Extended Linguistic Variable and Its Effective Set

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***Abstract*—The paper aims to expand the concept of linguistic variable. The study analyzes the common conclusion forms in the evaluation project. The evaluation often makes the final conclusion by analyzing the fuzzy number and its membership function. However, the information of modified word or phrase ("largely", "preliminary" and "unclear") has got lost in analysis. The research believe these modified words imply the inherent re-judgments on the mainly conclusions. And the study develops the extended linguistic variable by integrating the modified and traditional linguistic variable. Then the study defines the effective set of extended linguistic variable. The effective set suggest the valuable scale of the extended linguistic variable.**

***Index Terms*—extended linguistic variable; modified Linguistic variable; effective set.**

I. Introduction

The Linguistic Variable concept is already widely used in Fuzzy Theory. Zadeh(1975) developed the concept of Linguistic Variable[1], then he extended the concept in the following study[2][3]. The Linguistic Variable describe the intuitive feelings to the evaluation object with a set of predetermined natural language words, such as Very Good, Good, Mediocre, Bad, Very Bad[4]. The words may be transformed to many different sorts of fuzzy number. The fuzzy number can express the membership functions, such as the triangle or trapezoid membership function. Chen & Hwang(1992) give the most common used eight sets of Linguistic terms[4].

The project evaluation use these linguistics terms to describe its value. And some conclusions were given as following:

"The project - was largely good – natured"

"The procedure is largely right"

" The preliminary estimate was…, but some key details about its implementation remain unclear"

These conclusions were often treated as linguistic variable and transformed to fuzzy number. The evaluation makes the final conclusion by analyzing the fuzzy number and its membership function. However, the method does not take all information into account. The information of modified word or phrase ("largely", "preliminary" and "unclear") has got lost in analysis.

These modified words or phrases indicate some subconscious judgments of valuators in evaluation. They are not the completely unintentional and unmeaning adjunct on conclusion. In many cases, these elements imply the inherent re-judgments on their own conclusions. Sometimes, they are self-evaluation on the valuators themselves. Therefore, the information from the adjunct provides another view on evaluation in the different dimension.

It can be assumed that the valuators will be inclined to use lots of modified words or phrases such as "largely", "preliminary" or "unclear" while they does not maintain knowledge in relevant field. These words modified the mainly conclusion such as "good", "right". And the valuators will be inclined to use the positive words (obviously, certainly) to express the affirmative tone while they have very confident of their conclusion. Generally speaking, the adjunct reflects the judgment on the evaluation completeness and accuracy.

The study develops the concept of extended linguistic variable by analyzing the form of structure. Firstly, the normative linguistic terms set is given by the selected modified words. Then the modified linguistic terms set and traditional linguistic variable are integrated to build extended linguistic variable. Based on hedge operator and basic linguistic knowledge, the study develops the summary procedure of evaluation accuracy test. Secondly, the extended linguistic variable can be transformed to the grey fuzzy set or vague set.

The extended linguistic variable integrates the modified and traditional linguistic variable. The modified linguistic variable may be transformed to false membership degree or grey number. The extended linguistic can be transformed to vague set. Then extended linguistic variable can be divided into two parts according to the reliability of the result. The area with high reliability is defined as effective set of the extended linguistic variable.

A. Extended Linguistic Variable (ELV)

The quantitative and qualitative methods are widely used in evaluation. The objective measure method is effective in nature science field. It is easy to represent the object with quantitative model in this field. However, it is a very complex question to build an acceptable quantitative model in management science. In many cases, it is a more realistic goal to qualitative model to describe the characters of evaluation object. Therefore, it becomes a valuable research issue to improve the traditional linguistic variable and extend its covering domain.

The linguistic variable is the essential for fuzzy theory. It builds a bridge from the nature language words to fuzzy logic. Zadeh(1975) developed the concept of Linguistic Variable[1], and Chen & Hwang(1992) summarize the

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most common used linguistic terms[4]. These linguistic terms can be transformed to types of fuzzy number by the membership function. The valuator may select the word in linguistic term to describe the evaluation object easily. The result can be transformed to fuzzy number by predetermined membership function. And the fuzzy evaluation model was built to match the whole feeling of all valuators based on the fuzzy number.

In the specific work of evaluation, the valuators select a predetermined word to describe the utility values of project on the evaluation indicator . The project belongs to the evaluation project set . It supposed to get m linguistic evaluation matrix based on the selected words. And the m matrix can be transformed to m fuzzy number matrix by predetermined membership function. The fuzzy number matrix is the foundation of fuzzy evaluation. The fuzzy evaluation method is widely used in practice.

Another evaluation method is grey assessment method. It is approach to small sample or short-term conditions. In many case, the evaluation question has the feature of grey and fuzzy at the same time. And it is required to take these characters into account together [5]. The former study develop many grey fuzzy evaluation method to solve the problem[6][7].These methods try to build the evaluation model with the incomplete or short-term fuzzy information. And these methods take two features into account in aggregative model [5].

The study defines the word group as the extended linguistic variable. And the is the element of , i is the number of all group elements in . For example, it assumed there are 5 level words in , the extended linguistic terms is given in following table.

And B={Very Unclear, Unclear, Mainly, Obvious, Very Obvious}

The value domain of B has five levels. $b_i \in B$ is the adjunct of $s_i \in S$,and S is the traditional linguistic terms.

TABLE I.
FIVE LEVEL EXTENDED LINGUISTIC TERMS

Modified linguistic B	Linguistic S
Very Unclear	Very Bad
Unclear	Bad
Mainly	Medium
Obvious	Good
Very Obvious	Very Good

The integrated evaluation result is (b_i, s_j) . The 2-tuple data set (b_i, s_j) means the valuator use the word b_i modify the meaning of s_j .

Definition 1. Set $\tilde{\mu}_{ij}^e = (b_i, s_j)^e$ is the evaluation result of expert e. (b_i, s_j) is extended linguistic variable. And $\tilde{\mu}_{ij}^e$ means the expert e give linguistic evaluation result s_j , and he use the word b_i modify the meaning of s_j . There are m experts together. Their evaluation matrix is given as following:

$$\tilde{D}_m^e = [\tilde{\mu}_{ij}^e] = \begin{bmatrix} (b_i, s_j)^1 \\ (b_i, s_j)^2 \\ \cdots \\ (b_i, s_j)^e \end{bmatrix} and \quad e = 1,2,\ldots,m \tag{1}$$

Assume there are k evaluation indicators in practices. The evaluation is given as following:

$$\tilde{D}_m^e(k) = [\tilde{\mu}_{ij}^e(1),\ldots,\tilde{\mu}_{ij}^e(k)] \ and \ e = 1,2,\ldots,m \tag{2}$$

The modified linguistic variable b_i means the expert modify his evaluation result s_j in subconscious. In a sense, it is the judgment on the completeness of the evaluation result.

B. Vague Set based on Extended Linguistic Variable

The traditional fuzzy evaluation method is based on Fuzzy Sets by Zadeh (1965) and his linguistic variable concept[1][2][3]. However, some researchers believe the fuzzy set has some faultiness[8][9]. And the related studies try to improve the fuzzy sets. There are two important theories:Intuitionistic Fuzzy Set and Vague Set.

Intuitionistic Fuzzy Sets(IFS) is given by Atanassov[10].

Set U is universe domain, and the x is the element of U. The 3-tuple set $\{< x, t_A(x) >, f_A(x) \mid x \in U\}$ in U is an intuitionistic Fuzzy Set. The function $t_A(x), f_A(x)$ meet $t_A : U \to [0,1], f_A \to [0,1]$. $t_A(x), f_A(x)$ mean the true membership degree and false membership degree. And $t_A(x), f_A(x)$ meet $0 \le t_A(x) + f_A(x) \le 1$.

Gau and Buehrer(1993) define the concept of Vague set[11].

Set U is universe domain, and the x is the element of U. The vague set A in U has true membership function $t_A(x) : U \to [0,1]$ and false membership function $f_A(x) : U \to [0,1]$, and $0 \le t_A(x) + f_A(x) \le 1$. True membership function $t_A(x)$ means the lower bound of which the element x∈A. The function $\pi_A(x) = 1 - t_A(x) - f_A(x)$ means the unknown level which the element x relative to A. The greater $\pi_A(x)$ means there are more unknown information of element x relative to A. Set x∈U, the closed interval $[t_A(x),1-f_A(x)]$ is the value of Vague set A on the point x.

The vague set includes the true and false membership degree. It contains the unknown information about the x ∈U. Therefore, the vague set value $[t_A(x),1-f_A(x)]$ has more information than the fuzzy set value $\mu_A(x)$ on the point x.

The vague set degenerates to fuzzy set of Zadeh while the $\pi_A(x)=0, t_A(x)=1, f_A(x)=0$. Bustince and Burillo(1996) proved that the vague sets are equivalent to Intuitionistic Fuzzy Sets[12]. These two theories are in essence unified.

The extended can be transformed to vague set. In the extended linguistic variable (b_i, s_j) , the traditional linguistic variables s_j express the views of valuators on evaluation objects. And modified variable b_i suggest the fuzzy judgment on the traditional linguistic variables. It reflects the accuracy of the evaluation result. The linguistic variable s_j can be transformed to triangular fuzzy number by relevant membership function. The modified linguistic variable b_i can be transformed to fuzzy number in the same way. Then result

For example, the smaller b_i (b_i =Unclear) means the more unknown component part in the evaluation result. It suggests there is some uncertainty in evaluation result. And the bigger b_i (b_i =Obvious) means the valuator has more confidence on his evaluation result. It means the valuator believes he has the sufficient evidence to support his judgment.

It assumed there are 5 level words in extended

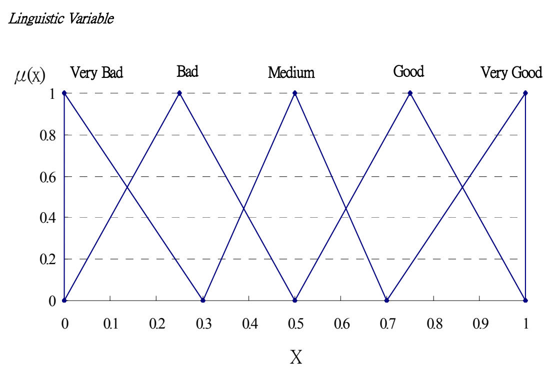

Figure1.five level grades linguistic variable and membership function

linguistic variable (B, S). The extended linguistic terms can be transformed to the triangle fuzzy number by the relevant fuzzy membership function. The relationship between the linguistic variable and fuzzy number is shown as the following figure [4].

The modified variable can be transformed to the fuzzy number in the same way. And the fuzzy number of modified variable has some relationship with the hesitancy degree of the valuator. It assumed that the fuzzy number of b_i is $\tilde{b}_i$.

While $\tilde{b}_i \ge \tilde{b}_j$,the hesitancy degree $\pi_i=(1-f_i-t_i)\le(1-f_j-t_j)=\pi_j$. Specially, while the $\tilde{b}_i > \tilde{b}_j$, the hesitancy degree π_i is strictly greater than π_j . So the study set (1- $\tilde{b}_i$) is approximately equal to the hesitancy degree π_i .

Definition2, set the expert i give his evaluation result in extended linguistic variable is (b_i, s_i) . The fuzzy number of (b_i, s_i) is $(\tilde{b}_i, \tilde{s}_i)$. Then:

$$\begin{cases} 1-\tilde{b}_i = 1-f_i-t_i \\ \tilde{s}_i = t_i \\ and \quad f_i \in [0,1], t_i \in [0,1] \end{cases} \quad (3)$$

According to formula (3), can be derived:

$$\begin{cases} f_i + t_i = \tilde{b}_i \\ t_i = \tilde{s}_i \\ and \quad f_i \in [0,1], t_i \in [0,1] \end{cases}$$

That is:

$$\begin{cases} f_i = \tilde{b}_i - \tilde{s}_i \\ t_i = \tilde{s}_i \\ \tilde{s}_i \in [0,1], \quad \tilde{b}_i \in [\tilde{s}_i, 1] \end{cases}$$

(4)

So the evaluation results (b_i, s_i) in extended linguistic variable of expert i can be transformed to vague set A_i according to formula (4). And:

$$A_i = (X_i, t_i, 1-f_i) = (X_i, \tilde{s}_i, 1-\tilde{b}_i + \tilde{s}_i)$$

Or $A_i = (X_i, t_i, f_i) = (X_i, \tilde{s}_i, \tilde{b}_i - \tilde{s}_i)$

And $\tilde{s}_i \in [0,1], \quad \tilde{b}_i \in [\tilde{s}_i, 1]$ (5)

In the formula, the X_i is the judgment of expert i on the evaluation indicator C. It is extended linguistic variable. The formula (5) satisfies the definition of vague set.

$$\forall\ 0 \le t_i + f_i = \widetilde{b}_i \le 1 \text{ ,and}$$

$$0 \le t_i = \widetilde{s}_i \le 1; 0 \le f_i = \widetilde{b}_i - \widetilde{s}_i \le 1 \quad (6)$$

To transform the extended linguistic variable to vague set, the study designs the following steps.

Step1: Transform the modified and traditional linguistic variable (b_i, s_i) to the triangle fuzzy number $(\widetilde{b}_i, \widetilde{s}_i)$ by the similar membership function in figure 1.

Step2: Compare the $\widetilde{b}_i$ and $\widetilde{s}$, if $\widetilde{b}_i < \widetilde{s}$, then the study believe the ELV is not an effective set. Otherwise, goto step 3.

Step3: Get the cut set on the level of the ELV, transform the triangle fuzzy number to interval fuzzy number.

Step4: Set the fuzzy number $\widetilde{s}_i$ represents the true membership degree t_i of vague set, and the modified fuzzy number $\widetilde{b}_i$ represents the sum of true and false membership degree $f_i + t_i$.

Step5: Set the vague set A_i is transformed from ELV (b_i, s_i). And (b_i, s_i) is the evaluation result of the expert i on the evaluation indicator C. Therefore, $A_i = (X_i, t_i, 1 - f_i) = (X_i, \widetilde{s}_i, 1 - \widetilde{b}_i + \widetilde{s}_i)$

In these procedures, step3 is the optional. It provides the convenient tool for following step. And it does not have substantial impact on these procedures. The vague set from ELV can be used in evaluation work.

C. The Effective Set of Extended Linguistic Variable

Definition3(Effective set), set the expert i give his evaluation result in extended linguistic variable is (b_i, s_i). The fuzzy number of (b_i, s_i) is $(\widetilde{b}_i, \widetilde{s}_i)$.

$\forall A_i(\widetilde{b}_i, \widetilde{s}_i)$, if and only if $\widetilde{s}_i \in [0,1]$, $\widetilde{b}_i \in [\widetilde{s}_i, 1]$, $A_i(\widetilde{b}_i, \widetilde{s}_i)$ is effective set of extended linguistic variable. Otherwise, the (b_i, s_i) has no meaning.

It assumes that there is an effective set which satisfies $\widetilde{b}_i \le \widetilde{s}_i$. It means the fuzzy number of modified linguistic variable is greater than that of linguistic variable. The expert i gives his linguistic judgment s_i, and he use a smaller linguistic variable b_i to modify his result. The most extreme example is $0 = \widetilde{b}_i < \widetilde{s}_i = 1$.

In this case, the expert i believe the object perform Very Good($\widetilde{s}_i = 1$). However, he use Very Unclear to modify his judgment. It is ambiguity. The study tends to think this judgment is (partly) invalid. So the concept of effective set defined in definition 3 is very important.

The effective sets have some special geometrical meaning. The study builds a Cartesian coordinate system to illustrate the geometrical meaning of ELV reference the method of Atanassov(1999) [13].

Assume vague set

$A = (X_i, t_i, f_i) = (X_i, \widetilde{s}_i, \widetilde{b}_i - \widetilde{s}_i)$ is come from ELV, and build a function g_A from the universe domain U to F. And:

1）If $x \in U$,Then $p = g_A \in F$;

2）The Cartesian Coordinates of point $p \in F$ is (a', b') ,and

$$0 \le a', b' \le 1, a' = t_A(x_i), b' = f_A(x_i)$$

3）$t_A(x_i) = \widetilde{s}_i$, $f_A(x_i) = \widetilde{b}_i - \widetilde{s}_i$

Reference the former study, assume the expert i give his judgment in extended linguistic variable[14][15][16]. And the Coordinates of ELV is $(t_i, f_i, \pi_i) = (\widetilde{s}_i, \widetilde{b}_i - \widetilde{s}_i, 1 - \widetilde{b})$ in figure 2. So the hesitancy degree π_i of expert i equal to $1 - t_i - f_i = 1 - \widetilde{s}_i - (\widetilde{b}_i - \widetilde{s}_i) = 1 - \widetilde{b}_i$.

It is same with formula(4) in form. The study makes further explain on hesitancy degree with the voting model.

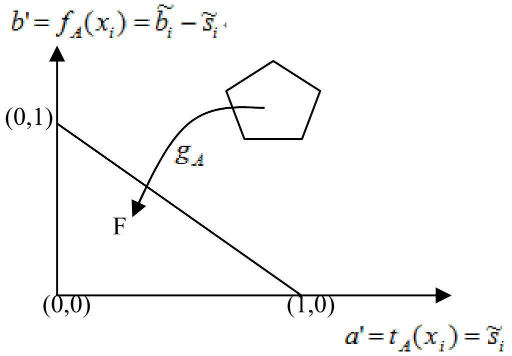

Figure2. Geometrical meaning of ELV

Reference the former study, assume the expert i give his judgment in extended linguistic variable [14][15][16]. And the coordinates of ELV is $(t_i, f_i, \pi_i) = (\widetilde{s}_i, \widetilde{b}_i - \widetilde{s}_i, 1 - \widetilde{b})$ in figure 2. So the hesitancy degree π_i of expert i equal to

$1-t_i-f_i=1-\tilde{s}_i-(\tilde{b}_i-\tilde{s}_i)=1-\tilde{b}_i$. It is same with formula(4) in form. The study makes further explain on hesitancy degree with the voting model.

Assume there are three experts A, B and C. they make their impendent judgment with extended linguistic variable. The coordinates of their result are A(1,0,0)、B(0,1,0)、C(0,0,1).

According to maximal subjection principle, the study gets the cut set in level λ =1. Then there are three possible situations:

1) Expert A(1,0,0) gives his evaluation result with words *Very Good*($s_i=1$),and he believe his conclusion is come from the large amount of information. So he use the words *Very Obvious* to modify(support) his judgment (*Very Good*).

2) Expert B(0,1,0) gives a *Very Bad* judgment to the object, and he also believe his conclusion is come from the large amount of information. So he use the words *Very Obvious* to suggest his judgment (*Very Bad*)is reliable.

3) Expert C(0,0,1) think the object is *Very Bad,* however, he has no confidence on his judgment because he only has very little "evidence"(information) to support his own conclusion. So he uses the words Very Unclear to suggest his hesitancy. And his hesitancy degree $\pi_i=1-\tilde{b}_i=1$ demonstrate his judgment is unreliable completely.

The rectangular projection of three-dimensional Graph of ELV is in figure3.

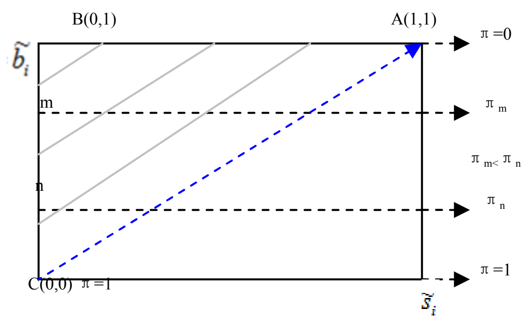

Figure3. rectangular projection of three-dimensional Graph of ELV

In figure3, there are some differences with the former study[17]. It is because of transformation $(t_i, f_i, \pi_i)=(\tilde{s}_i, \tilde{b}_i-\tilde{s}_i, 1-\tilde{b})$.

In this coordinate system, the bigger modified linguistic variable means the less hesitancy degree. Assume the fuzzy number of ELV is $(\tilde{s}_i,\tilde{b}_i)$, and the hesitancy degree is $\pi_i=1-\tilde{b}_i$. So,

$$\forall m,n \text{ ,If } \tilde{b}_m>\tilde{b}_n;$$

$$\text{Then } \pi_m=1-\tilde{b}_m<1-\tilde{b}_n=\pi_n \qquad (7)$$

And:

$$\forall c,d \text{ ,If } \tilde{s}_c>\tilde{s}_d;$$

$$\text{Then } t_c=\tilde{s}_c>\tilde{s}_d=t_d \qquad (8)$$

According to the formula (7) and (8), the reliability of evaluation result raise in the direction of the dotted line arrow in figure 3. Otherwise, the more uncertainty means the less completeness of evaluation result. In the field of △ABC,

there exist $\tilde{s}_i\in[0,1]$, $\tilde{b}_i\in[\tilde{s}_i,1]$, $\tilde{b}_i\ge\tilde{s}_i$. And it represent the effective sets.

The figure 4 show the three-dimensional Graph of ELV.

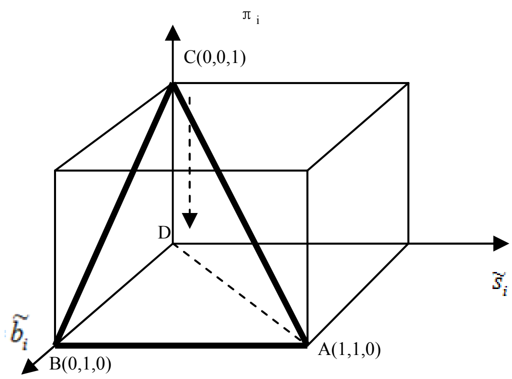

Figure4. Three-dimensional Graph of ELV

The effective sets are in areas beyond △ABC in the figure 11. The room beyond the tetrahedron ABCD in the three dimensional space satisfies the $\tilde{s}_i\in[0,1]$, $\tilde{b}_i\in[\tilde{s}_i,1]$, $\tilde{b}_i\ge\tilde{s}_i$. There is a formal contradiction that $\pi_i\ne1-\tilde{b}_i$ within the room ABCD outside the △ABC area. It has something with the algebraic system imperfection of fuzzy set[8][9].

The study explains the point D with voting model. The expert D (0, 0, 0) believe the object is very bad. And he uses the words *Very Unclear* to modify his evaluation result. However, the hesitancy degree π_i of expert D is equal to 0. It means he has great confidence on his evaluation result. There are two possible explanations.

One explanation is the expert D has very abundant of background knowledge. He finds the object has much uncertainty ($\tilde{b}_i=0$). He gives his evaluation result depends on his experience. And he is the authority in this field.

Another explanation is the expert make the judgment depends on his intuition. It is bounded rationality judgment.

Set $ELV_i=(b_i,s_i)$, and the ELV can be transformed to vague sets. The scale of five level grades ELV is given in the table 2.

TABEL 2
SCALE OF FIVE LEVEL GRADES ELV

s_i \ b_i	VU	U	M	O	VO
VB	$\tilde{b}_i = \tilde{s}_i$	$\tilde{b}_i > \tilde{s}_i$	$\tilde{b}_i > \tilde{s}_i$	$\tilde{b}_i > \tilde{s}_i$	$\tilde{b}_i > \tilde{s}_i$
B	$\tilde{b}_i < \tilde{s}_i$	$\tilde{b}_i = \tilde{s}_i$	$\tilde{b}_i > \tilde{s}_i$	$\tilde{b}_i > \tilde{s}_i$	$\tilde{b}_i > \tilde{s}_i$
M	$\tilde{b}_i < \tilde{s}_i$	$\tilde{b}_i < \tilde{s}_i$	$\tilde{b}_i = \tilde{s}_i$	$\tilde{b}_i > \tilde{s}_i$	$\tilde{b}_i > \tilde{s}_i$
G	$\tilde{b}_i < \tilde{s}_i$	$\tilde{b}_i < \tilde{s}_i$	$\tilde{b}_i < \tilde{s}_i$	$\tilde{b}_i = \tilde{s}_i$	$\tilde{b}_i > \tilde{s}_i$
VG	$\tilde{b}_i < \tilde{s}_i$	$\tilde{b}_i < \tilde{s}_i$	$\tilde{b}_i < \tilde{s}_i$	$\tilde{b}_i < \tilde{s}_i$	$\tilde{b}_i = \tilde{s}_i$

In the table2, the green areas are effective sets. And there is in grey field. While the evaluation result falls in the grey area, the result is at a lower level in reliability. It suggests that the valuator give the unsure evaluation result.

D. Summary

This paper briefly reviewed the concept of linguistic variable and its fuzzy set to investigate advantages of fuzzy set. Then the study analyzes the form of language structure. The normative linguistic terms set is given by the selected modified words. Then the modified linguistic terms set and traditional linguistic variable are integrated to build extended linguistic variable. The extended linguistic variable can be transformed to the grey fuzzy set or vague set. The modified linguistic variable may be transformed to false membership degree or grey number. The extended linguistic can be transformed to vague set. Then extended linguistic variable can be divided into two parts according to the reliability of the result. The area with high reliability is defined as effective set of the extended linguistic variable. The study discuss the effective sets are in three dimensional room. And the paper gives the theoretical analysis with voting model. The explanation is consistent with what happens in the real world.

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