Predicate Formal System based on 1-level Universal AND Operator and its Soundness

Yingcang Ma
School of Science, Xi'an Polytechnic University, Xi'an, Shaanxi, 710048, China
School of Electronics and information, Northwestern Polytechnical University, Xi'an, Shaanxi, 710048, China
Email: mayingcang@126.com

Huacan He
School of Computer Science, Northwestern Polytechnical University, Xi'an, Shaanxi, 710048, China
Email: hehuac@nwpu.edu.cn

Abstract—**The aim of this paper is solving the predicate calculus formal system based on 1-level universal AND operator. Firstly, universal logic and propositional calculus formal deductive system $UL_{h\in(0,1]}^{-}$ are introduced. Secondly, a predicate calculus formal deductive system $\forall UL_{h\in(0,1]}^{-}$ based on 1-level universal AND operator is built. Thirdly, the soundness theorem and deduction theorem of system $\forall UL_{h\in(0,1]}^{-}$ are given, which ensure that the theorems are tautologies and the reasoning rules are valid in system $\forall UL_{h\in(0,1]}^{-}$.**

Index Terms—**universal logic, predicate calculus formal system, universal AND operator**

I. Introduction

How to deal with various uncertainties and evolution problems have been critical issues for further development of artificial intelligence [1,2]. Mathematical logic is too rigid and it can only solve certainty problems, therefore, non-classical logic and modern logic develop rapidly, for example, fuzzy logic and universal logic.

Considerable progresses have been made in logical foundations of fuzzy logic in recent years, especially for logic system based on t-norm and its residua [3]. Some well-known logic systems have been built up, such as, the basic logic (BL) [4, 5] introduced by Hajek; the monoidal t-norm based logic [6, 7] introduced by Esteva and Godo; a formal deductive system L* introduced by Wang [8-10], Universal logic proposed by He [11], and so on.

Universal logic is a new continuous-valued logic system in studying flexible world's logical rule, which uses generalized correlation and generalized autocorrelation to describe the relationship between propositions, more studies can be found in [12-14]. For a logic system, the formalization's studies are very important, which include propositional calculus and predicate calculus. The propositional calculus formal systems are studied in [15-18]. But the studies of predicate calculus formal systems of universal logic are relatively rare, so we will mainly study the predicate calculus formal system in this paper, which can enrich the formalization's studies of universal logic.

Some predicate calculus formal deductive systems are built for fuzzy logic systems, for example, the predicate calculus formal deductive systems of Schweizer-Sklar t-norm in [19, 20]. The predicate calculus formal deductive systems of universal logic have been studies in [21-24], which mainly focus on the 0-level universal AND operator. In this paper, we focus on the formal system of universal logic based on 1-level universal AND operator. We will build predicate formal system $\forall UL_{h\in(0,1]}^{-}$ for 1-level universal AND operator, and its soundness and deduction theorem are given.

The paper is organized as follows. After this introduction, Section II contains necessary background knowledge about BL and UL. Section III we will build the predicate calculus formal deductive system $\forall UL_{h\in(0,1]}^{-}$ for 1-level universal AND operator. In Section IV the soundness and deduction theorem of system $\forall UL_{h\in(0,1]}^{-}$ will be given. The final section offers the conclusion.

II. Preliminaries

A. The Basic Fuzzy Logic BL and BL-algebra

The languages of BL [3] include two basic connectives $\to$ and $\&$, one truth constant $\bar{0}$. Further connectives are defined as follows:

$\varphi\wedge\psi$ is $\varphi\&(\varphi\to\psi)$,

$\varphi\vee\psi$ is $((\varphi\to\psi)\to\psi)\wedge(\psi\to\varphi)\to\varphi)$,

$\neg\varphi$ is $\varphi\to\bar{0}$,

$\varphi\equiv\psi$ is $(\varphi\to\psi)\&(\psi\to\varphi)$.

The following formulas are the axioms of BL:

(i) $(\varphi\to\psi)\to((\psi\to\chi)(\varphi\to\chi))$

(ii) $(\varphi\&\psi)\to\varphi$

(iii) $(\varphi\&\psi)\to(\psi\&\varphi)$

(iv) $\varphi\&(\varphi\to\psi)\to(\psi\&(\psi\to\varphi))$

(v) $(\varphi\to(\psi\to\chi))\to((\varphi\&\psi)\to\chi)$

(vi) $((\varphi\&\psi)\to\chi)\to(\varphi\to(\psi\to\chi))$

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(vii) $((\varphi \to \psi) \to \chi) \to (((\psi \to \varphi) \to \chi) \to \chi)$

(viii) $\bar{0} \to \varphi$

The deduction rule of BL is modus ponens.

Definition 1 [3] A BL-algebra is an algebra $L = (L, \cap, \cup, *, \Rightarrow, 0, 1)$ with four binary operations and two constants such that $(L, \cap, \cup, 0, 1)$ is a lattice with the greatest element 1 and the least element 0 (with respect to the lattice ordering $\leq$), $(L, *, 1)$ is a commutative semigroup with the unit element 1, i.e. $*$ is commutative, associative and $1 * x = x$ for all x, the following conditions hold for all x, y, z :

(i) $z \leq (x \Rightarrow y)$ iff $x * z \leq y$

(ii) $x \cap y = x * (x \Rightarrow y)$

(iii) $(x \Rightarrow y) \cup (y \Rightarrow x) = 1$.

B. Universal Logic

Universal logic was proposed by He [11], which thinks that all things in the world are correlative, that is, they are either mutually exclusive or mutually consistent, and we call this kind of relation generalized correlation.

The basic principles of universal logic show as follows:

A core objective. The objective is that any one of modern logics should include one or some dialectical contradictions, and which should exclude the logical contradictions. And different advanced logics have different dialectical contradictions.

Two basic methods. There are two ways to include dialectical contradictions (or uncertainty) in general. Firstly, the logical scope narrows to the sub-space that adapts to just include the dialectical contradictions (or uncertainty). Secondly, the logic system express the impact of the dialectical contradictions (or uncertainties) through continuously variable flexible parameters and functions in the logic operation model.

Three Break directions. There are three different break directions for the constraints of various modern logics relative to that of standard logic: the number of truth value of proposition, the dimension of truth value space, and the completeness of information reasoning.

Four logical elements. There are four logical elements to construct a logical system: domain, propositional connectives, quantifiers and reasoning rules. Universal logic discussed the possible forms of these elements, and put forward their general expression.

Universal logic includes four ways to contain dialectical contradictions (uncertainties) as follows:

1) The establishment of flexible domain.

The uncertainty firstly presents in the uncertainty of truth value of proposition. From the view of truth value domain and space dimension of proposition variable, the scope of uncertainty is fraction dimension space $[0, 1]^n$, n>0, which can include integer dimension space $[0, 1]^n$, n=2, 3, …, and which can also include 1-dimension continuous value space [0, 1]. This gives the possible to break the limitations of truth value domain of 1-dimaention two-valued logic.

The classical logic is a single granularity from the view of individual variable domain, that is, the logical properties of whole domain are identical. The future development trend of modern logic is introduced the concept of granularity computing into the logic. The domain is divided into different sub-domains according to some kind of equivalence relations, and the logical properties of different sub-domain may be different to express the uncertainty of domain. This gives the possible to break the limitations of single granularity of 1-dimension two-valued logic.

From the model domain, the classical logic is the single-mode. There are many different modes in the current modal logic. In the future continuous variable mode may be build, which can accurately describe the effect of modal area in uncertainty. This gives the possible to break the limitations of single mode of 1-dimension two-valued logic.

2) The definition of integrity cluster of operation model

The effect of all kinds of uncertainties on logic operations results can be expressed by all kinds of continuous-valued proposition conjunction integrity cluster of operation model. For example, in the propositional universal logics, Firstly, we narrow the logical scope to include fitness subspace of the contradictions of enemy/friends, loose/strict, light/heavy (include one, two or three) through time and space; secondly, we introduce two continuous variable flexible parameters k, $h \in [0, 1]$ into the logical operation models, and use the corresponding adjustment function to describe the full impact of the dialectical contradictions (or uncertainty) for the proposition conjunction computing model. Finally, we get the various types of propositional logics. It is obvious that if we can reduce the scope of logic to adapt to accommodate just a dialectical contradiction (or uncertainty) of the sub-space by time and space, then continuously variable flexible parameters and adjustment functions are introduced into the logic operation model, which can include effectively and deal with the dialectical contradictions (or uncertainties) in mathematics dialectical logic. This is foundation for continuous-valued logical algebra that will discuss below.

3) Defining a variety of flexible quantifiers to express the uncertainty of constraints (ranges).

The flexible quantifiers are: the universal quantifier $\forall$; the existential quantifier $\exists$; the threshold quantifier ♂^{k} symbolizing the threshold of propositional truth; the hypothesis quantifier $\$^k$ symbolizing hypothesis proposition; the scope quantifier $\oint^k$ constraining the scope of individual variables; the position quantifier ♀^{k} indicating the relative position of an individual variable and a specific point; the transition quantifier $\int^k$ changing the distribution transitional feature of the predicate truth. $k \in [0, 1]$ is a variable parameter, which express the change of constraints. When k=1, the constraints are the largest (strong), and when k=0, the constraints are the smallest (weak). So in this way the logic not only describe the uncertainty of constraints, but also control the degree of reasoning rules by adjusting the degree of k

value. For example, in the scope quantifier $\$^k$, k can be changed continuously to express the uncertainty of the scope of individual variables. In special case, k=1 indicates the universal quantifier∀; k>0 indicates existential quantifier ∃, k=! indicates the only existential quantifier ∃!; k=0 indicates the constraints of no scope quantifier.

4) All kinds of continuous-valued reasoning model

Because the truth value of flexible proposition, computing model and quantifiers of proposition conjunction are flexible, the reasoning rules based on them such as deductive reasoning, inductive reasoning, analogical reasoning, assuming reasoning, the evolution of reasoning are also flexible. The flexible reasoning rules are different from standard logic, which can coexist in a reasoning process. They transform each other by changing flexible parameters, and in which deductive reasoning mode is the most basic mode. Therefore, the theoretical framework can describe the unity of opposites and transformation process of contradictions, which provides the possibility to the symbolization and mathematization of dialectical logic.

The operators of universal logic as following:

1) Not operation:

NOT operation model $N(x)$ is unary operation on [0, 1]→[0, 1], which satisfies the following Not operation axiom.

Boundary condition N1: N (0)=1, N (1)=0.

Monotonicity N2: $N(x)$ is monotonously decreasing, iff ∀x, y∈[0, 1], if x<y, then $N(x)$≥$N(y)$.

The expression $\mathbf{N_3}$=ite{0|x=1; 1} is maximal Not operator. The expression $\mathbf{N_0}$=ite{1|x=0; 0} is minimal Not operator. The expression $\mathbf{N_1}$=1−x is central Not operator.

1-level universal NOT operators are mapping $N:[0,1]\to[0,1], N(x,k)=(1-x^n)^{1/n}$, which is usually denoted by $\neg_k$; the real number n has relation with the coefficient of generalized autocorrelation k as:

$$n=-1/\log_2 k\,, \tag{1}$$

where $k\in[0,1], n\in[0,+\infty)$.

Maximal Not operator is $\mathbf{N_3}$=$N(x, 1)$, and central Not operator is $\mathbf{N_1}$=$N(x, 0.5)$, and minimal Not operator is $\mathbf{N_0}$=$N(x, 0)$ (see Figure 1).

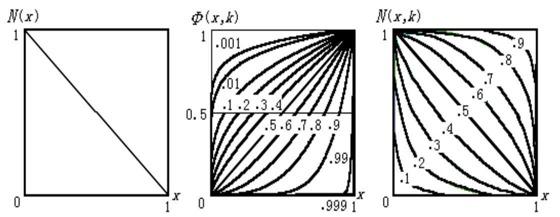

Figure 1. Not operator model integrity cluster and its generator integrity cluster

2) AND operation

AND operation model $T(x, y)$ is binary operation in $[0, 1]^2$→[0, 1], which satisfies the following operation axioms: x, y, z∈[0, 1]

Boundary condition T1 $T(0, y)$=0, $T(1, y)$=y.

Monotonicity T2 $T(x, y)$ increases monotonously along with x, y.

Association law T3 $T(T(x, y), z)$=$T(x, T(y, z))$.

Upper bound T4 $T(x, y)$≤min (x, y)

0-level universal AND operators are mapping $T:[0,1]\times[0,1]\to[0,1]$, $T(x,y,h)=\Gamma^1[(x^m+y^m-1)^{1/m}]$, which is usually denoted by $\wedge_h$; the real number m has relation with the coefficient of generalized correlation h as:

$$m=(3-4h)/(4h(1-h))\,, \tag{2}$$

$h\in[0,1], m\in R$. And $\Gamma^1[x]$ denotes x is restricted in $[0,1]$, if $x>1$ then its value will be 1, if $x<0$, its value will be 0.

1-level universal AND operators are mapping $T:[0,1]\times[0,1]\to[0,1]$ $T(x,y,h,k)=\Gamma^1[(x^{mn}+y^{mn}-1)^{1/mn}]$ which is usually denoted by $\wedge_{h,k}$. The relation m and h is as same as (2), the relation of n and k is the same as (1).

There are four special cases of $T(x, y, h)$ (see Figure 2) as follows:

Zadeh AND operator $T(x, y, 1)$=$\mathbf{T_3}$=min (x, y)

Probability AND operator $T(x, y, 0.75)$=$\mathbf{T_2}$=xy

Bounded AND operator $T(x, y, 0.5)$=$\mathbf{T_1}$=max $(0, x+y-1)$

Drastic AND operator $T(x, y, 0)$=$\mathbf{T_0}$=ite{min (x, y)|max (x, y)=1; 0}

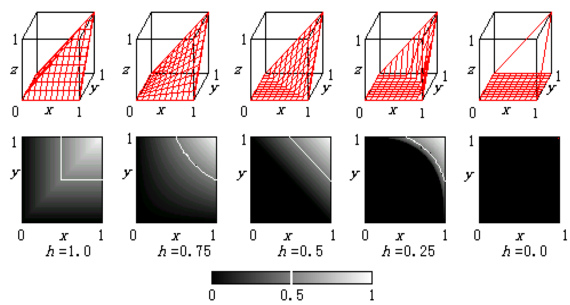

Figure 2. AND operator model figure for special h

3) OR operation:

OR operation model $S(x, y)$ is binary operation in $[0, 1]^2$→[0, 1], which satisfies the following operation axioms: x, y, z∈[0, 1].

Boundary condition $S(1, y)$=1, $S(0, y)$=y.

Monotonicity $S(x, y)$ increases monotonously along with x, y.

Association law $S(S(x, y), z)$=$S(x, S(y, z))$.

Lower bound $S(x, y)$≥max (x, y).

The Dualization law holds between $S(x, y, k, h)$ and $T(x, y, k, h)$.

$N(S(x, y, k, h), k)$=$T(N(x, k), N(y, k), k, h)$

$N(T(x, y, k, h), k)$=$S(N(x, k), N(y, k), k, h)$

There are four special cases of $S(x, y, h)$ (see Figure 3) as following:

Zadeh OR operator $S(x, y, 1)=\mathbf{S_3}=\max(x, y)$

Probability OR operator $S(x, y, 0.75)=\mathbf{S_2}=x+y-xy$

Bounded OR operator $S(x, y, 0.5)=\mathbf{S_1}=\min(1, x+y)$

Drastic OR operator $S(x, y, 0)=\mathbf{S_0}=\text{ite}\{\max(x, y)|\min(x, y)=0;1\}$

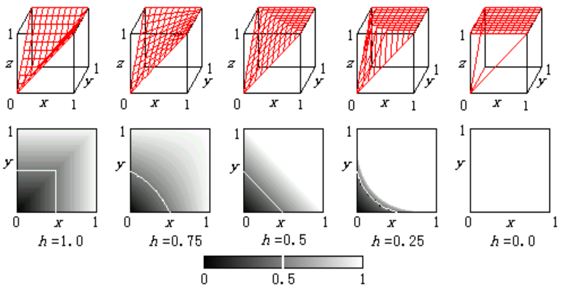

Figure 3. OR operator model figure for special h

4) IMPLICATION operation

IMPLICATION operation model $I(x, y)$ is binary operation in $[0, 1]^2 \to [0, 1]$, which satisfies the following operation axioms: $x, y, z \in [0, 1]$.

Boundary conditions I1 $I(0, y, h, k)=1$, $I(1, y, h, k)=y$, $I(x, 1, h, k)=1$.

Monotonicity I2 $I(x, y, h, k)$ is monotone increasing along with y, and is monotone decreasing along with x.

Continuity I3 When $h, k \in (0, 1)$, $I(x, y, h, k)$ is continuous along with x, y.

Order-preserving property I4 $I(x, y, h, k)=1$, iff $x \le y$ (except for $h=0$ and $k=1$).

Deduction I5 $T(x, I(x, y, h, k), h, k) \le y$ (Hypothetical consequence).

0-level universal IMPLICATION operators are mapping $I:[0,1]\times[0,1]\to[0,1], I(x,y,h)=ite\{1\,|\,x\le y;0\,|\,m\le 0\ and\ y=0;\Gamma^1[(1-x^m+y^m)^{1/m}]\}$, which is usually denoted by $\Rightarrow_h$. The relation m and h is the same as (1).

1-level universal IMPLICATION operators are mapping $I:[0,1]\times[0,1]\to[0,1], I(x,y,h)=ite\{1\,|\,x\le y;0\,|\,m\le 0\ and\ y=0;\Gamma^1[(1-x^{mn}+y^{mn})^{1/mn}]\}$, which is usually denoted by $\Rightarrow_h$. The relation m and h is the same as (1), the relation of n and k is the same as (2).

There are four special cases of $I(x, y, h)$ (see Figure 4) as following:

Zadeh IMPLICATION operator $I(x, y, 1)=\mathbf{I_3}=\text{ite}\{1|x\le y; y\}$

Probability IMPLICATION operator (Goguen Implication) $I(x, y, 0.75)=\mathbf{I_2}=\min(1, y/x)$

Bounded IMPLICATION operator (Lukasiewicz Implication) $I(x, y, 0.5)=\mathbf{I_1}=\min(1, 1-x+y)$

Drastic Implication operator $I(x, y, 0)=\mathbf{I_0}=\text{ite}\{y|x=1; 1\}$

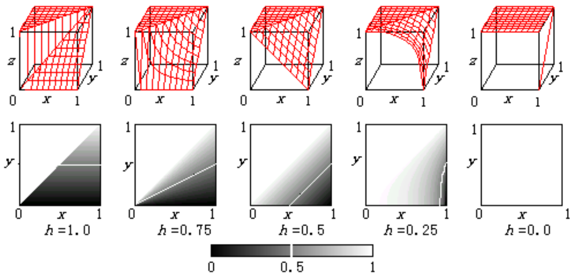

Figure 4. IMPLICATION operator model figure for special h

C. Universal Logic System $UL_{h\in(0,1]}$

The languages of 0-level UL system $UL_{h\in(0,1]}$ are based on two basic connectives $\to$ and & and one truth constant $\bar{0}$, which semantics are 0-level universal AND, 0-level universal IMPLICATION and 0 respectively (see [15]).

Axioms of the system $UL_{h\in(0,1]}$ are as following:

(i) $(\phi\to\psi)\to((\psi\to\chi)(\phi\to\chi))$

(ii) $(\phi\,\&\,\psi)\to\phi$

(iii) $(\phi\,\&\,\psi)\to(\psi\,\&\,\phi)$

(iv) $\phi\,\&\,(\phi\to\psi)\to(\psi\,\&\,(\psi\to\phi))$

(v) $(\phi\to(\psi\to\chi))\to((\phi\,\&\,\psi)\to\chi)$

(vi) $((\phi\,\&\,\psi)\to\chi)\to(\phi\to(\psi\to\chi))$

(vii) $((\phi\to\psi)\to\chi)\to(((\psi\to\phi)\to\chi)\to\chi)$

(viii) $\bar{0}\to\phi$

(ix) $(\phi\to\phi\,\&\,\psi)\to((\phi\to\bar{0})\vee\psi\vee((\phi\to\phi\,\&\,\phi)\wedge(\psi\to\psi\,\&\,\psi)))$.

The deduction rule of $UL_{h\in(0,1]}$ is modus ponens.

Definition 2 [15] A $\text{Ł}\Pi G$ algebra is a BL-algebra in which the identity

$(x\Rightarrow x*y)\Rightarrow((x\Rightarrow 0)\cup y\cup((x\Rightarrow x*x)\cap(y\Rightarrow y*y)))=1$

is valid.

For each $h\in(0,1]$, $([0,1],\min,\max,\wedge_h,\Rightarrow_h,0,1)$ which is called $\text{Ł}\Pi G$ unit interval is a linear ordering $\text{Ł}\Pi G$ algebra with its standard linear ordering.

Theorem 1 [16] (Soundness) All axioms of $UL_{h\in(0,1]}$ are 1-tautology in each PC (h). If ϕ and $\varphi\to\psi$ are 1-tautology of PC (h) then ψ is also a 1-tautology of PC (h). Consequently, each formula provable in $UL_{h\in(0,1]}$ is a 1-tautology of each PC (h), i.e. $\Gamma\vdash\phi$, then $\Gamma\vDash\phi$.

Theorem 2 [16] (Completeness) The system $UL_{h\in(0,1]}$ is complete, i.e. If $\vDash\phi$, then $\vdash\phi$. In more detail, for each formula ϕ φ, the following are equivalent:

(i) ϕ is provable in $UL_{h\in(0,1]}$, i.e. $\vdash\phi$;

(ii) ϕ is an L-tautology for each $\text{Ł}\Pi G$-algebra L;

(iii) ϕ is an L-tautology for each linearly ordered $\text{Ł}\Pi G$-algebra L;

(iv) ϕ is a tautology for each $\text{Ł}\Pi G$ unit interval, i.e. $\vDash\phi$.

D. Universal Logic System $UL^{-}_{h\in(0,1]}$

Definition 3 [17]Axioms of $UL^{-}_{h\in(0,1]}$ are those of $UL_{h\in(0,1]}$ plus

$(--\phi)\equiv\phi$ (Involution)

$\Delta(\phi \to \psi) \to \Delta(-\psi \to -\phi)$ (Order Reversing)
$\Delta\phi \vee \neg\Delta\phi$
$\Delta(\phi \vee \psi) \to (\Delta\phi \vee \Delta\psi)$
$\Delta\phi \to \phi$
$\Delta\phi \to \Delta\Delta\phi$
$\Delta(\phi \to \psi) \to (\Delta\phi \to \Delta\psi)$

where $\neg\phi$ is $\phi \to 0$. Deduction rules of $UL^{-}_{h\in(0,1]}$ are those of $UL^{\Delta}_{h\in(0,1]}$, that is, modus ponens and generalization: from φ derive $\Delta\varphi$.

Definition 4 [17] A $Ł\Pi G_{\Delta}$ -algebra is a structure $L =< L,*,\Rightarrow,\cap,\cup,0,1,\Delta >$ which is a $Ł\Pi G$ algebra expanded by an unary operation Δ in which the following formulas are true:

$\Delta x \cup (\Delta x \Rightarrow 0) = 1$
$\Delta(x \cup y) \le \Delta x \cup \Delta y$
$\Delta x \le x$
$\Delta x \le \Delta\Delta x$
$(\Delta x) * (\Delta(x \Rightarrow y)) \le \Delta y$
$\Delta 1 = 1$

Definition 5 [17] A $Ł\Pi G^{-}$ -algebra is a structure L $=< L,*,\Rightarrow,\cap,\cup,0,1,\Delta,- >$ which is a $Ł\Pi G_{\Delta}$ -algebra expanded by an unary operation -, and satisfying the following conditions:

(1) $--x = x$
(2) $\Delta(x \Rightarrow y) = \Delta(-y \Rightarrow -x)$
(3) $\Delta x \vee \neg\Delta x = 1$
(4) $\Delta(x \vee y) \le (\Delta x \vee \Delta y)$
(5) $\Delta x \le x$
(6) $\Delta x \le \Delta\Delta x$
(7) $(\Delta x) * (\Delta(x \Rightarrow y)) \le \Delta y$
(8) $\Delta 1 = 1$

Theorem 3 [17] (Soundness) Each formula provable in $UL^{-}_{h\in(0,1]}$ is a L-tautology for each $Ł\Pi G^{-}$ -algebra.

Theorem 4 [17] (Completeness) The system $UL^{-}_{h\in(0,1]}$ is complete, i.e. If $\vDash\phi$, then $\vdash\phi$. In more detail, for each formula ϕ, the following are equivalent:

(i) ϕ is provable in $UL^{-}_{h\in(0,1]}$, i.e. $\vdash\varphi$,
(ii) ϕ is an L-tautology for each $Ł\Pi G^{-}$ -algebra L,
(iii) ϕ is an L-tautology for each linearly ordered $Ł\Pi G^{-}$ -algebra L.

III. Predicate Formal System $\forall UL^{-}_{h\in(0,1]}$

In order to build first-order predicate formal deductive system based on 1-level universal AND operator, we give the first-order predicate language as following:

First-order language J consists of symbols set and generation rules:

The symbols set of J consist of as following:

(1) Object variables: $x, y, z, x_1, y_1, z_1, x_2, y_2, z_2,$;
(2) Object constants: $a, b, c, a_1, b_1, c_1,$, Truth constants: $\bar{0}, \bar{1}$;
(3) Predicate symbols: $P, Q, R, P_1, Q_1, R_1,$;
(4) Connectives: $\&, \to, \Delta, -$;
(5) Quantifiers: $\forall$ (universal quantifier), $\exists$ (existential quantifier);
(6) Auxiliary symbols: (,), , .

The symbols in (1)- (3) are called non-logical symbols of language J. The object variables and object constants of J are called terms. The set of all object constants is denoted by Var (J), The set of all object variables is denoted by Const (J), The set of all terms is denoted by Term (J). If P is n-ary predicate symbol, $t_1, t_2,, t_n$ are terms, then $P(t_1, t_2,, t_n)$ is called atomic formula.

The formula set of J is generated by the following three rules in finite times:

(i) If P is atomic formula, then $P \in J$;
(ii) If $P, Q \in J$, then $P \& Q, P \to Q, \Delta P \in J, -P \in J$;
(iii) If $P \in J$, and $x \in \mathrm{Var}(J)$, then $(\forall x)P, (\exists x)P \in J$.

The formulas of J can be denoted by $\phi, \varphi, \psi, \phi_1, \varphi_1, \psi_1,$. Further connectives are defined as following:

$\phi \wedge \psi$ is $\phi \& (\phi \to \psi)$,
$\phi \vee \psi$ is $((\phi \to \psi) \to \psi) \wedge (\psi \to \phi) \to \phi)$,
$\neg\phi$ is $\phi \to \bar{0}$,
$\phi \equiv \psi$ is $(\phi \to \psi) \& (\psi \to \phi)$.

Definition 6 The axioms and deduction rules of predicate formal system $\forall UL^{-}_{h\in(0,1]}$ as following:

(i)The following formulas are axioms of $\forall UL^{-}_{h\in(0,1]}$:

(U1) $(\phi \to \psi) \to ((\psi \to \chi) \to (\phi \to \chi))$
(U2) $(\phi \& \psi) \to \phi$
(U3) $(\phi \& \psi) \to (\psi \& \phi)$
(U4) $\phi \& (\phi \to \psi) \to (\psi \& (\psi \to \phi))$
(U5) $(\phi \to (\psi \to \chi)) \to ((\phi \& \psi) \to \chi)$
(U6) $((\phi \& \psi) \to \chi) \to (\phi \to (\psi \to \chi))$
(U7) $((\phi \to \psi) \to \chi) \to (((\psi \to \phi) \to \chi) \to \chi)$
(U8) $\bar{0} \to \phi$
(U9) $(\phi \to \phi \& \psi) \to ((\phi \to \bar{0}) \vee \psi \vee ((\phi \to \phi \& \phi) \wedge (\psi \to \psi \& \psi)))$
(U10) $(--\varphi) \equiv \varphi$
(U11) $\Delta(\varphi \to \psi) \to \Delta(-\psi \to -\varphi)$
(U12) $\Delta\phi \vee \neg\Delta\phi$
(U13) $\Delta(\phi \vee \psi) \to (\Delta\phi \vee \Delta\psi)$
(U14) $\Delta\phi \to \phi$
(U15) $\Delta\phi \to \Delta\Delta\phi$
(U16) $\Delta(\phi \to \psi) \to (\Delta\phi \to \Delta\psi)$
(U17) $(\forall x)\phi(x) \to \phi(t)$ (t substitutable for x in $\phi(x)$)
(U18) $\phi(t) \to (\exists x)\phi(x)$ (t substitutable for x in $\phi(x)$)
(U19) $(\forall x)(\chi \to \phi) \to (\chi \to (\forall x)\phi)$ (x is not free in χ)

(U20) $(\forall x)(\phi \to \chi) \to ((\exists x)\phi \to \chi)$ (x is not free in χ)

(U21) $(\forall x)(\phi \vee \chi) \to ((\forall x)\phi \vee \chi)$ (x is not free in χ)

Deduction rules of $\forall UL^{-}_{h\in(0,1]}$ are three rules. They are:

Modus Ponens (MP): from $\phi, \phi \to \psi$ infer ψ;

Necessitation: from ϕ infer $\Delta\phi$;

Generalization: from ϕ infer $(\forall x)\phi$.

The meaning of "t substitutable for x in $\phi(x)$" and "x is not free in χ" in the above definition have the same meaning in the classical first-order predicate logic, moreover, we can define the concepts such as proof, theorem, theory, deduction from a theory T, T-consequence in the system $\forall UL^{-}_{h\in(0,1]}$. $T \vdash \phi$ denotes that ϕ is provable in the theory T. $\vdash \phi$ denotes that ϕ is a theorem of system $\forall UL_{h\in(0,1]}$. Let $\mathrm{Thm}(\forall UL^{-}_{h\in(0,1]}) = \{\phi \in J \,|\vdash \phi\}, \mathrm{Ded}(T) = \{\phi \in J \mid T \vdash \phi\}$. Being the axioms of propositional system $UL^{-}_{h\in(0,1]}$ are in predicate system $\forall UL^{-}_{h\in(0,1]}$, then the theorems in $UL_{h\in(0,1]}$ are theorems in $\forall UL^{-}_{h\in(0,1]}$. According the similar proof in [3, 16, 17] we can get the following lemmas.

Lemma 1 The hypothetical syllogism holds in $\forall UL^{-}_{h\in(0,1]}$, i.e. let $\Gamma = \{\phi \to \psi, \psi \to \chi\}$, then $\Gamma \vdash \phi \to \chi$.

Lemma 2 $\forall UL^{-}_{h\in(0,1]}$ proves:

(1) $\phi \to \phi$;

(2) $\phi \to (\psi \to \phi)$;

(3) $(\phi \to \psi) \to ((\phi \to \gamma) \to (\psi \to \gamma))$;

(4) $(\phi \,\&\, (\phi \to \psi)) \to \psi$;

(5) $\Delta\phi \equiv \Delta\phi \,\&\, \Delta\phi$.

Lemma 3 If $T = \{\phi \to \psi, \chi \to \gamma\}$, then $T \vdash (\phi \,\&\, \chi) \to (\psi \,\&\, \gamma)$.

Let J is first-order predicate language, L is linearly ordered $Ł\Pi G^{-}$ algebra, $M = (M, (r_P)_P, (m_c)_c)$ is called a L-evaluation for first-order predicate language J, which M is non-empty domain, according to each n-ary predicate P and object constant c, r_P is L-fuzzy n-ary relation: $r_P : M^n \to$ L, m_c is an element of M.

Definition 7 Let J be predicate language, M is L-evaluation of J, x is object variable, $P \in J$.

(i) A mapping v: $Term(J) \to M$ is called M-evaluation, if for each $c \in Const\,(J)$, $v(c) = m_c$;

(ii)Two M-evaluation v, v' are called equal denoted by $v \equiv_x v'$ if for each $y \in \mathrm{Var}(J) \setminus x$, there is $v(y) = v'(y)$.

(iii) The value of a term given by M, v is defined by: $\|x\|_{\mathrm{M},v} = v(x)$; $\|c\|_{\mathrm{M},v} = m_c$. We define the truth value $\|\phi\|^{\mathrm{L}}_{\mathrm{M},v}$ of a formula ϕ as following. Clearly, $*, \Rightarrow, \Delta$ denote the operations of L.

$$\|P(t_1, t_2, , t_n)\|^{\mathrm{L}}_{\mathrm{M},v} = r_P(\|t_1\|_{\mathrm{M},v}, , \|t_n\|_{\mathrm{M},v})$$
$$\|\phi \to \psi\|^{\mathrm{L}}_{\mathrm{M},v} = \|\phi\|^{\mathrm{L}}_{\mathrm{M},v} \Rightarrow \|\psi\|^{\mathrm{L}}_{\mathrm{M},v}$$
$$\|\phi \,\&\, \psi\|^{\mathrm{L}}_{\mathrm{M},v} = \|\phi\|^{\mathrm{L}}_{\mathrm{M},v} * \|\psi\|^{\mathrm{L}}_{\mathrm{M},v}$$
$$\|\bar{0}\|^{\mathrm{L}}_{\mathrm{M},v} = 0; \quad \|\bar{1}\|^{\mathrm{L}}_{\mathrm{M},v} = 1$$
$$\|\Delta\phi\|^{\mathrm{L}}_{\mathrm{M},v} = \Delta\|\phi\|^{\mathrm{L}}_{\mathrm{M},v}$$
$$\|-\phi\|^{\mathrm{L}}_{\mathrm{M},v} = -\|\phi\|^{\mathrm{L}}_{\mathrm{M},v}$$
$$\|(\forall x)\phi\|^{\mathrm{L}}_{\mathrm{M},v} = \inf\{\|\phi\|^{\mathrm{L}}_{\mathrm{M},v'} \mid v \equiv_x v'\}$$
$$\|(\exists x)\phi\|^{\mathrm{L}}_{\mathrm{M},v} = \sup\{\|\phi\|^{\mathrm{L}}_{\mathrm{M},v'} \mid v \equiv_x v'\}$$

In order to the above definitions are reasonable, the infimum/supremum should exist in the sense of L. So the structure M is L-safe if all the needed infima and suprema exist, i.e. $\|\phi\|^{\mathrm{L}}_{\mathrm{M},v}$ is defined for all ϕ, v.

Definition 8 Let $\phi \in J$, M be a safe L-structure for J.

(i) The truth value of ϕ in M is $\|\phi\|^{\mathrm{L}}_{\mathrm{M}} = \inf\{\|\phi\|^{\mathrm{L}}_{\mathrm{M},v} \mid v \text{ M} - \text{evaluation}\}$.

(ii) A formula ϕ of a language J is an L-tautology if $\|\phi\|^{\mathrm{L}}_{\mathrm{M}} = 1_{\mathrm{L}}$ for each safe L-structure M. i.e. $\|\phi\|^{\mathrm{L}}_{\mathrm{M,v}} = 1$ for each safe L-structure M and each M-valuation of object variables.

Remark For each $h \in (0,1], k \in (0,1)$, $([0,1], \wedge_{h,k}, \Rightarrow_{h,k}, \min, \max, 0, 1, \Delta, -)$ is a $Ł\Pi G^{-}$-algebra. So the predicate system $\forall UL^{-}_{h\in(0,1]}$ can be considered the axiomatization for 1-level universal AND operator.

III. Soundness of System $\forall UL^{-}_{h\in(0,1]}$

Definition 9 A logic system is soundness if for its each theorem ϕ, we can get ϕ is a tautology.

Theorem 5 (Soundness of axioms) The axioms of $\forall UL^{-}_{h\in(0,1]}$ are L-tautologies for each linearly ordered $Ł\Pi G^{-}$-algebra L.

Proof. The axioms of (U1)- (U16) are L-tautologies can be get as in propositional calculus. We verify (U17)-(U21)

To verify (U17), (U18), let y is substitutable for x to ϕ, when $v'' \equiv_x v$ and $v''(x) = v(y)$, there is $\|\phi(y)\|^{\mathrm{L}}_{\mathrm{M},v} = \|\phi(x)\|^{\mathrm{L}}_{\mathrm{M},v''}$. So, $\|(\forall x)\phi(x)\|^{\mathrm{L}}_{\mathrm{M},v} = \inf_{v'\equiv v}\|\phi(x)\|^{\mathrm{L}}_{\mathrm{M},v'}$ $\leq \|\phi(y)\|^{\mathrm{L}}_{\mathrm{M},v''} \leq \sup_{v'}\|\phi(x)\|^{\mathrm{L}}_{\mathrm{M},v'} = \|(\exists x)\phi(x)\|^{\mathrm{L}}_{\mathrm{M},v}$, then $\|(\forall x)\phi(x) \to \phi(y)\|_{\mathrm{M},v} = \|(\forall x)\phi(x)\|_{\mathrm{M},v} \to \|\phi(y)\|_{\mathrm{M},v} = 1$.

For (U19), let x not free in χ, then for each M-valuation w, when $w \equiv_x v$, we have $\|v\|^{\mathrm{L}}_{\mathrm{M},w} = \|\phi(x)\|^{\mathrm{L}}_{\mathrm{M},v}$. We have to prove

$$\inf_w(\|v\|^{\mathrm{L}}_{\mathrm{M},w} \Rightarrow \|\phi\|^{\mathrm{L}}_{\mathrm{M},w}) \leq (\|v\|^{\mathrm{L}}_{\mathrm{M},v} \Rightarrow \inf_w\|\phi\|^{\mathrm{L}}_{\mathrm{M},w}).$$

Let $\|v\|^{\mathrm{L}}_{\mathrm{M},v}=\|v\|^{\mathrm{L}}_{\mathrm{M},w}=a$, $\|\phi\|^{\mathrm{L}}_{\mathrm{M},w}=b_w$, thus we must prove $\inf_w(a\Rightarrow b_w)\le(a\Rightarrow\inf_w b_w)$. On the one hand, $\inf_w b_w\le b_w$, thus $a\Rightarrow b_w\ge(a\Rightarrow\inf_w b_w)$ for each *w*, thus $\inf_w(a\Rightarrow b_w)\ge(a\Rightarrow\inf_w b_w)$. On the other hand if $z\le(a\Rightarrow b_w)$ for each *w,* then $z*a\le b_w$ for each *w*, $z*a\le\inf_w b_w$, $z\le(a\Rightarrow\inf_w b_w)$. Thus $(a\Rightarrow\inf_w b_w)$ is the infimum of all $(a\Rightarrow b_w)$. So (U19) holds.

For (U20), we need to verify $\inf_w(a_w\Rightarrow b_w)=(\sup_w a_w\Rightarrow b)$. Indeed, $\sup_w\ge a_w$, thus $(\sup_w a_w\Rightarrow b)\le(a_w\Rightarrow b)$, hence $(\sup_w a_w\Rightarrow b)\le\inf_w(a_w\Rightarrow b)$, If $z\le a_w\Rightarrow b$ for all w, then $a_w\le(z\Rightarrow b)$ for all *w*, then $\sup_w a_w\le(z\Rightarrow b)$, $z\le(\sup_w a_w\Rightarrow b)$, so $\sup_w a_w\Rightarrow b$ is the infimum. So (U20) holds.

Finally we verify (U21), we need to verify $\inf_w(a\vee b_w)=a\vee\inf_w b_w$. Indeed, $a\le a\vee b_w$, thus $a\le\inf_w(a\vee b_w)$; similarly, $\inf_w b_w\le\inf_w(a\vee b_w)$, thus $a\vee\inf_w b_w\le\inf_w(a\vee b)$. Conversely, let $z\le a\vee b_w$ for all w, we prove $z\le a\vee\inf_w b_w$.

Case 1: Let $a\le\inf_w b_w$. Then $z\le b_w$ for each w , $z\le\inf_w b_w$ and $z\le a\vee\inf_w b_w$.

Case 2: Let $a\ge\inf_w b_w$. Then for some w_0 , $a\ge b_{w_0}$, thus $z\le a$ and $z\le a\vee\inf_w b_w$.

So we prove the soundness of axioms.

Theorem 6 (Soundness of deduction rules) (1) For arbitrary formulas ϕ,ψ , safe-structure *M* and evaluation v , $\|\psi\|^{\mathrm{L}}_{\mathrm{M},v}\ge\|\phi\|^{\mathrm{L}}_{\mathrm{M},v}*\|\phi\to\psi\|^{\mathrm{L}}_{\mathrm{M},v}$. In particular, if $\|\phi\|^{\mathrm{L}}_{\mathrm{M},v}=\|\phi\to\psi\|^{\mathrm{L}}_{\mathrm{M},v}=1_{\mathrm{L}}$ then $\|\psi\|^{\mathrm{L}}_{\mathrm{M},v}=1_{\mathrm{L}}$.

(2) Consequently, $\|\psi\|^{\mathrm{L}}_{\mathrm{M}}\ge\|\phi\|^{\mathrm{L}}_{\mathrm{M}}*\|\phi\to\psi\|^{\mathrm{L}}_{\mathrm{M}}$, thus if $\phi,\phi\to\psi$ are then ψ is 1_L -true in *M*.

(3) If ϕ is 1_L -true in *M* then $\Delta\phi$ is 1_L -true in *M* .

(4) If ϕ is 1_L -true in *M* then $(\forall x)\phi$ is 1_L -true in *M*.

Proof. (1) is just as in propositional calculus.

To prove (2) put $\|\phi\|_w=a_w,\|\psi\|_w=b_w,\inf_w a_w=a$. We have to prove $\inf_w(a_w\Rightarrow b_w)\le\inf_w a_w\Rightarrow\inf_w b_w$. Observe the following:

$\inf(a_w\Rightarrow b_w)\le(a_w\Rightarrow b_w)\le(a\Rightarrow b_w)$,

thus $\inf_w(a_w\Rightarrow b_w)\le\inf_w(a\Rightarrow b_w)$. It remains to prove $\inf_w(a\Rightarrow b_w)\le a\Rightarrow\inf_w b_w$, this is holds from Theorem 5.

(3) If ϕ is 1_L -true in M then $\phi^L_M=1_L$, so $\|\Delta\phi\|^{\mathrm{L}}_{\mathrm{M},v}=\Delta\|\phi\|^{\mathrm{L}}_{\mathrm{M},v}=1_L$. Then (3) holds.

(4) Being $\|\phi\|^{\mathrm{L}}_{\mathrm{M}}=\inf\{\|\phi\|^{\mathrm{L}}_{\mathrm{M},v}\mid v\ M-evaluation\}$

$\le\inf\{\phi_{M,v'}\mid v'\equiv v\}=\|(\forall x)\phi\|^{\mathrm{L}}_{\mathrm{M}}\}$, So (4) holds.

So we can get the following soundness theorem.

Theorem 7 (Soundness) Let *L* is linearly ordered $Ł\Pi G^{-}$-algebra and ϕ is a formula in *J*, if $\vdash\phi$, then ϕ is *L*-tautology, i.e. $\|\phi\|^{\mathrm{L}}_{\mathrm{M}}=1_{\mathrm{L}}$.

Similarly, we can get the following strong soundness theorem.

Definition 10 Let *T* be a theory, *L* be a linearly ordered $Ł\Pi G^{-}$-algebra and *M* a safe *L*-structure for the language of *T*. *M* is an *L*-model of *T* if all axioms of *T* are 1_L -true in *M*, i.e. $\phi=1_L$ in each $\phi\in T$.

Theorem 8 (Strong Soundness) Let *T* be a theory, *L* is linearly ordered $Ł\Pi G^{-}$-algebra and ϕ is a formula in *J*, if $T\vdash\phi$ (ϕ is provable in *T*), then $\phi^L_M=1_L$ for each linearly ordered $Ł\Pi G^{-}$-algebra *L* and each *L*-model M of *T*.

Proof. In fact, from the proof of Theorem 5, for each *L*-model M of *T*, the axioms are true, and the formulas in *T* are true, from the proof of Theorem 6, the deduction rules preserve true. So the theorem holds.

Theorem 9 (Deduction Theorem) Let *T* be a theory, ϕ,ψ are closed formulas. Then $(T\cup\phi)\vdash\psi$ iff $T\vdash\Delta\phi\to\psi$.

Proof. Sufficiency: Let $T\vdash\Delta\phi\to\psi$, from ϕ ($\phi\in(T\cup\phi)$), then $\Delta\phi$ by necessitation, so we can get ψ by MP rules.

Necessity: Let *m* is the proof length from $T\cup\phi$ to ψ , we prove by induction for the length *m*.

When $m=1$, $\psi\in T\cup\phi\cup\mathrm{Axm}(\mathrm{C}\forall)$, if $\psi=\phi$, The result holds. If $\psi\in T$ or ψ is axiom, from Lemma2 (2), we have $\psi\to(\Delta\phi\to\psi)$, then by $\psi,\psi\to(\Delta\phi\to\psi)$, we get $\Delta\phi\to\psi$, thus $T\vdash\Delta\phi\to\psi$.

Assume that the result holds when $m\le k$, i.e. we get γ at k step, then $T\vdash\Delta\phi\to\gamma$. Now Let $m=k+1$.

If ψ is obtained from MP rule by the above results $\gamma,\gamma\to\psi$ in the proof sequence, then by induction hypothesis, we get $T\vdash\Delta\phi\to\gamma,T\vdash\Delta\phi\to(\gamma\to\psi)$. From Lemma 3, we can get $T\vdash(\Delta\phi\,\&\,\Delta\phi)\to(\gamma\,\&\,(\gamma\to\psi))$. Being $T\vdash(\Delta\phi)\,\&\,(\Delta\phi)\equiv\Delta\phi$, so $T\vdash\Delta\phi\to(\gamma\,\&\,(\gamma\to\psi))$. From lemma 2 (4) we have $(\gamma\,\&\,(\gamma\to\psi)\to\psi$, so we get $T\vdash\Delta\phi\to\psi$ by the hypothetical syllogism.

If ψ is obtained from necessitation rule by the above step γ in the proof sequence, i.e. $\Delta\gamma=\psi$, then by induction hypothesis, we get $T\vdash\Delta\phi\to\gamma$. $T\vdash\Delta(\Delta\phi\to\gamma)$, from (U16) we can get $T\vdash\Delta\Delta\phi\to\Delta\gamma$, from (U15) we can get $\Delta\phi\to\Delta\Delta\phi$, thus by the hypothetical syllogism we can get $T\vdash\Delta\phi\to\Delta\gamma$, i.e. $T\vdash\Delta\phi\to\psi$.

If ψ is obtained from generalization rule by the above step γ in the proof sequence, i.e. $(\forall x)\gamma=\psi$, then by induction hypothesis, we get $T \vdash \Delta\phi \to \gamma$, From generalization rule we can get $T \vdash (\forall x)(\Delta\phi \to \gamma)$, being $\Delta\phi, \gamma$ are closed formula and from (U19), we can get $T \vdash \Delta\phi \to (\forall x)\gamma$, i.e. $T \vdash \Delta\phi \to \psi$.

So the theorem holds.

IV. Conclusion

In this paper a predicate calculus formal deductive system $\forall UL^{-}_{h\in(0,1]}$ based on the propositional system $UL^{-}_{h\in(0,1]}$ for 1-level universal AND operator is built up. We prove the system $\forall UL^{-}_{h\in(0,1]}$ is sound. The deduction theorem is also given. Next we will discuss the completeness of system $\forall UL^{-}_{h\in(0,1]}$.

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Yingcang Ma, is a professor in school of science, Xi'an Polytechnic University. He received the PhD. degree from School of Computer Science, Northwestern Polytechnical University, in July 2006. His main researchinterests are in the areas of fuzzy set, rough set and non-classical mathematical logic.

Hucan He, male, Professor and Ph.D. tutor from the Department of Computer Science and Engineering of Northwestern Polytechnical University, interested in the foundation and application of AI, universal logic and uncertainties reasoning.