_1	m_2	m_3	m_4	m_5
A	0.5	0	0.55	0.55	0.55
B	0.2	0.9	0.1	0.1	0.1
C	0.3	0.1	0.35	0.35	0.35

TABLE 4
FUSION RESULTS

	$m(A)$	$m(B)$	$m(C)$
DST using positive sequential evidence sources	0	0.1228	0.8772
DST using inverse sequential evidence sources	0	0.1228	0.8772
centralized DST fusion	0	0.1228	0.8772

The fusion results are shown in Table 4. The results of using methods of positive sequential sort, inverse sequential sort and the centralized DST fusion are the same exactly. That is to say, sequential fusion of DST combination rule conforms to commutative law and associative law, sequential DST fusion and centralized DST fusion are equivalent, and the order of fusion among evidence sources has no effects on the results. For the algorithm's time complexity, as shown in Table 2, when the number of evidence sources is 5, algorithm's time complexity of centralized fusion is 965, algorithm's time complexity of sequential fusion is 32. At the same time, algorithm's time complexity of positive sequential sort and inverse sequential sort are the same. Due to these advantages above, under the same set of evidence sources conditions, sequential fusion is more convenient and concise than centralized fusion apparently. If there are conditions of parallel computing, parallel DST algorithm can further improve the time efficiency.

VII CONCLUSION

DST combination rule meets commutative law and associative law, and sequential fusion and centralized fusion of DST combination rule are equivalent. In this paper, these two properties of DST have been proved. In addition, in order to enhance the time efficiency of DST information fusion further, the parallel DST algorithm is given. According to the conclusions of this work, the use of sequential DST fusion for parallel computing will greatly save time, and can get the same fusion results. DST combination rule has the better fusion effect when low conflict combines with parallel algorithm of sequential fusion of DST combination rule, which provides a more effective information fusion method. Our next research will focus on more effective information fusion methods.

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Dr. Jinguang Chen (1977-): He was born in Henan Province, China. He obtained Bachelor and Master degree from the Xi'an Polytechnic University in 2000 and 2005, respectively. In 2011, He graduated from School of Electronic Engineering, Xidian University, and obtained PHD. His interests include information fusion, evidence theory, state estimation, target tracking, etc. Now, he is an associate Professor of Xi'an Polytechnic University.

Fen Zhang (1988-): She was born in Shaanxi Province, China. She obtained Bachelor degree from the Xi'an Polytechnic University in 2011. Her interests include information fusion, evidence theory, etc. Now, she is pursing Master degree in Xi'an Polytechnic University.