

# Oscillation Criteria for Second Order Nonlinear Neutral Perturbed Dynamic Equations on Time Scales

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**Abstract**—To investigate the oscillatory and asymptotic behavior for a certain class of second order nonlinear neutral perturbed dynamic equations on time scales. By employing the time scales theory and some necessary analytic techniques, and introducing the class of parameter functions and generalized Riccati transformation, some new sufficient conditions for oscillation of such dynamic equations on time scales were established. The results not only improve and extend some known results in the literature, but also unify the oscillation of second order nonlinear neutral perturbed differential equations and second order nonlinear neutral perturbed difference equations. In particular, the results are essentially new under the relaxed conditions for the parameter function. Some examples are given to illustrate the main results. Dynamic equations on time scales are widely used in many fields such as computer, electrical engineering, population dynamics, and neural network, etc.

**Index Terms**—oscillation, nonlinear neutral perturbed dynamic equation, time scales, Riccati transformation.

## I. INTRODUCTION

The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Higher in his Ph.D. thesis [1] in 1988 in order to unify continuous and discrete analysis. Not only can this theory of so-called “dynamic equations” unify the theories of differential equations and of difference equations, but also it is able to extend these classical cases “in between”, e.g., to so-called q-difference equations. Several authors have expounded on various aspects of this new theory, see the survey paper by Agarwal [2] and references cited therein. A book on the subject of time scales by Bohner and Peterson [3] summarizes and organizes much of the time scale calculus. A time scales  $\mathbb{T}$  is an arbitrary nonempty closed subset of the real numbers  $\mathbb{R}$ . There are many

interesting time scales and they give rise to plenty of applications, the cases when the time scale is equal to reals or the integers represent the classical theories of differential and of difference equations. Another useful time scale a time scale  $\mathbb{P}_{a,b} \cup_{n=0}^{\infty} [n(a+b), n(a+b)+a]$  is widely used to study population in biological communities, electric circuit and so on [3].

In recent years, there has been much research activity concerning the oscillation and nonoscillation of solutions of some dynamic equations on time scales, and we refer the reader to the papers [4-17] and references cited therein. Regarding neutral dynamic equations, Argarwal et al [6] considered the second order neutral delay dynamic equation

$$\{\alpha(t)[(x(t) + c(t)x(t-\tau))^{\Delta}]^{\gamma}\}^{\Delta} + f(t, x(t-\delta)) = 0. \quad (1)$$

where  $\gamma > 0$  is an odd positive integer,  $\tau$  and  $\delta$  are position constants,  $\alpha^{\Delta}(t) > 0$ , and proved that the oscillation of (1) is equivalent to the oscillation of a first order delay dynamic inequality. Saker [7] considered (1) where  $\gamma \geq 1$ , is an odd positive integer, the condition  $\alpha^{\Delta}(t) > 0$  is abolished and established some new sufficient conditions for oscillation of (1). However the results established in [6-7] are only valid for the time scales  $\mathbb{R}, \mathbb{N}$ , or  $h\mathbb{N}, q^{\mathbb{N}}$ , where  $q^{\mathbb{N}} = \{t : t = q^k, k \in \mathbb{N}, q > 1\}$ .

Sahiner et al [8] considered the general equation

$$\{\alpha(t)[(x(t) + c(t)x(\tau(t))^{\Delta}]^{\gamma}\}^{\Delta} + f(t, x(\delta(t))) = 0. \quad (2)$$

on a time scale  $\mathbb{T}$ , where  $\gamma \geq 1$  and  $\tau(t) \leq t, \delta(t) \leq t$ , and followed the argument in [6-7] by reducing the oscillation of (2) to the oscillation of a first order delay dynamic inequality and established some sufficient conditions for the oscillation. However one can easily see that the two

examples presented in [8] to illustrate the main results are valid only when  $\mathbb{T} = \mathbb{R}$  and cannot be applied when  $\mathbb{T} = \mathbb{N}$ . Agarwal, O'Regan and Saker [3] considered (2) where  $\gamma \geq 1$  is an odd positive integer and  $\alpha^\Delta(t) > 0$ , and established some new oscillation criteria by employing the Riccati transformation technique which can be applied on any time scale  $\mathbb{T}$  and improved the results in [6, 8].

Bohner and Saker [9] considered perturbed nonlinear dynamic equation

$$\{\alpha(t)((x(t))^\Delta)^\gamma\}^\Delta + F(t, x^\sigma) = G(t, x^\sigma, x^\Delta). \tag{3}$$

on a time scales  $\mathbb{T}$ . Where  $\gamma > 0$  is an odd positive integer, using Riccati transformation techniques, they obtained some sufficient conditions for the solution to be oscillatory or converge to zero.

Following this trend, we shall study the oscillation for the second-order neutral nonlinear perturbed dynamic equations of the form

$$\{\alpha(t)((x(t) + c(t)x(\tau(t)))^\Delta)^\gamma\}^\Delta + F(t, x(\delta(t))) = G(t, x(\delta(t)), x^\Delta), \tag{4}$$

and

$$\{\alpha(t)((x(t) - c(t)x(\tau(t)))^\Delta)^\gamma\}^\Delta + F(t, x(\delta(t))) = G(t, x(\delta(t)), x^\Delta). \tag{5}$$

on an arbitrary time scales  $\mathbb{T}$ , where  $\gamma$  is a quotient of positive odd integer,  $\alpha, c$  is a positive real-valued rd-continuous function defined on a time scales  $\mathbb{T}$  and the following conditions are satisfied:

- (H1)  $0 \leq c(t) \leq c_0 < 1$ ,  $\int_0^{+\infty} \Delta t / (\alpha(t))^{1/\gamma} = \infty$ , for all  $t \in \mathbb{T}$ ;
- (H2)  $\tau, \delta: \mathbb{T} \rightarrow \mathbb{T}$  satisfies  $\tau(t) \leq t$ , for all  $t \in \mathbb{T}$ , either  $\delta(t) \geq t$  or  $\delta(t) \leq t$  for all sufficiently large  $t$ , and  $\lim_{t \rightarrow \infty} \tau(t) = \lim_{t \rightarrow \infty} \delta(t) = \infty$ ;
- (H3)  $p, q: \mathbb{T} \rightarrow \mathbb{R}$  are rd-continuous function, such that  $q(t) - p(t) > 0$ , for all  $t \in \mathbb{T}$ ;
- (H4)  $F: \mathbb{T} \times \mathbb{R} \rightarrow \mathbb{R}$  and  $G: \mathbb{T} \times \mathbb{R}^2 \rightarrow \mathbb{R}$  are functions such that  $uF(t, u) > 0$  and  $uG(t, u, v) > 0$ , for all  $u \in \mathbb{R} - \{0\}$ ,  $v \in \mathbb{R}$ ,  $t \in \mathbb{T}$ ;
- (H5)  $F(t, u)/u^\gamma \geq q(t)$ , and  $G(t, u, v)/u^\gamma \leq p(t)$  for all  $u, v \in \mathbb{R} - \{0\}$ ,  $t \in \mathbb{T}$ .

We note that in all the above results the conditions  $0 \leq c(t) < 1$ ,  $\gamma \geq 1$  and  $\delta(t) \leq t$  are required. And some authors utilized the kernel function  $(t - s)^m$  or the general class of functions  $H(t, s)$  and obtained some oscillation criteria, but the condition  $H^\Delta(t, s) \leq 0$  is required. In this paper the study is free of these restrictions and contains the cases when  $0 < \gamma < 1$ ,  $\delta(t) \geq t$ , and  $-1 < c(t) \leq 0$ . In particular, by utilizing the general class of functions  $H(t, s)$ , we shall derive some sufficient conditions for the solutions of (4) and (5) to be oscillatory or converge

to zero when the condition  $H^\Delta(t, s) \leq 0$  is relaxed. Our results are different from the existing results for neutral equations on time scales that were established in [6-11, 13-17]. Also, we give some examples to illustrate the main results.

Since we are interested in the oscillatory and asymptotic behavior of solutions near infinity, we assume that  $\sup \mathbb{T} = \infty$ , and define the time scale interval  $[t_0, \infty)_{\mathbb{T}}$  by  $[t_0, \infty)_{\mathbb{T}} := [t_0, \infty) \cap \mathbb{T}$ . By a solution of (4), we mean a nontrivial real-valued function  $x(t)$  satisfying (4) for  $t \geq t_0$ . A solution  $x(t)$  of (4) is said to be oscillatory if it is neither eventually positive nor eventually negative, otherwise it is called nonoscillatory. Equation (4) is said to be oscillatory if all its solutions are oscillatory. Our attention is restricted to those solutions of (4) which exist on some half line  $[t_0, \infty)$  and satisfy  $\sup\{|x(t)| : t \geq t_x\} > 0$ , for any  $t_x \geq t_0$ .

The paper is organized as follows. In next section, we present some basic formula and lemma concerning the calculus on time scales. In Section 3, we will use Riccati transformation techniques and the general class of functions  $H(t, s)$  and give some sufficient conditions for the oscillatory behavior of solutions of (4) and (5). In last section, we give some examples to illustrate our main results.

Through this paper, we let

$$d_+(t) = \max[0, d(t)], Q(t) = (q(t) - p(t))(1 - c(\delta(t)))^\gamma, \\ d_-(t) = \max[0, -d(t)], \rho(t, u) := \frac{\int_u^{\delta(t)} \Delta s / \alpha^{1/\gamma}(s)}{\int_u^t \Delta s / \alpha^{1/\gamma}(s)},$$

and for sufficiently large  $T^*$ ,

$$\beta(t, T^*) = \begin{cases} 1, & \delta(t) \geq t, \\ \rho^\gamma(t, T^*), & \delta(t) \leq t. \end{cases}$$

## II. SOME PRELIMINARIES ON TIME SCALES

A time scales  $\mathbb{T}$  is an arbitrary nonempty closed subset of the real numbers  $\mathbb{R}$ . In this paper, we only consider time scales interval of form  $[t_0, \infty)_{\mathbb{T}}$ , on  $\mathbb{T}$  we define the forward jump operator  $\sigma$  and the graininess  $\mu$  by

$$\sigma(t) := \inf \{s \in \mathbb{T} : s > t\} \text{ and } \mu(t) := \sigma(t) - t.$$

A point  $t \in \mathbb{T}$  with  $\sigma(t) = t$  is called right-dense, while  $t$  is referred to as being right-scattered if  $\sigma(t) > t$ . A function  $f: \mathbb{T} \rightarrow \mathbb{R}$  is said to be rd-continuous if it is continuous at each right-dense point and if there exists a left limit in all left-dense points. The ( $\Delta$  derivative)  $f^\Delta$  of  $f$  is defined by

$$f^\Delta(t) = \lim_{\substack{s \rightarrow t \\ s \in U(t)}} \frac{f(\sigma(t)) - f(s)}{\sigma(t) - s}, \text{ where } U(t) = \mathbb{T} \setminus \{\sigma(t)\}.$$

The derivative and the forward jump operator are related by the useful formula

$$f^\sigma = f + \mu f^\Delta, \text{ where } f^\sigma := f \circ \sigma.$$

We will also make use of the following product and quotient rules for the derivative of the product  $f g$  and the quotient  $f/g$  ( $g g^\sigma \neq 0$ ) of two differentiable functions  $f$  and  $g$  :

$$(f g)^\Delta = f^\Delta g + f^\sigma g^\Delta, \text{ and } \left(\frac{f}{g}\right)^\Delta = \frac{f^\Delta g - f g^\Delta}{g g^\sigma}. \quad (6)$$

By using the product rule, the derivative of  $f(t) = (t - \alpha)^m$  for  $m \in \mathbb{N}$  and  $\alpha \in \mathbb{T}$  can be calculated as

$$f^\Delta(t) = \sum_{\nu=0}^{m-1} (\sigma(t) - \alpha)^\nu (t - \alpha)^{m-\nu-1}. \quad (7)$$

For  $a, b \in \mathbb{T}$  and a differentiable function  $f$ , the Cauchy integral of  $f^\Delta$  is defined by

$$\int_a^b f^\Delta(t) \Delta t = f(b) - f(a).$$

The integration by parts formula follows from (6) and reads

$$\int_a^b f^\Delta(t) g(t) \Delta t = f(t) g(t) \Big|_a^b - \int_a^b f^\sigma(t) g^\Delta(t) \Delta t.$$

To prove our main results, we will use the formula

$$(x^\gamma(t))^\Delta = \gamma \int_0^1 [h x^\sigma + (1-h)x]^\gamma dh x^\Delta(t). \quad (8)$$

which is a simple consequence of Keller's chain rule [2]. Also, we need the following lemma [5].

Lemma 1 Assume  $A$  and  $B$  are nonnegative constants,  $\lambda > 1$ , then

$$\lambda A B^{\lambda-1} - A^\lambda \leq (\lambda - 1) B^\lambda.$$

The reader is referred to [2] for more detailed and extensive developments in calculus on time scales.

### III. MAIN RESULTS

First, we state the oscillation criteria for (4).

Set

$$y(t) = x(t) + c(t)x(\tau(t)). \quad (9)$$

Theorem 1 Assume that (H1) - (H5) hold, Furthermore, suppose that there exists a positive  $\Delta$ -differentiable function  $g(t)$  such that for all sufficiently large  $T^*$ , and for all  $\delta(T) > T^*$ , we have

$$\limsup_{t \rightarrow \infty} \int_t^{\delta(t)} (\beta(s, T^*) g(s) Q(s) - \frac{\alpha(s) ((g^\Delta(s))_+)^{\gamma+1}}{(\gamma+1)^{\gamma+1} g^\gamma(s)}) \Delta s = \infty. \quad (10)$$

Then every solution of (4) is oscillatory on  $[t_0, \infty)_\mathbb{T}$ .

proof Suppose (4) has a nonoscillatory solution  $x(t)$ . without loss of generality, there exists some  $t_1 \geq t_0$ , sufficiently large such that  $x(t) > 0$ ,  $x(\tau(t)) > 0$ ,  $x(\delta(t)) > 0$  for all  $t \geq t_1$ . Hence In the view of (9), by (H1) we get  $y(t) > 0$ . from (4) and by (H2) - (H5), we have that

$$\alpha(y^\Delta)^\gamma \leq -(q(t) - p(t))x^\gamma(\delta(t)) < 0,$$

and using the same proof of Theorem 1 [4], there exists  $t_2 \geq t_1$  such that for all  $t \geq t_2$ , we have

$$\begin{cases} y(t) > 0, y^\Delta(t) > 0, \\ (\alpha(y^\Delta)^\gamma)^\Delta \leq -(q(t) - p(t))(1 - c(\delta(t)))y^\gamma(\delta(t)) < 0. \end{cases} \quad (11)$$

By the definition of  $Q(t)$ , we get

$$(\alpha(y^\Delta)^\gamma)^\Delta \leq -Q(t)y^\gamma(\delta(t)) < 0. \quad (12)$$

Make the generalized Riccati substitution

$$w(t) = g(t) \frac{\alpha(t)(y^\Delta(t))^\gamma}{y^\gamma(t)}. \quad (13)$$

By the product and quotient rules, we have for all  $t \geq t_2$

$$\begin{aligned} w^\Delta(t) &= \frac{g(t)(\alpha(t)(y^\Delta(t))^\gamma)^\Delta}{y^\gamma(t)} + \left(\frac{g(t)}{y^\gamma(t)}\right)^\Delta (\alpha(t)(y^\Delta(t))^\gamma)^\sigma \\ &= \frac{g(t)(\alpha(t)(y^\Delta(t))^\gamma)^\Delta}{y^\gamma(t)} + \\ &\quad \left(\frac{g^\Delta(t)}{y^{\gamma\sigma}(t)} - \frac{g(t)(y^\gamma(t))^\Delta}{y^\gamma(t)y^{\gamma\sigma}(t)}\right) (\alpha(t)(y^\Delta(t))^\gamma)^\sigma. \end{aligned} \quad (14)$$

From (12) - (14), we obtain

$$\begin{aligned} w^\Delta(t) &\leq -g(t)Q(t) \left(\frac{y(\delta(t))}{y(t)}\right)^\gamma + \frac{g^\Delta(t)}{g^\sigma(t)} w^\sigma(t) \\ &\quad - \frac{g(t)w^\sigma(t)}{g^\sigma(t)} \frac{(y^\gamma(t))^\Delta}{y^\gamma(t)}. \end{aligned} \quad (15)$$

First consider the case when  $\delta(t) \geq t$ . For all large  $t$ , from  $y^\Delta(t) > 0$ , we have

$$\frac{y(\delta(t))}{y(t)} \geq 1,$$

which implies that

$$w^\Delta(t) \leq -g(t)Q(t) + \frac{g^\Delta(t)}{g^\sigma(t)} w^\sigma(t) - \frac{g(t)w^\sigma(t)}{g^\sigma(t)} \frac{(y^\gamma(t))^\Delta}{y^\gamma(t)}. \quad (16)$$

Next consider the case when  $\delta(t) \leq t$ , for all large  $t$ . By using  $\alpha(y^\Delta)^\gamma$  is strictly decreasing on  $[t_2, \infty)$ , we can choose  $t_3 \geq t_2$  such that  $\delta(t) \geq t_2$ , for  $t \geq t_3$ . Then we obtain

$$y(t) - y(\delta(t)) = \int_{\delta(t)}^t \frac{(\alpha(s)y^\Delta(s))^{1/\gamma}}{\alpha^{1/\gamma}(s)} \Delta s$$

$$\leq (\alpha(\delta(t))(y^\Delta(\delta(t)))^\gamma)^{1/\gamma} \int_{\delta(t)}^t \frac{\Delta s}{\alpha^{1/\gamma}(s)},$$

and hence

$$\frac{y(t)}{y(\delta(t))} \leq 1 + \frac{(\alpha(\delta(t))(y^\Delta(\delta(t)))^\gamma)^{1/\gamma}}{y(\delta(t))} \int_{\delta(t)}^t \frac{\Delta s}{\alpha^{1/\gamma}(s)}. \tag{17}$$

Also, for  $t \geq t_3$ , we can see that

$$y(\delta(t)) > y(\delta(t)) - y(t_2) = \int_{t_2}^{\delta(t)} \frac{(\alpha(s)y^\Delta(s))^{1/\gamma}}{\alpha^{1/\gamma}(s)} \Delta s$$

$$\geq (\alpha(\delta(t))(y^\Delta(\delta(t)))^\gamma)^{1/\gamma} \int_{t_2}^{\delta(t)} \frac{\Delta s}{\alpha^{1/\gamma}(s)},$$

and therefore

$$\frac{(\alpha(\delta(t))(y^\Delta(\delta(t)))^\gamma)^{1/\gamma}}{y(\delta(t))} \leq \left( \int_{t_2}^{\delta(t)} \frac{\Delta s}{\alpha^{1/\gamma}(s)} \right)^{-1}.$$

From (17) and the above inequality, we have

$$\frac{y(t)}{y(\delta(t))} \leq \int_{t_2}^t \frac{\Delta s}{\alpha^{1/\gamma}(s)} \left( \int_{t_2}^{\delta(t)} \frac{\Delta s}{\alpha^{1/\gamma}(s)} \right)^{-1}, \tag{18}$$

therefore we get the desired inequality

$$\frac{y(\delta(t))}{y(t)} \geq \rho(t, t_2), \text{ for } t \geq t_3. \tag{19}$$

Using (19) in (15), when  $\delta(t) \leq t$ , we get

$$w^\Delta(t) \leq -\rho^\gamma(t, t_2)g(t)Q(t) + \frac{g^\Delta(t)}{g^\sigma(t)}w^\sigma(t)$$

$$- \frac{g(t)w^\sigma(t)}{g^\sigma(t)} \frac{(y^\gamma(t))^\Delta}{y^\gamma(t)}. \tag{20}$$

From (16), (20) and the definition of  $\beta(t, t_2)$ , we have

$$w^\Delta(t) \leq -\beta(t, t_2)g(t)Q(t) + \frac{g^\Delta(t)}{g^\sigma(t)}w^\sigma(t)$$

$$- \frac{g(t)w^\sigma(t)}{g^\sigma(t)} \frac{(y^\gamma(t))^\Delta}{y^\gamma(t)}. \tag{21}$$

By (8), we obtain

$$(y^\gamma(t))^\Delta = \gamma \int_0^1 [hy^\sigma + (1-h)y]^{\gamma-1} dh y^\Delta(t)$$

$$\geq \begin{cases} \gamma(y^\sigma(t))^{\gamma-1} y^\Delta(t), & 0 < \gamma \leq 1, \\ \gamma(y(t))^{\gamma-1} y^\Delta(t), & \gamma \geq 1. \end{cases}$$

Since  $\alpha(y^\Delta)^\gamma$  is strictly decreasing on  $[t_2, \infty)$ , we get

$$(y^\gamma(t))^\Delta \geq \begin{cases} \frac{\gamma(\alpha^\sigma(t))^{1/\gamma} (y^\sigma(t))^{\gamma-1} (y^\Delta(t))^\sigma}{\alpha^{1/\gamma}(t)}, & 0 < \gamma \leq 1, \\ \frac{\gamma(\alpha^\sigma(t))^{1/\gamma} (y(t))^{\gamma-1} (y^\Delta(t))^\sigma}{\alpha^{1/\gamma}(t)}, & \gamma \geq 1. \end{cases}$$

From the last inequality and (21), if  $0 < \gamma \leq 1$ , we have

$$w^\Delta(t) \leq -\beta(t, t_2)g(t)Q(t) + \frac{g^\Delta(t)}{g^\sigma(t)}w^\sigma(t) -$$

$$\frac{\gamma g(t)(w^\sigma(t))^{1+1/\gamma}}{\alpha^{1/\gamma}(t)(g^\sigma(t))^{1+1/\gamma}} \left( \frac{y^\sigma(t)}{y(t)} \right)^\gamma,$$

whereas if  $\gamma > 1$ , we find that

$$w^\Delta(t) \leq -\beta(t, t_2)g(t)Q(t) + \frac{g^\Delta(t)}{g^\sigma(t)}w^\sigma(t)$$

$$- \frac{\gamma g(t)(w^\sigma(t))^{1+1/\gamma}}{\alpha^{1/\gamma}(t)(g^\sigma(t))^{1+1/\gamma}} \frac{y^\sigma(t)}{y(t)}.$$

And by using  $y^\Delta(t) > 0$ , we obtain that

$$w^\Delta(t) \leq -\beta(t, t_2)g(t)Q(t) + \frac{g^\Delta(t)_+}{g^\sigma(t)}w^\sigma(t)$$

$$- \frac{\gamma g(t)}{\alpha^{1/\gamma}(t)(g^\sigma(t))^\lambda} (w^\sigma(t))^\lambda, \tag{22}$$

where  $\lambda := (\gamma + 1)/\gamma$ . Define  $A \geq 0$  and  $B \geq 0$  by

$$A^\lambda := \frac{\gamma g(t)(w^\sigma(t))^\lambda}{\alpha^{1/\gamma}(t)(g^\sigma(t))^\lambda}, \quad B^{\lambda-1} := \frac{\alpha^{1/(\gamma+1)}(t)(g^\Delta(t))_+}{\lambda(\gamma g(t))^{1/\lambda}},$$

then using Lemma 1, we obtain

$$\frac{g^\Delta(t)_+}{g^\sigma(t)}w^\sigma(t) - \frac{\gamma g(t)}{\alpha^{1/\gamma}(t)(g^\sigma(t))^\lambda} (w^\sigma(t))^\lambda \leq \frac{\alpha(t)((g^\Delta(t))_+)^{\gamma+1}}{(\gamma+1)^{\gamma+1} g^\gamma(t)}.$$

From the last inequality and (22), we have

$$w^\Delta(t) \leq \frac{\alpha(t)((g^\Delta(t))_+)^{\gamma+1}}{(\gamma+1)^{\gamma+1} g^\gamma(t)} - \beta(t, t_2)g(t)Q(t).$$

Integrating both sides from  $t_3$  to  $t$ , we get

$$\int_{t_3}^t [\beta(s, t_2)g(s)Q(s) - \frac{\alpha(s)((g^\Delta(s))_+)^{\gamma+1}}{(\gamma+1)^{\gamma+1} g^\gamma(s)}] \Delta s$$

$$\leq w(t_3) - w(t) \leq w(t_3),$$

which leads to a contradiction to (10). This completes the proof.

Corollary 1 Assume that (H1) - (H5) hold, Furthermore, suppose that for all sufficiently large  $T^*$ , and for  $\delta(T) > T^*$ , we have

$$\limsup_{t \rightarrow \infty} \int_T^t (s\beta(s, T^*)Q(s) - \frac{\alpha(s)}{(\gamma+1)^{\gamma+1} s^\gamma}) \Delta s = \infty.$$

Then every solution of (4) is oscillatory on  $[t_0, \infty)_T$ .

Corollary 2 Assume that (H1) - (H5) hold, Furthermore, suppose that for all sufficiently large  $T^*$ , and for  $\delta(T) > T^*$ , we have

$$\limsup_{t \rightarrow \infty} \int_T^t \beta(s, T^*)Q(s) \Delta s = \infty.$$

Then every solution of (4) is oscillatory on  $[t_0, \infty)_{\mathbb{T}}$ .

We next study a Philos-type oscillation criteria for (4). First, Let us introduce the class of functions  $\mathfrak{R}$  which will be extensively used in the sequel.

Let  $\mathbb{D} = \{(t, s) \in \mathbb{T}^2 : t \geq s \geq t_0\}$ . The function  $H \in C_{rd}(\mathbb{D}, \mathbb{R})$  is said to belong to the class  $\mathfrak{R}$  by  $H \in \mathfrak{R}$ , if

$$H(t, t) = 0, t \geq t_0; H(t, s) > 0, t > s \geq t_0, \quad (23)$$

and  $H$  has a continuous  $\Delta$ -partial derivative  $H^{\Delta_s}(t, s)$  with respect to the second variable.

Theorem 2 Assume that (H1) - (H5) hold. Let  $g(t)$  be as defined in Theorem 1, and  $H, h \in C_{rd}(\mathbb{D}, \mathbb{R})$  such that  $H \in \mathfrak{R}$ . Furthermore, suppose that there exists a positive rd-continuous function  $\varphi(t)$  satisfies

$$\frac{H(t, s)}{H(t, t_0)} \leq \varphi(s), \quad (24)$$

$$-H^{\Delta_s}(t, s) - H(t, s) \frac{g^{\Delta}(s)}{g^{\sigma}(s)} = \frac{h(t, s)}{g^{\sigma}(s)} (H(t, s))^{\gamma/(\gamma+1)}, \quad (25)$$

and for all sufficiently large  $T^*$ , we have

$$\limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t [\beta(s, T^*) g(s) Q(s) H(t, s) - \frac{\alpha(s) (h_-(t, s))^{\gamma+1}}{(\gamma+1)^{\gamma+1} g^{\gamma}(s)}] \Delta s = \infty. \quad (26)$$

Then every solution of (4) is oscillatory on  $[t_0, \infty)_{\mathbb{T}}$ .

Proof Suppose (4) has a nonoscillatory solution  $x(t)$ , without loss of generality, say  $x(t) > 0, x(\tau(t)) > 0, x(\delta(t)) > 0$ , for all  $t \geq t_1$ , for some  $t_1 \geq t_0$ . By (H2) - (H5), proceed as in the proof of Theorem 1, we get that (11) holds for all  $t \geq t_1$ . Again we define  $w(t)$  as in the proof of Theorem 1, then there exists  $t_2 \geq t_1$ , sufficiently large such that for all  $t^* \geq t_2$  and for  $t \geq t^*$ , (22) holds and let  $g^{\Delta}(t)$ , be replaced by  $g^{\Delta}(t)$  in (22), thus

$$\beta(t, t_2) g(t) Q(t) \leq -w^{\Delta}(t) + \frac{g^{\Delta}(t)}{g^{\sigma}(t)} w^{\sigma}(t) - \frac{\gamma g(t)}{\alpha^{1/\gamma}(t) (g^{\sigma}(t))^{\gamma}} (w^{\sigma}(t))^{\gamma}. \quad (27)$$

Multiplying both the sides of (27), with  $t$  replaced by  $s$ , by  $H(t, s)$  and integrating with respect to  $s$  from  $t^*$  to  $t$ , we obtain

$$\int_{t^*}^t H(t, s) \beta(s, t_2) g(s) Q(s) \Delta s \leq -\int_{t^*}^t H(t, s) w^{\Delta}(s) \Delta s + \int_{t^*}^t H(t, s) \frac{g^{\Delta}(s)}{g^{\sigma}(s)} w^{\sigma}(s) \Delta s - \int_{t^*}^t H(t, s) \frac{\gamma g(s)}{\alpha^{1/\gamma}(s) (g^{\sigma}(s))^{\gamma}} (w^{\sigma}(s))^{\gamma} \Delta s.$$

Integrating by parts formula and using (23) and (25), we get

$$\int_{t^*}^t H(t, s) \beta(s, t_2) g(s) Q(s) \Delta s \leq H(t, t^*) w(t^*) + \int_{t^*}^t \left[ \frac{h(t, s) (H(t, s))^{\gamma/2}}{g^{\sigma}(s)} w^{\sigma}(s) - \frac{\gamma H(t, s) g(s)}{\alpha^{1/\gamma}(s) (g^{\sigma}(s))^{\gamma}} (w^{\sigma}(s))^{\gamma} \right] \Delta s. \quad (28)$$

And applying Lemma 1, we obtain

$$\frac{h_-(t, s) (H(t, s))^{\gamma/2}}{g^{\sigma}(s)} w^{\sigma}(s) - \frac{\gamma H(t, s) g(s)}{\alpha^{1/\gamma}(s) (g^{\sigma}(s))^{\gamma}} (w^{\sigma}(s))^{\gamma} \leq \frac{(h_-(t, s))^{\gamma+1} \alpha(s)}{(\gamma+1)^{\gamma+1} g^{\gamma}(s)}.$$

From the last inequality and (24), (28), we have

$$\frac{1}{H(t, t_0)} \int_{t_0}^t [\beta(s, t_2) g(s) Q(s) H(t, s) - \frac{(h_-(t, s))^{\gamma+1} \alpha(s)}{(\gamma+1)^{\gamma+1} g^{\gamma}(s)}] \Delta s \leq \varphi(t^*) w(t^*) + \int_{t_0}^{t^*} \varphi(s) \beta(s, t_2) g(s) Q(s) \Delta s < \infty,$$

which contradicts with (26). The proof is completed

Remark 1 If  $H^{\Delta_s}(t, s) \leq 0$  holds for  $t \geq s \geq t_0$ , then (24) holds (It is easily proved). But, the converse is not true. For instance, let  $H(t, s) = (t-s)^m \mu(s), t \geq s \geq t_0$ , where  $\mu: \mathbb{T} \rightarrow \mathbb{R}^+$  is rd-continuously differential function,  $m \geq 1$  is an integer. Clearly  $H \in \mathfrak{R}$ , and for all  $t \geq s \geq t_0$ , from (6) and (7), we have

$$\frac{H(t, s)}{H(t, t_0)} \leq \frac{(t-s)^m \mu(s)}{(t-t_0)^m \mu(t_0)} = \varphi(s),$$

$$H^{\Delta_s}(t, s) = -\sum_{\nu=0}^{m-1} (t-\sigma(s))^{\nu} (t-s)^{m-\nu-1} \mu(s) + (t-\sigma(s))^m \mu^{\Delta}(s) \leq -m(t-\sigma(s))^{m-1} \mu(s) + (t-\sigma(s))^m \mu^{\Delta}(s),$$

by Remark 3.3 in [7]. Clearly the right side of the second inequality is not necessarily nonpositive. Therefore  $H^{\Delta_s}(t, s)$  is not necessarily nonpositive for  $t \geq s \geq t_0$ .

In Theorem 2, let  $g(t) = 1$  and  $H(t, s) = (t-s)^m$ , we have the following result.

Corollary 3 Assume that (H1) - (H5) hold, and  $m \geq 1$ , for all sufficiently large  $T^*$ , we have

$$\limsup_{t \rightarrow \infty} \frac{1}{t^m} \int_{t_0}^t (t-s)^m \beta(s, T^*) Q(s) \Delta s = \infty.$$

Then every solution of (4) is oscillatory on  $[t_0, \infty)_{\mathbb{T}}$ .

Next, we state the oscillation criteria for (5). Set

$$z(t) = x(t) - c(t)x(\tau(t)).$$

Theorem 3 Assume that (H1) - (H5) hold, Furthermore, suppose that there exists a positive  $\Delta$ -differentiable function  $g(t)$  such that for all sufficiently large  $T^*$ , and for all  $\delta(T) > T^*$ , we have

$$\limsup_{t \rightarrow \infty} \int_T^t [\beta(s, T^*)g(s)(q(s) - p(s)) - \frac{\alpha(s)((g^\Delta(s))_+)^{\gamma+1}}{(\gamma+1)^{\gamma+1}g^\gamma(s)}] \Delta s = \infty. \tag{29}$$

Then every solution of (5) is either oscillatory on  $[t_0, \infty)_T$  or tends to zero.

proof Suppose that  $x$  is an eventually positive solution of (5), say  $x(t) > 0$ ,  $x(\tau(t)) > 0$ ,  $x(\delta(t)) > 0$  for all  $t \geq t_1$  for some  $t_1 \geq t_0$ . We consider only this case, because the proof for the case that  $x$  is eventually negative is similar. In the view of (5), by (H2) - (H5), and there exists  $t_2 \geq t_1$  such that for all  $t \geq t_2$ , we have

$$(\alpha(z^\Delta)^\gamma)^\Delta \leq -(q(t) - p(t))x^\gamma(\delta(t)) < 0, \tag{30}$$

then  $\alpha(z^\Delta)^\gamma$  is strictly decreasing on  $[t_2, \infty)$ . Hence  $z(t)$  and  $z^\Delta(t)$  are of constant sign eventually. We claim that  $x(t)$  is bounded. If not, there exists  $\{t_k\} \subseteq [t_2, \infty)$ , such that  $\lim_{k \rightarrow \infty} t_k = \infty$ ,  $\lim_{k \rightarrow \infty} x(t_k) = \infty$ , and

$$x(t_k) = \max\{x(s) : t_0 \leq s \leq t_k\}.$$

Since  $\lim_{k \rightarrow \infty} \tau(t_k) = \infty$ , we can choose a large  $k$  such that  $\tau(t_k) > t_0$ , and by (H2), we obtain that

$$\begin{aligned} x(\tau(t_k)) &= \max\{x(s) : t_0 \leq s \leq \tau(t_k)\} \\ &\leq \max\{x(s) : t_0 \leq s \leq t_k\} = x(t_k). \end{aligned}$$

Therefore, for all large  $k$ ,

$$z(\tau(t_k)) \geq x(t_k) - c_0 x(\tau(t_k)) \geq (1 - c_0)x(t_k),$$

and  $\lim_{k \rightarrow \infty} z(t_k) = \infty$ . From (H1) and (30), as in the proof of Theorem 1 [4], there exists  $t_3 \geq t_2$  such that for all  $t \geq t_3$ , we have

$$z(t) > 0, z^\Delta(t) > 0. \tag{31}$$

In view of (5), (30) and (31), we get

$$(\alpha(z^\Delta)^\gamma)^\Delta \leq -(q(t) - p(t))z^\gamma(\delta(t)) < 0. \tag{32}$$

Now by using the same proof of Theorem 1, we get a contradiction with (29). Thus  $x(t)$  is bounded and hence  $z(t)$  is bounded.

Also, by using (H1) and the same proof of Theorem 1 in [4], there exist  $t_4 \geq t_3$  such that  $z^\Delta(t) > 0$  on  $[t_4, \infty)$ . There are two cases.

Case 1  $z(t) > 0$  and  $z^\Delta(t) > 0$ . As in the proof of Theorem 1, we get a contradiction with (29).

Case 2  $z(t) < 0$  and  $z^\Delta(t) > 0$ . We claim  $\lim_{t \rightarrow \infty} x(t) = 0$ .

Assume not, then there exists  $\{t_k\} \subseteq [t_5, \infty)$  such that  $\lim_{k \rightarrow \infty} t_k = \infty$ ,  $\lim_{k \rightarrow \infty} x(t_k) =: b > 0$  and  $x(t_k) = \max\{x(s) : t_0 \leq s \leq t_k\}$ . But, by  $x(\tau(t_k)) \leq x(t_k)$ , we get

$$0 > z(t_k) \geq x(t_k)(1 - c_0) \rightarrow b(1 - c_0) > 0, \text{ as } k \rightarrow \infty.$$

Which is a contradiction. This completes the proof.

Corollary 4 Assume that (H1) - (H5) hold, Furthermore, suppose that for all sufficiently large  $T^*$ , and for  $\delta(T) > T^*$ , we have

$$\limsup_{t \rightarrow \infty} \int_T^t (s\beta(s, T^*)(q(s) - p(s)) - \frac{\alpha(s)}{(\gamma+1)^{\gamma+1}s^\gamma}) \Delta s = \infty.$$

Then every solution of (5) is either oscillatory on  $[t_0, \infty)_T$  or tends to zero.

Corollary 5 Assume that (H1) - (H5) hold, Furthermore, suppose that for all sufficiently large  $T^*$ , and for  $\delta(T) > T^*$ , we have

$$\limsup_{t \rightarrow \infty} \int_T^t \beta(s, T^*)(q(s) - p(s)) \Delta s = \infty.$$

Then every solution of (5) is either oscillatory on  $[t_0, \infty)_T$  or tends to zero.

We next study a Philos-type oscillation criteria for (5).

Theorem 4 Assume that (H1) - (H5) hold. Let  $g(t)$  be as defined in Theorem 1, and  $H, h \in C_{rd}(\mathbb{D}, \mathbb{R})$  such that  $H \in \mathfrak{R}$ . Furthermore, suppose that there exists a positive rd-continuous function  $\varphi(t)$  such that (24), (25) hold, and for all sufficiently large  $T^*$ , we have

$$\begin{aligned} \limsup_{t \rightarrow \infty} \frac{1}{H(t, t_0)} \int_{t_0}^t \{ \beta(s, T^*)g(s)(q(s) - p(s))H(t, s) \\ - \frac{\alpha(s)(h_-(t, s))^{\gamma+1}}{(\gamma+1)^{\gamma+1}g^\gamma(s)} \} \Delta s = \infty. \end{aligned} \tag{33}$$

Then every solution of (5) is either oscillatory on  $[t_0, \infty)_T$  or tends to zero.

Proof Suppose that (5) has a nonoscillatory solution  $x(t)$ , without loss of generality, say  $x(t) > 0$ ,  $x(\tau(t)) > 0$ ,  $x(\delta(t)) > 0$ , for all  $t \geq t_1$ , for some  $t_1 \geq t_0$ . By (H2) - (H5), we obtain that (30) holds for all  $t \geq t_1$ , and  $z(t)$  and  $z^\Delta(t)$  are of constant sign eventually. Similar to the proof of Theorem 3, we claim that  $x(t)$  is bounded. If not, there exists  $\{t_k\} \subseteq [t_1, \infty)$ , for all large  $k$ , there exists  $t_2 \geq t_1$ , such that (31) and (32) hold for  $t \geq t_2$ . Again we define  $w(t)$  as in the proof of Theorem 1, then there exists  $t_3 \geq t_2$ , sufficiently large such that for  $t^* \geq t_3$  and for  $t \geq t^*$ , we find

$$\begin{aligned} &\beta(t, t_3)g(t)(q(t) - p(t)) \\ &\leq -w^\Delta(t) + \frac{g^\Delta(t)}{g^\sigma(t)}w^\sigma(t) - \frac{\gamma g(t)}{\alpha^{1/\gamma}(t)(g^\sigma(t))^\Delta} (w^\sigma(t))^\Delta. \end{aligned} \tag{34}$$

And similar to the proof of the theorem 3, we obtain

$$\begin{aligned} &\frac{1}{H(t, t_0)} \int_{t_0}^t [\beta(s, t_3)g(s)(q(s) - p(s))H(t, s) \\ &- \frac{(h_-(t, s))^{\gamma+1} \alpha(s)}{(\gamma+1)^{\gamma+1} g^\gamma(s)}] \Delta s \leq \varphi(t^*)w(t^*) + \end{aligned}$$

$$\int_{t_0}^t \varphi(s)\beta(s, t_3)g(s)(q(s) - p(s))\Delta s < \infty,$$

which contradicts with (33). Thus  $x(t)$  is bounded and hence  $z(t)$  is bounded.

Similar to the proof of Theorem in [4], there exists  $t_4 \geq t^*$  such that  $z^\Delta(t) > 0$  on  $[t_4, \infty)$ . And then there are two cases of Theorem 3. As in the proof of Theorem 3, if the case 1 holds, we get a contradiction with (33); if the case 2 holds, we obtain  $\lim_{t \rightarrow \infty} x(t) = 0$ . This completes the proof.

In Theorem 4, let  $g(t) = 1$  and  $H(t, s) = (t - s)^m$ , we have the following result.

Corollary 6 Assume that (H1) - (H5) hold, and  $m \geq 1$ , for all sufficiently large  $T^*$ , we have

$$\limsup_{t \rightarrow \infty} \frac{1}{t^m} \int_{t_0}^t (t - s)^m \beta(s, T^*)(q(s) - p(s))\Delta s = \infty.$$

Then every solution of (5) is either oscillatory on  $[t_0, \infty)_T$  or tends to zero.

IV. EXAMPLES

In this section, we give some examples to illustrate our main results. Define

$$\xi(t) = \begin{cases} 1, & \delta(t) \geq t, \\ \rho^\gamma(t, t_0), & \delta(t) \leq t. \end{cases}$$

Note that  $\int_{t_0}^\infty \Delta t / (\alpha(t))^{1/\gamma} = \infty$ , implies  $\lim_{t \rightarrow \infty} \frac{\beta(t, T^*)}{\xi(t)} = 1$ .

Example 1 Consider the nonlinear neutral perturbed dynamic equation

$$\begin{aligned} & (t^{\gamma-1}((x(t) \pm \frac{1}{t+1}x(\tau(t)))^\Delta)^\gamma)^\Delta \\ & + F(t, x(\delta(t))) = G(t, x(\delta(t)), x^\Delta), \end{aligned} \tag{35}$$

for  $t \in [1, \infty)_T$ , where  $\gamma$  is the quotient of odd positive integers. Let

$$\begin{aligned} \alpha(t) &= t^{\gamma-1}, \quad F(t, u) = \left( \frac{k(1 + \delta(t))^\gamma}{t^2 \delta^\gamma(t) \xi(t)} + \frac{1}{t^4} + u^2 \right) u^\gamma, \\ c(t) &= \frac{1}{t+1}, \quad G(t, u, v) = \frac{k(1 + \delta(t))^\gamma}{2t^2 \delta^\gamma(t) \xi(t)} \frac{u^{\gamma+2}}{(u^2 + v^2 + 1)}, \end{aligned}$$

where  $k$  is a positive constant. Then

$$Q(t) = k/2t^2 \xi(t).$$

Since  $\int_{t_0}^\infty \Delta t / (\alpha(t))^{1/\gamma} = \int_{t_0}^\infty \Delta t / t^{\gamma-1/\gamma} = \infty$ , hence the conditions (H1) - (H5) are clearly satisfied. And,

$$\begin{aligned} & \limsup_{t \rightarrow \infty} \int_T^t (s\beta(s, T^*)g(s)Q(s) - \frac{\alpha(s)}{(\gamma+1)^{\gamma+1} s^\gamma})\Delta s \\ &= \left( \frac{k}{2} - \frac{1}{(\gamma+1)^{\gamma+1}} \right) \limsup_{t \rightarrow \infty} \int_T^t \frac{\Delta s}{s} = \infty, \end{aligned}$$

if  $k > 2/(\gamma+1)^{\gamma+1}$ . Also,

$$\begin{aligned} & \limsup_{t \rightarrow \infty} \int_T^t (s\beta(s, T^*)g(s)(q(s) - p(s)) - \frac{\alpha(s)}{(\gamma+1)^{\gamma+1} s^\gamma})\Delta s \\ &= \left( \frac{k}{2} - \frac{1}{(\gamma+1)^{\gamma+1}} \right) \limsup_{t \rightarrow \infty} \int_T^t \frac{\Delta s}{s} = \infty, \end{aligned}$$

if  $k > 2/(\gamma+1)^{\gamma+1}$ . Thus it follows from Corollary 1 that every solution of (35) + is oscillatory on  $[1, \infty)_T$  if  $k > 2/(\gamma+1)^{\gamma+1}$ , and it follows from Corollary 4 that every solution of (35) - is either oscillatory on  $[1, \infty)_T$  or tends to zero if  $k > 2/(\gamma+1)^{\gamma+1}$ .

Example 2 Consider the nonlinear neutral perturbed dynamic equation

$$\begin{aligned} & (t^{2/3}((x(t) - \frac{1}{2 + \sin^2 t}x(\tau(t)))^\Delta)^{5/3})^\Delta \\ & + F(t, x(\delta(t))) = G(t, x(\delta(t)), x^\Delta). \end{aligned} \tag{36}$$

for  $t \in [2, \infty)_T$ , where  $\alpha(t) = t^{2/3}$ ,  $\gamma = 5/3$ ,  $c(t) = 1/(2 + \sin^2 t)$ . Let

$$F(t, u) = \left( \frac{1}{t \xi(t)} + t^4 + u^2 \right) u^{5/3},$$

and

$$G(t, u, v) = \frac{1}{2t \xi(t)} \frac{u^{11/3}}{(u^2 + v^4 + 2)}.$$

Then  $q(t) - p(t) = 1/2t \xi(t)$ . The conditions (H1) - (H5) are clearly satisfied. For all  $t > s \geq 2$ , let  $m=2$ , we have

$$\begin{aligned} & \limsup_{t \rightarrow \infty} \frac{1}{t^2} \int_2^t (t-s)^2 \beta(s, T^*)(q(s) - p(s))\Delta s \\ &= \limsup_{t \rightarrow \infty} \frac{1}{t^2} \int_2^t \frac{(t-s)^2}{2s} \Delta s \\ &= \limsup_{t \rightarrow \infty} \left[ \frac{1}{t^2} \int_2^t \frac{s}{2} \Delta s + \int_2^t \frac{1}{2s} \Delta s - \frac{t-2}{t} \right] = \infty. \end{aligned}$$

Thus it follows from Corollary 6 that every solution of (36) is either oscillatory on  $[2, \infty)_T$  or tends to zero.

IV. CONCLUSIONS

To investigate the oscillatory and asymptotic behavior for a certain class of second order nonlinear neutral perturbed dynamic equations on time scales. This paper proposed some new sufficient conditions for oscillation of such dynamic equations on time scales were established. The results not only improve and extend some known results in the literature, but also unify the oscillation of second order nonlinear neutral perturbed differential equations and second order nonlinear neutral perturbed difference equations. In particular, the results are essentially new under the relaxed conditions for the parameter function.

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