Contour Error Coupled-Control Strategy based on Line Interpolation and Curve Interpolation

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***Abstract*—In practical machining, the multi-axis actual dynamic performances don't match well, which reduces the profile precision greatly. The computer numerical control (CNC) machine tools contour error coupled-control strategy based on line interpolation and curve interpolation is developed in the paper. After analyze the conventional CNC contour error control scheme, put forward the contour error coupled-control scheme based on line interpolation and curve interpolation; Then bring forward the contour error computing models based on line interpolation and curve interpolation; Furthermore, add the obtained contour error to the following error of current sampling period, and send the results to CNC PID position controller to calculate position controlled quantity in order to compensate contour error. The contour error compensation control experimentation results show that the developed approach can reduce contour error effectively and enhance profile precision further.**

***Index Terms*—machine tools, contour error, complex parts, linear interpolation, curve interpolation**

I. Introduction

In manufacturing fields many parts have complex profile, and the profile includes analytic curve, piecewise curve, listing curve and so on [1, 2]. In general, multi-axis CNC machine tools are adopted to process these complex parts, after approximating complex cutter position track instruction curve with straightway [3, 4]. To multi-axis CNC machine tools, the profile precision is the important factor to determine its machining accuracy [5, 6]. But the profile precision relates with the matching degree of all the linked axes dynamic performances, and is decided by both each-axis position accuracy and the multi-axis linked accuracy [7, 8]. Because CNC machine tools have complicated servo drive equipments, and the CNC system parameters may change in practical machining, the multi-axis actual dynamic performances don't match well, this reduces the profile precision [9, 10, 11]. In contrast to the advanced single-axis servo controller, the cross-coupled-controller is more effective to enhance profile precision [12, 13, 14], which computes the contour error and compensates each axis servo motor on each sampling period [15].

Some research results in point have been achieved recently. For instance, after introducing contour error transfer function, Syh-Shiuh Yeh transforms the multi-axis cross-coupled control to a single-input-single-output system, and defines the distance of actual cutter position to the tangent on reference curve current position as contour error [16]. Myung-Hoon LEE puts forward a multi-axis contour controller based on a contour error vector using parametric curve interpolation, which is a vector from the actual tool position to the nearest point on the desired path [17]. Peng Chao-Chung introduces a new contour index (CI) aimed to arc and line profile, which can be looked as an equivalent contour error such that a reduction in CI implies a reduction in contour error [18]. Aimed to profile curve in plane and space, Gen Lirong and Wang Baoren look the distance of actual position to the line which links the dots of the current and the last sampling period as the current contour error respectively [19-20]. Zhao Ximei and Guo Qingding achieve three-axis linked contour error control on basis of calculating XY, YZ, XZ axes plane coupling model [21]. Liu Yi and Cong Shuang develop a Frenet coordinate frame on a desired trajectory as the task coordinate frame, and the contour error is computed by the normal component of tracking error in the task coordinate frame [22]. Zhao Guoyong defines the distance between the actual cutter position and the nearest interpolation dot on cutter path curve as contour error on each sampling period [23]. However, because of inertia and frictional force, the hysteresis phenomena exist in truly CNC machine tool each axis movement, which is difficult to be foreseen accurately. As a result, the calculation error is uneasy to control if the hysteresis time is much longer than a sampling period.

Consequently, in the CNC machining on complex parts, how to compute contour error with high precision and distribute contour error correction quantity to enhance

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profile accuracy on each sampling period, has been a crucial problem for the researchers to settle. The CNC machine tools contour error coupled-control strategy based on line interpolation and curve interpolation is developed in the paper, which is with stable calculation error, high computing precision and satisfied real-time characteristic. Above all, analyze the conventional CNC contour error control scheme; Secondly, put forward the contour error coupled-control scheme based on line interpolation and curve interpolation; Thirdly, bring forward the contour error computing models based on line interpolation and curve interpolation; Then add the obtained contour error to the following error of current sampling period, and send the results to CNC PID position controller to calculate position controlled quantity in order to compensate contour error; Finally, the contour error compensation control experimentations are done on the three-axis linked CNC test table.

II. Conventional CNC Contour Error Control Scheme

A. Definition of Contour Error

The contour error is defined as the distance between the actual cutter trajectory and desired trajectory on the direction of trajectory normal. Considering a 2D arbitrary curve shown in Figure 1, let P* be the desired position vector, P be the actual position vector corresponding to P* on the desired contour, P_1 be position vector on the desired contour along the direction of curve normal that is closest to P, L be the tangent through P* on the desired contour, and θ be the angle between L and X axis. Then E is the following error between actual position and the instantaneous desired position of the cutter, i.e.,

$$E = P^* - P\,. \tag{1}$$

Let E_x be the part along X axis and E_y along Y axis of E. And the contour error can be expressed as:

$$\varepsilon = P_1 - P\,. \tag{2}$$

Let vector P plumbs tangent L on point P_1*, when the following error E is small on low federate. The contour error ε is approximately equal to ε^*, i.e.,

$$\varepsilon \approx \varepsilon^* = P_1{}^* - P = -E_x C_x + E_y C_y\,. \tag{3}$$

where C_x and C_y are computed by the following equations:

$$c_x = \sin\theta - E_x/(2\rho) \tag{4}$$

$$c_y = \cos\theta + E_y/(2\rho) \tag{5}$$

where ρ is the instantaneous radius of curvature.

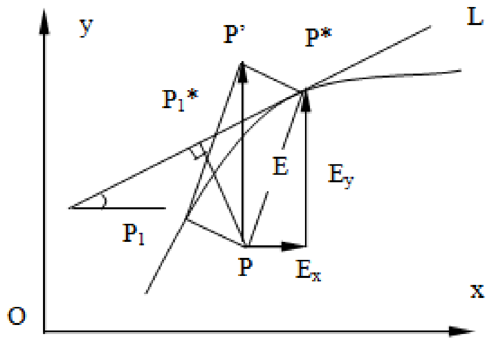

Figure 1. Definition of contour error

B. The Conventional CNC Contour Machining Scheme

Contour error is the maximal influence factor in CNC machine system. When machining on complex profile parts, conventionally, CAD/CAM systems have to segment a complex curve into a huge number of small linear segments and send them to CNC systems for linear interpolation machining. But the linear interpolation approach isn't able to achieve high speed and high accuracy at the same time. Conventional CNC contour machining scheme usually adopts position feedback controller to minimize following error, adopts feed forward controller to minimize tracking lag and contour deviation. In conventional cross-coupled control, the equation (3), which approximately computes contour error ε according to E, is adopted to establish the contour error model. Then the cross-coupled controller computes and distributes the correction signals to individual axis through some PID control algorithms. The cross-coupled control system is a multivariable, nonlinear and time-varying system, so it is very difficult to compute ε, θ and ρ. What is more, the approach to compute ε is only suited to condition when following error E is small in the low feed rate. Especially, this approach is difficult to compute contour error on multi-axes motion. So there are some difficulties in applying the approach to practical NC machining. The conventional two-axis CNC contour control scheme is shown in Figure 2.

III. Contour Error Coupled-control Scheme based on Line Interpolation and Curve Interpolation

After analyzing the conventional two-axis CNC contour machining scheme, put forward the contour error coupled-control scheme based on line interpolation and curve interpolation. As shown in Figure 3, firstly, adopt the linear interpolation or curve interpolation on the complex parts cutter path instruction curve, and measure the real worktable position; Secondly, compute the contour error based on interpolation dots and actual worktable position; Thirdly, compute the contour error correction quantity for x, y, z axes, and output the correction quantity to the x, y, z axes drivers and worktable.

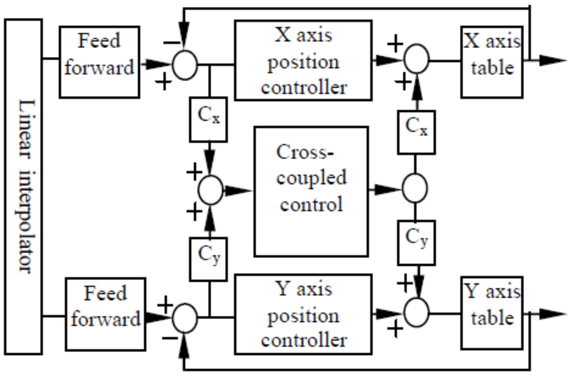

Figure 2. Conventional two-axis NC contour machining scheme

IV. Contour Error Computing Models based on Line Interpolation and Curve Interpolation

A. Contour Error Computing Model Based on Line Interpolation

The key idea of the developed contour error computing model is as followed: After approximating complex parts cutter position track instruction curve with straightway according to equi-error method, calculate the current actual cutter position coordinates owing to the position measure feedback from each axis and worktable on each line interpolation sampling period; Compute the minimum distance from current actual cutter position to cutter position track instruction curve according to the actual cutter position dots and the approximate nodes, in other words, to calculate the contour error.

As shown in Figure 4, the contour error computing model is explained more detailedly. Suppose to approximate part cutter track instruction curve L under the precision requirement with straightway AB, BC…, and define the actual cutter position as dot R on certain sampling period. Above all, obtain the three approximate nodes A, B, C nearest to actual cutter position R on the cutter position instruction curve L, and then calculate the distance $|RM|$, $|RN|$ from actual cutter position R to straightway AB, BC. It is noticed that the calculation is complicated if transform the distance from dot to line, to the maximum distance from dot to plane pencil through the line. Consequently, the vector method with the space analytic geometry and vector algebra theory is adopted to compute the distance $|RM|$, $|RN|$ from dot R to straightway AB, BC:

$$|RM| = \frac{|AB \times AR|}{|AB|} . \quad (6)$$

$$|RN| = \frac{|BC \times BR|}{|BC|} . \quad (7)$$

The coordinates of both approximate nodes A, B, C and actual cutter position R are known, so the calculations of Equation (6) and (7) are simple. After obtaining $|RM|$ and $|RN|$, the contour error ε is calculated according to two kinds of conditions.

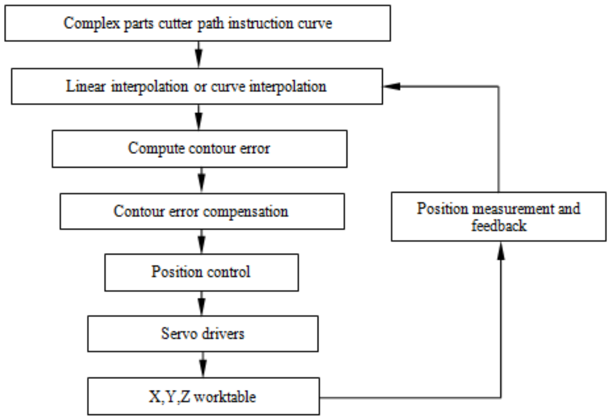

Figure 3. Contour error coupled-control scheme based on line interpolation and curve interpolation

If $|RM| \le |RN|$, obtain the contour error with Equation (8):

$$\varepsilon \approx RM . \quad (8)$$

As shown in Figure 4, the approximate error ST is constant, suppose the intersection point of RM and curve L be dot P, then the calculation error of Equation (8) is MP.

Because $|MP| \le |ST|$, the calculation error of contour error computing model is less than or equal to approximate error.

If $|RM| > |RN|$, obtain the contour error with Equation (9):

$$\varepsilon \approx RN . \quad (9)$$

In like manner, the calculation error of contour error computing model is less than or equal to approximate error.

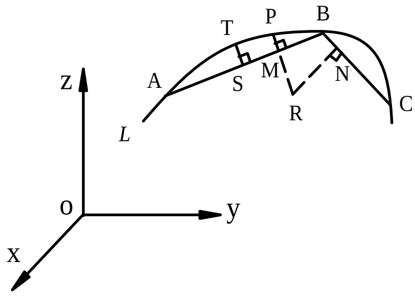

Figure 4. The contour error computing model based on line interpolation

B. NURBS Curve Interpolation Approach

NURBS can express free and analytical curve and surface unified with the advantages of smoothness and local controllability, and has been applied in the CAD/CAM fields successfully. So it's significant to investigate the NURBS curve direct interpolator in the CNC fields. At present except the FANUC and Siemens

CNC system with the NURBS curve direct interpolator, most of the CNC systems only have linear interpolation and arc interpolation function. The NURBS curve interpolation CNC machining flow is shown in Figure 5.

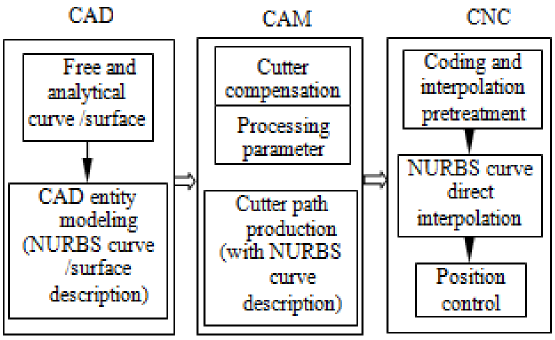

Figure 5. The NURBS curve interpolation CNC machining flow

The NURBS curve representation is given by

$$P(u) = \frac{\sum_{i=0}^{n} B_{i,k}(u) W_i V_i}{\sum_{i=0}^{n} B_{i,k}(u) W_i}. \quad (10)$$

where V_i is the control point, W_i is its weighting factor. By manipulating the values of control points and weights factor, a wise variety of part shapes can be designed using NURBS. Each point on the curve is corresponding to a certain knot parameter u.

In the I[th] interpolation period, NURBS curve interpolator computes the point $P(u_{i+1})$ and send $|P(u_{i+1}) - P(u_i)|$ as feed increment to servo controller. How to determine successive values of u_{i+1} such that appropriate feed increment length can be accurately generated is important and complicated in NURBS curve interpolation.

Taylor's second-order expansion is introduced in the NURBS curve interpolation algorithm in this paper aimed at the demands of high speed, high accuracy and real-time. The procedure for determining successive values of u is summarized in the following.

Let the NURBS curve be defined as $P(u) = (x(u), y(u), z(u))'$, then the knot factor u_{i+1} in $i+1$ interpolation period can be obtained:

$$u_{i+1} \approx u_i + \Delta T \frac{du}{dt}\Big|_{u=u_i} + \frac{\Delta T^2}{2} \frac{d^2u}{dt^2}\Big|_{u=u_i}. \quad (11)$$

As shown in Equation (11), the key is to compute $\frac{du}{dt}$ and real-time. Define the feed rate along the NURBS curve as:

$$v = \frac{ds}{dt} = \left|\frac{dP(u)}{dt}\right| = \left|\frac{dP(u)}{du}\right| \times \frac{du}{dt}. \quad (12)$$

Therefore,

$$\frac{du}{dt} = \frac{v}{|dP(u)/du|} = \frac{v}{\sqrt{(\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2}}. \quad (13)$$

$$\frac{d^2u}{dt^2} = \frac{dv/dt}{\sqrt{(\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2}} - \frac{v^2(\dot{x}\ddot{x} + \dot{y}\ddot{y} + \dot{z}\ddot{z})}{((\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2)^2} \quad (14)$$

Substituting the computed $\frac{du}{dt}$ and $\frac{d^2u}{dt^2}$ above into Equation (11), u_{i+1} will be obtained.

C. Contour Error Computing Model Based on Curve Interpolation

Conventionally, CAD/CAM systems have to segment a complex curve into a huge number of small linear segments and send them to CNC systems for linear interpolation machining. However, the experimental results show this approach can't achieve high speed and high accuracy at the same time. Especially, the interpolation dots aren't on the tracking curve. To overcome this problem, curve direct interpolation has to be adopted. Furthermore, according to interpolation dots in reference profile, a "three dots arc algorithm" contour error computing model is developed to calculate the minimal distance between actual dot and complex profile in each sampling period.

As shown in Figure 6, suppose the reference curve be L and cutter actual position be M (M_x, M_y, M_z) in some sampling period. Firstly, find the nearest interpolation point C_i from M on the curve, and suppose the two adjacent interpolation dots from C_i be C_{i-1} and C_{i+1} points. It is noteworthiness that all of the three dots are on curve L. Secondly, suppose the centre of the circle through C_{i-1}, C_i and C_{i+1} be point O (O_x, O_y, O_z), and the radius be r. Compute the contour error with Eq. (15):

$$|\varepsilon| = r - \overline{OM} = r - \sqrt{(M_x - O_x)^2 + (M_y - O_y)^2 + (M_z - O_z)^2}. \quad (15)$$

where, the contour error ε is along the $\overrightarrow{OM}$ direction.

Finally, decompose vector ε along x, y and z coordinate axis

$$\varepsilon_x = \varepsilon * \frac{M_x - O_x}{\sqrt{(M_x - O_x)^2 + (M_y - O_y)^2 + (M_z - O_z)^2}}. \quad (16)$$

$$\varepsilon_y = \varepsilon * \frac{M_y - O_y}{\sqrt{(M_x - O_x)^2 + (M_y - O_y)^2 + (M_z - O_z)^2}} . \quad (17)$$

$$\varepsilon_z = \varepsilon * \frac{M_z - O_z}{\sqrt{(M_x - O_x)^2 + (M_y - O_y)^2 + (M_z - O_z)^2}} . \quad (18)$$

In conclusion, the contour error computing model approximates curve L in locality with arc properly, so higher precision will be achieved.

V. Contour Error Compensation Approach

Except for the three PID position controller for X axis, Y axis and Z axis, Myung-Hoon LEE sets up an additional PID contour error controller [17]. The calculation approach is rather complicated. In the paper the contour error control compensation approach is developed, which adds the obtained contour error to the following error of current sampling period, and sends the result to CNC PID position controller to calculate position controlled quantity. The CNC contour error calculation and compensation program flow chart is shown in Figure 7.

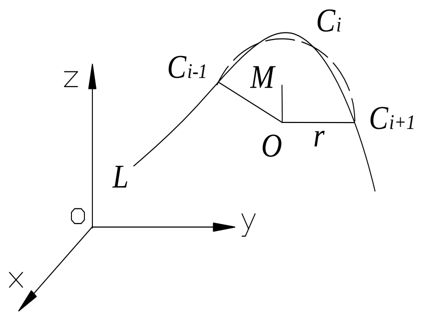

Figure 6. The contour error computing model based on curve interpolation

Firstly, after receiving the N[th] machining program segment coding and pretreatment results on the K[th] sampling period, interpolate and obtain following error E_x, E_y, E_z; Secondly, adopt the contour error computing model, and calculate contour error ε with Equation (8), Equation (9) or Equation (15) ; Thirdly, decompose ε to ε_x, ε_y, ε_z along X, Y, Z coordinate axes, and compute each axis optimal displacement of current sampling period after contour error compensation, which is μ_x, μ_y, μ_z; Finally, input the μ_x, μ_y, μ_z to X, Y, Z coordinate axes PID position controller respectively, and compute the correction quantity to control X, Y, Z coordinate axes servo motors.

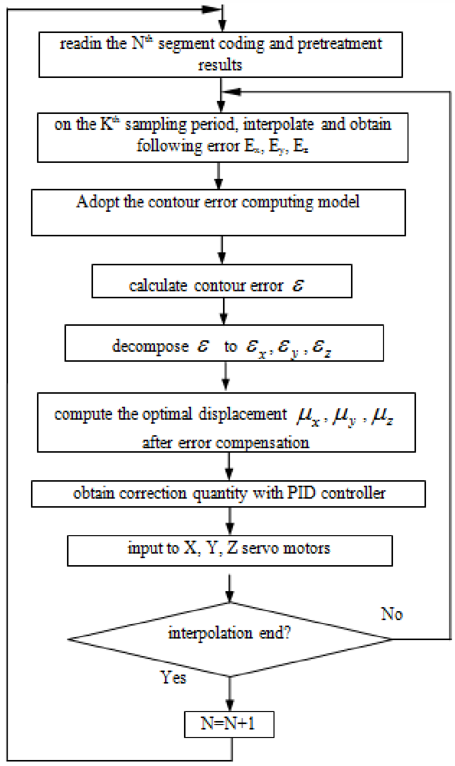

Figure 7. The CNC contour error calculation and compensation program flow chart

VI. Experimentations on Contour Error Compensation Control

A. The Three-axis Linked CNC Test Table

The three-axis linked CNC test table hardware structure is shown in Figure 8. The CNC controller is made up of PC and programmable DSP movement control card. The PC and DSP movement control card communicate through USB2.0.

The PC acts as the man-machine interface, which implements instruction control, code compilation, states display and other functions; And the interpolation, position control and contour error compensation control function are carried out on the programmable DSP movement control card. The Panasonic servo drivers and motors are adopted in the X, Y, Z axes. Both the interpolation period and sampling period are 4 ms.

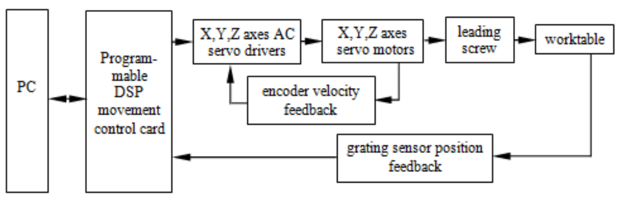

Figure 8. The three-axis linked CNC test table hardware structure

B. The Contrast Experimentations on Contour Error Compensation

Interpolate and machine a block of three order NURBS curve. The control knots are:

A (15, 0, 15)
B (15, 15, 15)
C (0, 15, 0)
D (-15, 15, -15)
E (-15, 0, -15)
F (-15, -15, -15)
G (0, -15, 0)
H (15, -15, 15)
I (15, 0, 15) ;
The scale factors are:
(1, 0.6, 1, 0.5, 1, 0.5, 1, 0.6, 1) ;
The knot vector is:
(0, 0, 0, 0, 0.25, 0.375, 0.5, 0.625, 0.75, 1, 1, 1, 1).

Firstly, interpolate and machine this NURBS profile when not adopting the introduced contour error coupled-control approach. The ideal profile curve and real profile are shown in Figure 9, where the contour error is magnified 12 times to display. The contour error when machining the curve is shown in Figure 10. From Figure 10 it can be seen, the maximal contour error is near to 0.104mm.

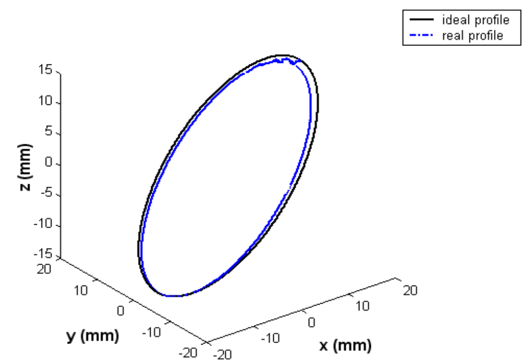

Figure 9. The profile when not adopting the introduced contour error coupled-control approach

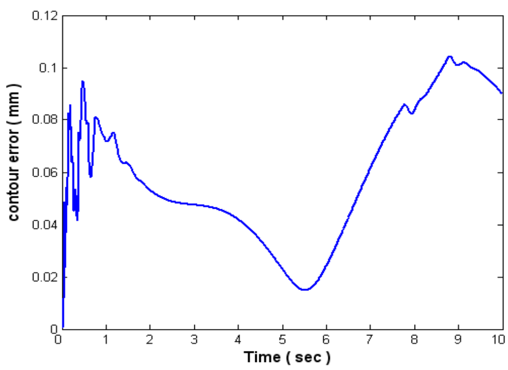

Figure 10. The contour error when not adopting the introduced contour error coupled-control approach

Secondly, interpolate and machine this NURBS profile when adopting the introduced contour error coupled-control approach based on linear interpolation. The ideal profile curve and real profile are shown in Figure 11, where the contour error is magnified 12 times to display. The contour error when machining the curve is shown in Figure 12. From Figure 12 it can be seen, the maximal contour error is near to 0.054mm.

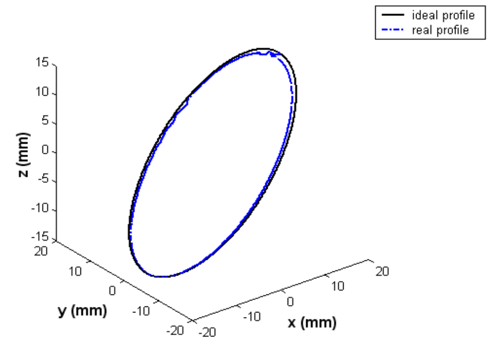

Figure 11. The profile when adopting the introduced contour error coupled-control approach based on linear interpolation

Finally, interpolate and machine this NURBS profile when adopting the introduced contour error coupled-control approach based on curve interpolation. The ideal profile curve and real profile are shown in Figure 13, where the contour error is magnified 12 times to display. The contour error when machining the curve is shown in Figure 14. From Figure 14 it can be seen, the maximal contour error is near to 0.044mm.

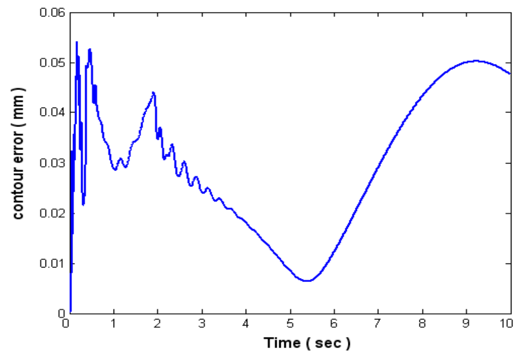

Figure 12. The contour error when adopting the introduced contour error coupled-control approach based on linear interpolation

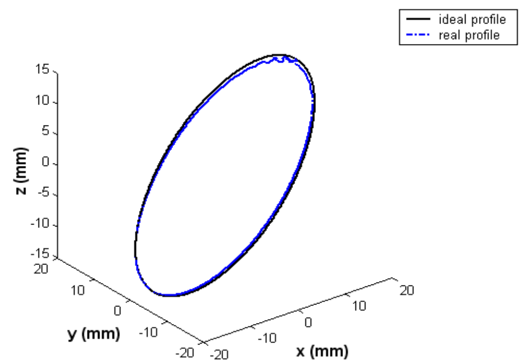

Figure 13. The profile when adopting the introduced contour error coupled-control approach based on curve interpolation

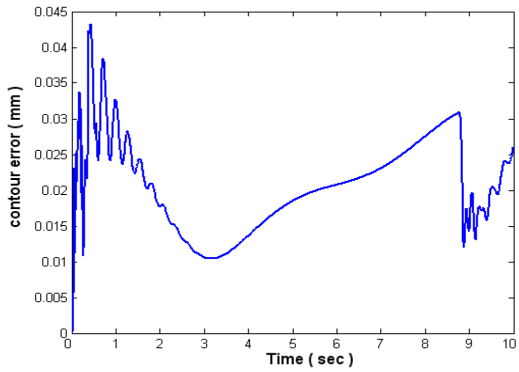

Figure 14. The contour error when adopting the introduced contour error coupled-control approach based on curve interpolation

VII. CONCLUSIONS

The CNC machine tools contour error coupled-control strategy based on line interpolation and curve interpolation is developed in the paper, which is with stable calculation error, high computing precision and satisfied real-time characteristic.

For one thing, bring forward the contour error computing model based on line interpolation and the contour error computing model based on curve interpolation; For another thing, add the obtained contour error to the following error of current sampling period, and send the results to CNC PID position controller to calculate position controlled quantity. The contour error compensation control experimentation results show that the developed approach can reduce contour error effectively and enhance profile precision further.

ACKNOWLEDGMENT

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