

Multi-Step Prediction Algorithm of Traffic Flow Chaotic Time Series based on Volterra Neural Network

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Abstract—The accurate traffic flow time series prediction is the prerequisite for achieving traffic flow inducible system. Aiming at the issue about multi-step prediction traffic flow chaotic time series, the traffic flow Volterra Neural Network (VNN) rapid learning algorithm is proposed. Combing with the chaos theory and the Volterra functional analysis, method of the truncation order and the truncation items is given and the VNN model of traffic flow time series is built. Then the mechanism of the chaotic learning algorithm is described, and the adaptive learning algorithm of VNN for traffic flow time series is designed. Last, a multi-step prediction of traffic flow chaotic time series is researched by traffic flow VNN network model, Volterra prediction filter and the BP neural network based on chaotic algorithm. The simulations show that the VNNTF network model predictive performance is better than the Volterra prediction filter and the BP neural network by the simulation results and root-mean-square value.

Index Terms—Chaos Theory, Phase Space Reconstruction, Time Series Prediction, VNN Neural Networks, Algorithm

I. INTRODUCTION

The Volterra series is a model for non-linear behavior similar to the Taylor series. It differs from the Taylor series in its ability to capture 'memory' effects. It has the advantages of high precision and clear physical meaning, has become one of the very effective non-parametric model of nonlinear system [1-4]. Traffic flow chaotic time series with the nonlinear behavior of the response and memory function, the Volterra series to become one of the primary means of traffic flow in nonlinear system identification [5-6]. Many scholars and technology developers have proposed a lot of Volterra identification algorithm, but the establishment of nonlinear systems on Volterra Series model is very difficult [7-9]. Volterra series has an obvious drawback is that if you want to achieve a satisfactory accuracy may require a considerable number of estimated parameters. The high-level nuclear estimates are facing the greatest difficulties. Therefore, the Volterra functional model of the

application is to be greatly restricted, and sometimes in order to avoid solving the higher-order kernel function and Volterra functional model artificially simplified, resulting in the modeling inaccuracy.

With the rapid development of computer technology, the neural network is more deeply and widely used in nonlinear systems [11-13]. The neural network not only has the self-adaptive, parallelism and fault tolerance characteristics, but also has the ability to approximate any nonlinear function. Based on these advantages, the neural network model of the nonlinear system has a very wide range of applications [14-16]. Due to the consistency of the Volterra model and the three-layer ANN model, combined with the traffic flow chaotic time series chaotic characteristics, how to make use the Volterra accurate modeling of the advantages to overcome the shortcomings of Solutions of Higher Order kernel function; and how to use the advantages of ANN neural network model for learning and training network to overcome the blindness of the ANN neural network modeling is worth exploring.

Based on the above considerations, the physical significance of the truncation order of the Volterra series model and the truncated number and the mathematical properties of the minimum embedding dimension and delay time in traffic flow chaotic time series reconstructed phase space, thus, traffic flow chaotic time series VNNTF network model and the corresponding algorithm has been established [17-20]. VNNIF neural network model to learn the advantages of the Volterra series to establish an accurate traffic prediction model and the ANN network training is easy to solve the Volterra model kernel function; thus, to overcome the difficulties on Volterra series model for solving the higher order kernel function and the blindness of the ANN network model, in the traffic flow chaotic time series prediction, obtained good results.

II. TRAFFIC FLOW CHAOTIC TIME SERIES VOLTERRA MODEL

For nonlinear systems, discretization of the Volterra model is as follows:

$$y(n) = \sum_i \sum_{l_1, \dots, l_i=0}^{\infty} h_i(l_1, \dots, l_i) x(n-l_1) \dots x(n-l_i) \quad (1)$$

Where, $n, l_i \in R$, $y(n)$ is the output of the nonlinear system; $x(n-l_i)$ is the input of the nonlinear system and $h_i(l_1, l_2, \dots, l_i)$ ($i = 1, 2, \dots, n$) is Volterra kernel function of order i .

A. Model of Chaotic Time Series Prediction

The chaotic time series prediction is based on the Takens' delay-coordinate phase reconstruct theory. If the time series of one of the variables is available, based on the fact that the interaction between the variables is such that every component contains information on the complex dynamics of the system, a smooth function can be found to model the portraits of time series. If the chaotic time series are $\{x(t)\}$, then the reconstruct state vector is $\mathbf{x}(t) = (x(t), x(t+\tau), \dots, x(t+(m-1)\tau))$, Where m ($m = 2, 3, \dots$) is called the embedding dimension ($m = 2d + 1$, d is called the freedom of dynamics of the system), and τ is the delay time. The predictive reconstruct of chaotic series is a inverse problem to the dynamics of the system essentially. There exists a smooth function defined on the reconstructed manifold in R^m to interpret the dynamics $x(t+T) = F(x(t))$, where T ($T > 0$) is forward predictive step length, and $F(\cdot)$ is the reconstructed predictive model.

B. The Determination of the Truncation Order on Traffic Flow Chaotic Time Series Volterra Model

Assume that the measured traffic flow chaotic time series is $\{x(t) | (t = 1, 2, 3, \dots)\}$, the traffic flow chaotic time series phase space reconstruction based on Takens Theorem, you can get the input of the nonlinear system is $x(t), x(t+\tau), x(t+2\tau), \dots, x(t+(m-1)\tau)$, where, m is the embedding dimension, namely the reconstruction of phase space dimension, τ is the delay time. Here, m corresponds to the number of finite order in the discretization of the Volterra model, and to predict the traffic flow is predicted on the basis of the m item, then the traffic flow chaotic time series phase space reconstruction model with m-order truncation Volterra series model can be characterized as follows:

$$\begin{aligned} x(t'+T) = F(X(t)) = & h_0 + \sum_{l_0=0}^{\infty} h_1(l_0)x(t-l_0\tau) \\ & + \sum_{l_1=0}^{\infty} \sum_{l_2=0}^{\infty} h_2(l_1, l_2)x(t-l_1\tau)x(t-l_2\tau) + \dots \\ & + \sum_{l_1=0}^{\infty} \sum_{l_2=0}^{\infty} \dots \sum_{l_p=0}^{\infty} h_m(l_1, l_2, \dots, l_m)x(t-l_1\tau)x(t-l_2\tau) \dots x(t-l_m\tau) \end{aligned} \quad (2)$$

where, $h_m(l_1, l_2, \dots, l_m)$ is the m order Volterra kernel function, $t' = t + (m-1)\tau$, T ($T > 0$) is the forward prediction step. This infinite series, theoretically, can be very accurately predicting traffic flow chaotic time series, but difficult to achieve in practical applications, it must be a finite order truncation and the finite sum in the form.

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For traffic flow chaotic time series prediction from equation (2), it is the m-order truncated infinite item summation form. For example, when $m = 3$, it is a finite sum of the third order intercept Volterra series model:

$$\begin{aligned} x(t'+T) = F(X(t)) = & h_0 + \sum_{l_0=0}^{N_1-1} h_1(l_0)x(t-l_0\tau) \\ & + \sum_{l_1=0}^{N_2-1} \sum_{l_2=0}^{N_2-1} h_2(l_1, l_2)x(t-l_1\tau)x(t-l_2\tau) \\ & + \sum_{l_1=0}^{N_3-1} \sum_{l_2=0}^{N_3-1} \sum_{l_3=0}^{N_3-1} h_3(l_1, l_2, l_3)x(t-l_1\tau)x(t-l_2\tau)x(t-l_3\tau) \end{aligned} \quad (3)$$

so, actually want to calculate the total number of coefficients is $1 + N_1 + N_2^2 + N_3^3$. Be seen with the increase of m in the Volterra series, the number of items of Volterra Series will power rapid increase; the corresponding required number of calculations also showed exponential growth, which makes the actual traffic flow chaotic time series predicted to achieve more and more difficult. The total number of items of Volterra series number decreases exponentially growth. In practice, the truncation order is generally the second-order truncation or third order intercept.

C. The Determination of the Truncation Items on Traffic Flow Chaotic Time Series Volterra Model

In the form of the flow chaotic time series Volterra series model is (2), assume that the truncated form of limited items are as follows:

$$\begin{aligned} x(t'+T) = F(X(t)) = & h_0 + \sum_{l_0=0}^{N_1-1} h_1(l_0)x(t-l_0\tau) \\ & + \sum_{l_1=0}^{N_2-1} \sum_{l_2=0}^{N_2-1} h_2(l_1, l_2)x(t-l_1\tau)x(t-l_2\tau) + \dots \\ & + \sum_{l_1=0}^{N_m-1} \sum_{l_2=0}^{N_m-1} \dots \sum_{l_p=0}^{N_m-1} h_m(l_1, l_2, \dots, l_m)x(t-l_1\tau)x(t-l_2\tau) \dots x(t-l_m\tau) \end{aligned} \quad (4)$$

For traffic flow chaotic time series, it is assumed that $x(t)$ and $y(t)$ are the input and output signals of the functional system $f(t, x(t'), t' \leq t)$ in the traffic flow, the input signal of the functional system in the traffic flow to meet:

- ① Traffic flow input signal is a causal relationship is met when $t < 0$, then $x(t) = 0$.
- ② Traffic flow functional system $f(t, x(t'), t' \leq t)$ is the limited memory, that is, for the t time in the system,

t_0 time very far from the t time, $t_0 \rightarrow \infty$, $x(t-t_0)$ has no effect on $y(t)$; means that the predicted value of $y(t)$ is irrelevant to $x(t-t_0)$.

In the prediction of chaos traffic flow chaotic time series, $t' = t + (m-1)\tau$, T ($T > 0$) is forward prediction step, $x(t'+T)$ represents the output associated with the input signal $x(t)$ and the delay time τ , then

$$x(t'+T) = f(x_{l_1}, x_{l_2}, \dots, x_{N_{l_i}}) = h_0 + \sum_{l_1=0}^{N_{l_1}-1} h_1(l_1)x(t-l_1\tau) + \sum_{l_1=0}^{N_{l_2}-1} \sum_{l_2=0}^{N_{l_2}-1} h_2(l_1, l_2)x(t-l_1\tau)x(t-l_2\tau) + \sum_{l_1=0}^{N_{l_3}-1} \sum_{l_2=0}^{N_{l_3}-1} \sum_{l_3=0}^{N_{l_3}-1} h_3(l_1, l_2, l_3)x(t-l_1\tau)x(t-l_2\tau)x(t-l_3\tau) + \dots \quad (5)$$

note $N_{\max} = \max(N_{l_1}, N_{l_2}, N_{l_3}, \dots, N_{l_i})$, ($i = 1, 2, 3, \dots$), when $n \geq N_{\max}$, the same to meet the input traffic flow signal $x_i = x(t-l_i\tau)$ is irrelevant to $y(t)$, then the formula (4) can be written as:

$$x(t'+T) = f(x_{l_1}, x_{l_2}, \dots, x_{N_{l_i}}) = h_0 + \sum_{l_1=0}^{N_{\max}-1} h_1(l_1)x(t-l_1\tau) + \sum_{l_1=0}^{N_{\max}-1} \sum_{l_2=0}^{N_{\max}-1} h_2(l_1, l_2)x(t-l_1\tau)x(t-l_2\tau) + \sum_{l_1=0}^{N_{\max}-1} \sum_{l_2=0}^{N_{\max}-1} \sum_{l_3=0}^{N_{\max}-1} h_3(l_1, l_2, l_3)x(t-l_1\tau)x(t-l_2\tau)x(t-l_3\tau) + \dots \quad (6)$$

Know from the above analysis of the traffic flow functional systems, the power series expansion item of prediction results are in fact only related to Know from the above analysis of the traffic flow functional systems, the power series expansion item of prediction results are in fact only related to summation form all the products of the Input signal and the first power delay time signal. This means that the value of $N_{\max} = \max(N_{l_1}, N_{l_2}, N_{l_3}, \dots, N_{l_i})$, ($i = 1, 2, 3, \dots$) is only related with the number of input signal and the delay time signal, which is the minimum embedding dimension m of phase space, so $N_{\max} = \max(N_{l_1}, N_{l_2}, N_{l_3}, \dots, N_{l_i}) = m$.

Such traffic flow chaotic time series Volterra series model is finalized by the formula (5) as follows:

$$x(t'+T) = f(x_{l_1}, x_{l_2}, \dots, x_{N_{l_i}}) = h_0 + \sum_{l_1=0}^{m-1} h_1(l_1)x(t-l_1\tau) + \sum_{l_1=0}^{m-1} \sum_{l_2=0}^{m-1} h_2(l_1, l_2)x(t-l_1\tau)x(t-l_2\tau) + \sum_{l_1=0}^{m-1} \sum_{l_2=0}^{m-1} \sum_{l_3=0}^{m-1} h_3(l_1, l_2, l_3)x(t-l_1\tau)x(t-l_2\tau)x(t-l_3\tau) + \dots + \sum_{l_1=0}^{m-1} \sum_{l_2=0}^{m-1} \sum_{l_3=0}^{m-1} \dots \sum_{l_m=0}^{m-1} h_m(l_1, l_2, l_3, \dots, l_m)x(t-l_1\tau)x(t-l_2\tau)x(t-l_3\tau) \dots x(t-l_m\tau) \quad (7)$$

III. TRAFFIC FLOW TIME SERIES VOLTERRA NEURAL NETWORK MODEL (VNNTF)

A. Representation of Nonlinear Systems Using Artificial Neural Network

Has proven that the BP neural network with one hidden layer can approximate any continuous bounded non-linear system, therefore, generally selected to contain a three-layer back propagation BP network with one hidden layer to approximate nonlinear systems. A single output three-layer back propagation neural network is shown in Figure 1. In the figure, the input vector $x_k^T = [x_{k,0}, x_{k,1}, \dots, x_{k,M}]$ at moment n can obtain by the delay of $x(k)$, where $x_{k,m} = x(k-m)$, the input of the l hidden unit ($l = 1, 2, \dots, L$) is

$$Z_{l,k} = S_l(u_{l,k}); u_{l,k} = \sum_{m=0}^M w_{l,m}x_{k,m} \quad (8)$$

A single output three-layer back propagation neural network is shown in Figure 1.

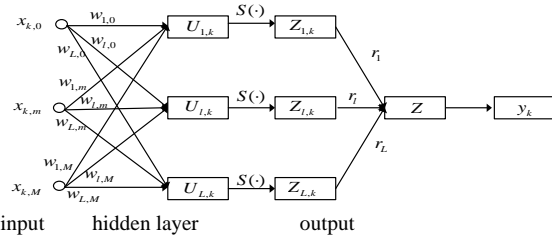


Figure 1. Three layer neural networks in response to M+1 input and single output system

If the implicit function selected the sigmoid function, then

$$S_l(u_{l,k}) = \frac{1}{1 + \exp[-\lambda(u_{l,k} - \theta_l)]} \quad (9)$$

Where, θ_l is the threshold of the unit n , If the output unit is linear summation unit, the output at moment n is

$$y_k = \sum_{l=1}^L r_l Z_{l,k} \quad (10)$$

The output of each hidden unit to expand into a Taylor series at the threshold θ_l :

$$Z_{l,k} = \varphi_l(u_{l,k}) = \sum_{i=0}^{\infty} d_i(\theta_l)u_{l,k}^i \quad (11)$$

where, $d_i(\theta_l)$ is the commencement of the coefficient, the value associated with θ_l , and because of

$u_{l,k} = \sum_{m=0}^M w_{l,m}x_{k,m}$, then the output of the neural network is

$$y_k = \sum_{l=1}^L r_l \sum_{i=0}^{\infty} d_i(\theta_l) \cdot \sum_{m_1=0}^M \dots \sum_{m_i=0}^M w_{l,m_1} \dots w_{l,m_i} x_{k,m_1} \dots x_{k,m_i} \quad (12)$$

B. Traffic Flow Volterra Neural Network Model

Analysis and comparison of traffic flow Volterra series model in equation (6) and three-layer BP neural network in equation (12), if the input vector in equation (12) in VNNTF to take the traffic flow chaotic time series, then between them in the function, structure and method for solving are inherently close contact and similarity.

1) From a functional point of view, the traffic flow chaotic time series, Volterra series model and ANN model can be measured traffic flow chaotic time series, to simulate and predict the traffic flow process. Traffic flow chaotic time series Volterra model can determine the model truncation order of the truncation by the characteristics analysis of the traffic flow time series. Then, it can use the system identification to strike a nuclear function of the Volterra series model, or proper orthogonal decomposition method, stepwise multiple regression method, iterative decline in the gradient method, Volterra filter and constraints orthogonal approximation method to solve the nuclear function or Volterra series, which reflect the chaotic nonlinear law of the traffic flow.

2) From a structural point of view, the traffic flow chaotic time series Volterra model and ANN model is also isomorphic. Length of the storage memory of past traffic flow relative to chaotic time series in the traffic flow Volterra model, that is, the minimum embedding dimension in phase space reconstruction of is equivalent to the number of neurons of the ANN model input layer.

3) From a method for solving point of view, Traffic flow chaotic time series Volterra model is based on orthogonal polynomials for the numerical approximation to find the approximate solution.the Meixner function systems and network weights have the same effect.

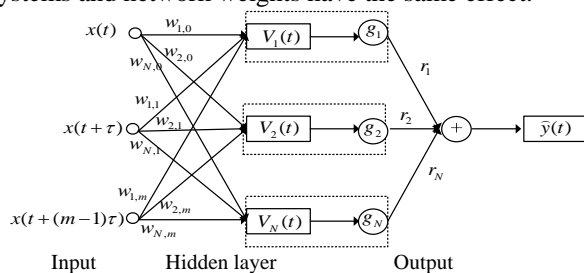


Figure 2. The chaotic time series Volterra neural network traffic flow model (VNNTF)

Through consistency of traffic flow chaotic time series Volterra model and ANN model, in this paper, the traffic flow chaotic time series Volterra neural network model (VNNTF) has been proposed in Figure 2. In the figure, $X(t) = (x(t), x(t + \tau), \dots, x(t + (m - 1)\tau))^T$ ($t = 1, 2, \dots$) is the traffic flow chaotic time series reconstructed phase space vector; $w_{i,j}$ ($i = 1, 2, \dots; j = 1, 2, \dots$), r_n is the traffic flow chaotic time series Volterra neural network weights parameters; g_s , ($s = 1, 2, \dots, N$) is the activation function and $V_s(k)$ is the traffic flow of the convolution of the input signal:

$$V_N(t) = \sum_{i=0}^m w_{Ni} x(t + (i-1)\tau) \tag{13}$$

Thus, the traffic flow chaotic time series Volterra neural network expression is

$$\begin{aligned} \hat{y}(t) &= f(X(t)) = f(\bar{x}(t)) = \sum_{s=1}^N r_s g_s(V_N(t)) \\ &= \sum_{s=1}^N r_s g_s\left(\sum_{i=0}^m w_{si} x(t + (i-1)\tau)\right) \end{aligned} \tag{14}$$

IV. TRAFFIC FLOW VOLTERRA NEURAL NETWORK RAPID LEARNING ALGORITHM

A. Activation Function Analysis of Traffic Flow Volterra Neural Network

Activation function of hidden layer to the VNNTF model designed for the following polynomial function:

$$g_s = a_{0,s} + a_{1,s}x + a_{2,s}x^2 + \dots + a_{i,s}x^i + \dots \tag{15}$$

where $a_{i,s} \in R$ is the polynomial coefficients, and then

$$\begin{aligned} \hat{y}(t) &= \sum_{s=1}^N r_s g_s(V_N(t)) = \sum_{s=1}^N \sum_{i=1}^{+\infty} r_s a_{i,s} (V_N(t))^i \\ &= \sum_{s=1}^N \sum_{i=1}^{+\infty} r_s a_{i,s} \left(\sum_{i=0}^m w_{si} x(t + (i-1)\tau)\right)^i \end{aligned}$$

So, to get:

$$h_j(l_1, l_2, \dots, l_j) = \sum_{s=1}^N r_s a_{j,s} w_{s,l_1} w_{s,l_2} \dots w_{s,l_j} \quad (j = 1, 2, \dots, m) \tag{16}$$

In the VNNTF model, the sigmoid function or other functions as the activation function $g_s(V_s(t))$ training VNNTF network, the weights and thresholds are obtained, the activation function $g_s(V_s(t))$ is expanded into a Taylor series, you can obtain the polynomial coefficients:

$$a_{j,s} = \frac{g_s^{(j)}(\theta_s)}{j!} \tag{17}$$

Among them, $g_s^{(j)}(\theta_s)$ is the j -order derivative of function $g_s(V_s(t))$ in θ_s ; that is a different activation function, you can get $a_{j,s}$. VNNTF network learning and training, according to the connection weights of the network of neurons and the coefficients of $a_{j,s}$, you can solve any order kernel function, which would address the difficulties of solving high-level nuclear function in the Volterra model. In general, if directly using the polynomial function for the activation function, the polynomial order is taken as m , the same Taylor expansion of the Taylor series, the order is taken to the m , so VNNTF model by setting different order of the activation functions to reflect the effect equivalent to the Volterra model higher order kernel function.

B. Traffic Flow Volterra Neural Network Rapid Learning Algorithm

On the establishment of traffic flow chaotic time series WNN, Network input the number of neurons, hidden layers and the number of neurons in the hidden layer are to be considered. The following traffic flow data used are from "Chongqing Road Traffic Management Data Sheet I" and "Chongqing Road Traffic Management Data Sheet II" in 2006. There is the study of traffic volume time series of two-lane road 28 hours and 5 minutes every 5 minutes, including mini-vehicles, passenger cars, light trucks, midsize, large cars, trailer, micro van, not stereotypes, such as vehicles, and its sequence length $n = 337$. First, Pretreatment of the traffic flow time series, the minimum embedding dimension $m = 4$ and delay time $\tau = 3$ are obtained by calculation. Then, the traffic flow Volterra neural network can be constructed: traffic flow Volterra neural network is designed to be three layers: input layer, single hidden layer and output layer; the number of hidden layer wavelet neural taken as 9 by Kolmogorov Theorem, the number of input layer neurons equal to the minimum embedding dimension ($m = 4$), the number of output layer is 1, so that the 4-9-1 structure of traffic flow Volterra neural network was obtained, specifically shown in Figure 2. The hidden layer activation function can be used sigmoid function or other commonly used functions, and here it be used with polynomial activation functions $g_s = a_{0,s} + a_{1,s}x + a_{2,s}x^2 + \dots + a_{i,s}x^i + \dots$, $a_{i,s} \in R$ is polynomial coefficients. Optimal network parameter $w_{s,j}$ and r_s ($s = 1, 2, \dots, N$, $j = 1, 2, \dots, m$) can be obtained by learning and training the network for reducing the error E , and further $h_j(l_1, l_2, \dots, l_j)$ ($j = 1, 2, \dots, m$) can be calculated by combining the polynomial coefficients.

The steps of traffic flow chaotic time series Volterra Neural Network fast learning algorithm is showed and the specific steps are as follows:

Algorithm VNNTF model fast learning algorithm

Step1) The hidden neurons number is 9 by Kolmogorov Theorem, so that the 4-9-1 structure of traffic flow VNNTF neural networks was obtained. The traffic flow time series input signal is $(x(t), x(t + \tau), \dots, x(t + (m - 1)\tau))^T$, ($t = 1, 2, \dots$); the output signal is $\hat{y}(t)$; the weight coefficient matrix of the hidden layer is $w = (w_{s,l_j})_{N \times m} = (w_{s,i})_{N \times m}$, ($s = 1, 2, \dots, 9$, $i, j = 1, 2, \dots, 4$) and the parameter is r_s ($s = 1, 2, \dots, 9$).

Step2) The traffic flow chaotic time series Volterra Neural Network parameters $w = (w_{s,i})_{N \times m}$ and r_s ($s = 1, 2, \dots, 9$, $i = 1, 2, \dots, 4$) are initialized, where the parameters $w = (w_{s,i})_{N \times m}$ in each component take random function between 0 and 1; and r_s are initialized to take 9 number between 0 and 1 by the random function.

Step3) Using phase space reconstruction theory to preprocess the traffic flow chaotic time series, and perform normalization for the reconstructed network input signal. Based on Takens theorem, the minimum

embedding dimension $m = 4$, and the delay time $\tau = 3$. The reconstruction phase space vector number is $N - 1 - (m - 1)\tau = 327$, which the top 250 vector are used as network input signals. the form is $(x(t), x(t + \tau), \dots, x(t + (m - 1)\tau))^T$, where $t = 1, 2, \dots, 250$, $m = 4$ and $\tau = 3$.

Then, the 250 phase space vectors to make a simple normalized, the normalized as $[x(t) - \text{mean}(x(t))]/[\max(x(t)) - \min(x(t))]$, $t = 1, 2, \dots, 250$ and, making the value is owned by a range of $-1/2$ to $1/2$.

Step4) Using the initialized network and the preprocessed traffic flow time series, the first VNNTF neural network training begin with the function

$$\hat{y}(t) = \sum_{s=1}^N \sum_{i=1}^{+\infty} r_s a_{i,s} \left(\sum_{i=0}^m w_{s,i} x(t + (i - 1)\tau) \right)^i,$$

and the assumed activation function is a polynomial activation function g_s , here $a_{i,s} \in R$ are polynomial coefficients.

Step5) Calculate error function, the function formula:

$$E(\theta) = \frac{1}{2} \sum_{t=1}^{250} (y(t) - \hat{y}(t))^2$$

Set the maximum error is $E_{\max} = 0.035$, if $E < E_{\max}$, the storage VNNTF neural network parameter use $w = (w_{s,i})_{N \times m}$ and r_s ($s = 1, 2, \dots, 9$, $i = 1, 2, \dots, 4$); and further $h_j(l_1, l_2, \dots, l_j)$ ($j = 1, 2, \dots, m$) can be calculated by combining the polynomial coefficients, otherwise, transferred to step6).

Step6) Calculate local gradient of the traffic flow chaotic time series Volterra neural network. Specifically, according to the formula $\delta_j(t) = (y(t) - \hat{y}_j(t))g'_s(V_j(t))$ (j is the output layer) and the formula

$$\delta_j(t) = -\frac{\partial E(t)}{\partial y_j(t)} g'_s(V_j(t)) \tag{18}$$

where, the local gradients are calculated in the hidden layer.

Step7) By introducing the momentum term, to adjust the learning weights of the traffic flow chaotic time series Volterra neural network. Introduce nonlinear feedback into the weighting formal to adopt Chaos Mechanisms, due to the nonlinear feedback is vector form of weighting variables. In order to facilitate understanding, respectively, gives the vector w and its weighting formal, as follows. Note $\Delta w_{ji}^l(t + 1) = w_{ji}^l(t + 1) - w_{ji}^l(t)$, which represents the current value of weighting variables, then

$$\Delta w_{ji}^l(t + 1) = w_{ji}^l(t + 1) - w_{ji}^l(t) = -\eta \delta_j^{l+1}(t) x_i^l(t).$$

In order to speed up the learning process, in the right to join a momentum term $\alpha \Delta w_{ji}^l(t)$, then

$$\Delta w_{ji}^l(t + 1) = -\eta \delta_j^{l+1}(t) x_i^l(t) + \alpha \Delta w_{ji}^l(t)$$

where α is inertia factor; η is learning step; $\alpha\Delta w_{j,i}(t+1)$ is the introduction of the momentum and $\delta_j(t)$ is calculated with the formula (9).

Expand this equation into scalar form as follow:

$$\begin{cases} \Delta w_{j,i}^l(t+1) = -\eta\delta_j^{s+1}(t)x_i^l(t) + g(\Delta w_{j,i}^l(t)) \\ \Delta w_{j,i}^l(t+1+\tau) = -\eta\delta_j^{s+1}(t+\tau)x_i^l(t+\tau) + g(\Delta w_{j,i}^l(t+\tau)) \\ \Delta w_{j,i}^l(t+1+2\tau) = -\eta\delta_j^{s+1}(t+2\tau)x_i^l(t+2\tau) + g(\Delta w_{j,i}^l(t+2\tau)) \\ \dots\dots\dots \\ \Delta w_{j,i}^l(t+1+(m-1)\tau) = -\eta\delta_j^{s+1}(t+(m-1)\tau)x_i^l(t+(m-1)\tau) + g(\Delta w_{j,i}^l(t+(m-1)\tau)) \end{cases} \quad (19)$$

where, feedback can take a variety of vector functions, for example:

$$g(x) = \tanh(px)\exp(-qx^2) \text{ OR } g(x) = px\exp(-q|x|),$$

in the study, $p = 0.7$, $q = 0.1$.

Step8) Calculating the modified weights in the traffic flow chaotic time series Volterra neural network in Step8) and transferred to step4), and train network again, then calculate the network output $\hat{y}(t)$ and the error E , repeated training until the relative error in traffic meet $E < E_{\max} = 0.035$.

Step9) Output of every training storage of network parameters $w = (w_{s,i})_{N \times m}$ and r_s ($s = 1, 2, \dots, 9$, $i = 1, 2, \dots, 4$) in the traffic flow chaotic time series Volterra neural network. The activation function $g_s(V_s(t))$ is expanded into a Taylor series at the threshold θ_s and the expansion coefficient $d_i(\theta_s)$ is obtained. If the activation function is a polynomial, then $d_i(\theta_s) = a_{i,s}$ ($s = 1, 2, \dots, 9$, $i = 1, 2, \dots, 4$).

Step10) According to the formula

$$h_j(l_1, l_2, \dots, l_j) = \sum_{s=1}^N r_s d_i(\theta_s) w_{s,l_1} w_{s,l_2} \dots w_{s,l_j} \quad (20)$$

the kernel function ($s = 1, 2, \dots, 9$, $i = 1, 2, \dots, 4$) of the output system is calculated.

V. EXPERIMENTAL RESULTS AND ANALYSIS

Experimental objective is study how much extent does the prediction performance in VNNTF neural network improve from the aspects of model construction and algorithm application.

In order to study the prediction performance of traffic flow time series in traffic flow VNNTF network, respectively the VNNTF network model, Volterra prediction filter and ANN to predict the network traffic flow chaotic time series, and analyze and compare their predictions.

Multi-step prediction is a major aspect to reflect the performance of predictive model. Traffic flow time series Multi-step prediction is as follows: If the sample size is N , in the new data point cannot be used or only the sample points N , It can be predicted beyond $N+1$ values, can also predict the $N+2$ values,

$N+3$ values... $N+T$ values ($T > 0$). That is, for the known sample set can be extrapolated to predict T step. The following multi-step prediction of the traffic flow VNNTF network, and the results compare with the multi-step prediction of the BP neural network and filter Volterra, further, analyzing the causes of the different predictions. In fact, the multi-step prediction results can also be compared with the prediction results of wavelet neural network Algorithm and Wavelet Neural Network Based on Chaotic Algorithm. Can also be an attempt of the analysis for the prediction of the different results.

Where, the minimum embedding dimension in phase space is $m = 4$, the delay time is $\tau = 3$, and the vector number of phase space reconstruction which can be used to train and predict is $N - (m - 1)\tau = 327$.

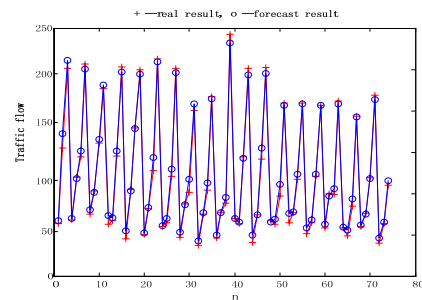


Figure 4. The 2-step forecast result and real result

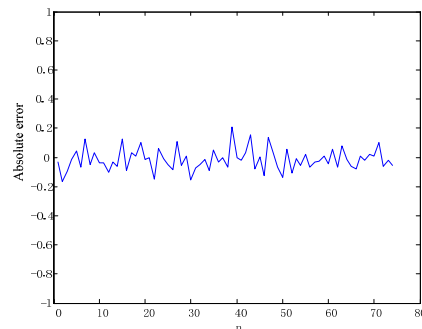


Figure 5. The 2-step forecast error curve

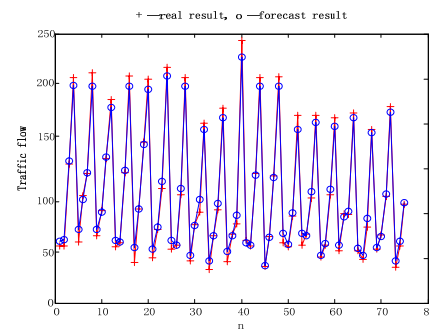


Figure 6. The 3-step forecast result and real result

In the network training of the multi-step prediction, such as Step 2 to Step 4, the training objectives of the 250 reconstructed vector among the 327 reconstructed phase space vector are the traffic flow signals from t' to $t' + 249$ ($t' = 12, 13, 14$). Network training, in order to compare with the measured traffic flow signal, the $T(T = 2, 3, 4)$ step forecast traffic flow signal

corresponding $260+T$ ($T=2,3,4$) to 337 traffic flow signal, that is, if the forecast number of steps each one more, then its projection is reduced by one. If not to make a prediction comparison with the measured signal, it does not have this restriction.

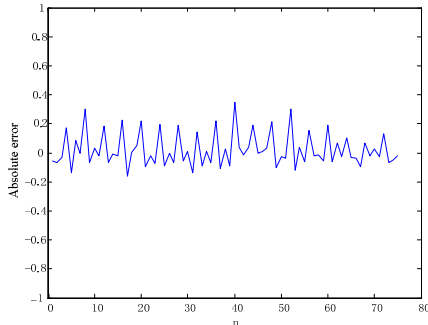


Figure 7. The 3-step forecast error curve

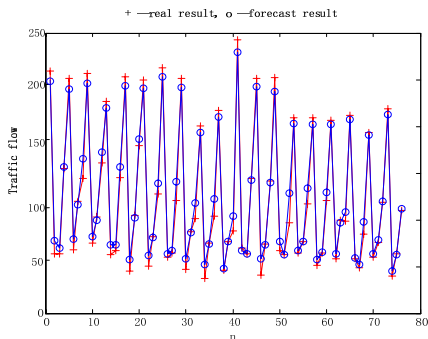


Figure 8. The 4-step forecast result and real result

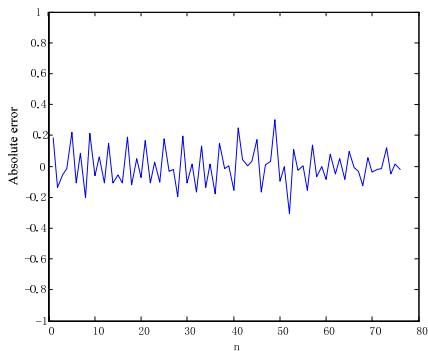


Figure 9. The 4-step forecast error curve

Were calculated the error root mean square in Figure 5, 7 and 9, and these results are compared with the error root mean square of the BP network and the wavelet neural network based on the non-chaotic algorithm, and the compare results are shown in Table 1. From Table 1, with the increasing number of prediction steps, in which the same prediction step, the root mean square of the wavelet neural network based on chaotic algorithm is significantly less than the root mean square of BP neural network and the wavelet neural network based on non-chaotic algorithm.

Figure 4, 6 and 8, respectively, which corresponds to 2-step, 3-step and 4-step predicted and actual comparison curves of VNNTF network based VNNTF network rapid learning algorithm; and “+” shows the true value, “o” shows the forecasted value Figure 3, 7, and 9 respectively,

which corresponds to 2-step, 3-step and 4-step absolute error curve. Figure 4 to Figure 9 shows the effect of 3-step prediction results is worse than the 2-step prediction, the effect of 4-step prediction results is worse than the 3-step prediction; and the general trend is to predict the longer the step, the prediction performance has become getting worse.

Analysis of multi-step prediction results to VNNTF network, the 2-step, 3-step and 4-step predictable performance overall is better than the BP neural network prediction and the Volterra filter prediction; this is because the network VNNTF combines the Volterra series and ANN network advantages, to overcome the difficulties of solving the Volterra kernel function and the blindness of ANN network modeling. In fact, the prediction results to VNNTF network is better than the wavelet neural network prediction based on chaotic algorithm, and this may be the establishment of a good traffic flow time series prediction model is relatively more important than to choose a good algorithm, from this sense, the establishment of traffic flow prediction model is the most critical.

TABLE I.
NORMALIZATION OF RMSE COMPARISON

prediction step	BP network	Volterra filter	VNNTF network
1 step	0.7014	0.3567	0.1368
2 step	0.8074	0.3941	0.1507
3 step	0.8653	0.4225	0.2322
4 step	0.9799	0.4782	0.2417

VI. CONCLUSIONS

In the paper traffic flow chaotic time series VNNTF model was designed. A traffic flow VNNTF fast learning algorithm based on chaos theory was proposed. The method of model selection and algorithm design, are considered the chaotic characteristics of traffic flow time series, which is a theoretical value. Simulation results show that the method can reduce network training time and improve the forecast accuracy, and show better predictive effectiveness and reliability.

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