Research on Translation of Index π-Net based on Index π-Calculus

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***Abstract*— In terms of the definitions and features of index π-calculus, a new kind of index π-net is proposed in this paper, which can reflect the system actions of index π-calculus system. This article presents definitions of Index π-calculus places, transitions and arcs. Meanwhile, the conditions to activate a transition and the effects on the tokens of its successor place are given as well. We try not to disturb the nature of petri net while defining index π-nets and expound the structural congruence relationship between them which verifies the semantic correctness of index π-nets. Finally, a simplified model of index π-nets is built, and a complete procedure about how the internal channel can be exposed to the external system is offered according to the model.**

***Index Terms*—Index π-Calculus, Bismilarity, Index π-Net, Structural congruence, place**

I. Introduction

Pi calculus has been proved to be more capable in modeling interactions and mobility [1]. This article mainly aims to build the model of Index π-net based on the Index π-calculus places and the related transitions.

As a structure drawn more close to Colored Petri Net [2], π-net which is a modular net[3] reforms the places, transitions and the arcs of the Petri Nets [10] through a set of mappings which maintains the nature of Petri Nets in maximum. The names of free channels can be separated from the restricted ones by injecting the specific character of π-calculus into the mappings. In order to keep the equivalent of the transformation, the mapping functions are built as bijective functions, so that it facilitates the state comparison of these two systems during the certification of bisimulation.

Since there are no new name types added in index π-calculus, channel names are still divided into the restricted ones [4,5] and free ones, which makes no new types of places increased when the index π-calculus is mapped to the π-nets, and that's the primary reason for the mapping mentioned above in this article. Relative to the classical π-calculus, index π-calculus has its own characteristics, the main one of them is the actions of the label places: the label places can expose a once restricted name outward as a free system name [6], i.e., transmitting the name from internal to external through a special channel.

In this paper, the definition of the mapping will be proposed where index π-calculus is mapped to the π-nets which temporarily called index π-nets here, and the definitions of structure, the related places, arcs and transitions of index π-nets are given as well. All the notions above are obtained from the corresponding definitions of petri nets under the mapping functions.

The label places and their relevant actions play an important role in the index π-calculus. At the end of the paper a model of the relevant actions of label places in index π-nets is addressed, and the bisimilarity certification will be given to prove the validity of the model.

II. Model of Π-nets based on Index Π-calculus

A. Definitions of Place in Index π-nets

Being similar to the definition of π-nets, the index π-net is defined as a quaternion, as in:

$$N = \{S_\pi, T_\pi, F_\pi, \zeta_\pi\} . \tag{1}$$

The mapping function ζ_π is needed to operate on S_π , T_π and F_π compared with the quaternion definition of Petri net. Semantic properties of index π-nets are added to the Colored Petri Net by mappings [7].

Here the definition of place in index π-nets is given:

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We use S_π represents the label places of index π-nets, for $\forall s \in S_\pi$, there is a mapping $\zeta(s) = \{\lambda_s, \alpha_s, \tau_s\}$, where s stands for names of places. Each place is a ternary, where:

(1) λ_s calculates the number of the tokens existed in the places, this paper clarifies that only the same type of tokens can be hold in one place. The type of a token is determined by τ_s, which makes every place has its own τ_s.

(2) α_s is an identifier. In index π-calculus there is a need to make specific channel names exposed external, and that differs in petri nets which treat the same kind of tokens indiscriminately. In index π-nets, a once restricted channel will be made exposed. The system is about to let the token which is the representation of a channel name exposure to external when the token is identified. However, not all the restricted channels can be exposed. For instance, some exposure cannot be done in order to ensure the integrity and the black-box characteristics of the system. In addition, the α operation, which is a name replacing process, is not available since a restricted channel name is exposed, then the requirement is proposed that the external system need to recognize the precise name of the exposed channel instead of its type. And α_s is defined to identify the channel names to be exposed.

(3) τ_s is a measure to distinguish the type of tokens standing for different channel names in places. Here's the definition of τ_s:

$$\tau_s = \{fn, bn\}. \tag{2}$$

The tokens of channel names hold in current place are free when $\tau_s = fn$ and are restricted when $\tau_s = bn$.

B. Definition of Arcs in Index π-nets

Arcs in petri nets are divided into two categories: one is from places to transitions, and the other is on the contrary [8]. Generally speaking, definition distinction is not made on these two kinds of arcs.

Arcs in Index π-nets not only have the efficacy and the feature the same with that in the petri nets but also have their unique properties.

Therefore it's necessary to treat these two kinds of arcs differently. An identifier is needed when $F_s = S \times T$, so that when $\tau_s = bn$, the identifier can mark the channel name which is required to expose itself and guarantee that only the restricted channel which has the need of exposure can be selected without affecting other restricted ones. On the other hand, the identifier on the arc represents the name of a known channel when $\tau_s = fn$, which is used to transmit a restricted chosen channel outward. We can't conduct the α operation so as to choose the proper restricted channels when $F_s = S \times T$.

At the condition of $F_s = T \times S$, the arc injects new tokens stands for channel names to the connected place, hence it's important to identify the chosen token. Moreover, actions of the places and the transitions of index π-nets are completed in one step, for this reason the system must fulfill the α operation on the restricted names when $F_s = T \times S$. As soon as the operation is accomplished, the exposed channel appears under this name and cannot be changed again.

Based on views above, we are about to give the definition of arcs in index π-nets as follows:

$$F_\pi = (S_\pi \times T_\pi) \cup (T_\pi \times S_\pi). \tag{3}$$

Where S_π is on behalf of the set of place names and T_π represents the set of transition names. $\zeta(F_\pi)$ is a trail and $\zeta(F_\pi) = \{\lambda, \sigma, \tau\}$. λ is a positive integer value which counts the number of changed tokens generated through a transition; and σ is a set in which the amount of elements equals the value of λ, σ stands for the name set of channels that already dealt with by the arcs during the transition; τ represents the α operation on the arcs, if $F_s = S \times T$, there is $\tau = \{\varphi\}$, it means that there's no α operation when there is only the null element in the set, if $F_s = T \times S$ there is $\tau \subseteq \sigma$, which means τ is a subset of σ and elements in τ stand for the sets where the α operation is required.

For example:

$$\zeta(F_{(t,s)}) = \{3, \{name_i, name_j, name_k\}, \{name_j, name_k\}\} \tag{4}$$

Equation (4) shows that F is an arc point from t to s which can deal with 3 tokens at one time, we assume them as $name_i$, $name_j$ and $name_k$. The α operation is carried on $name_j$ and $name_k$ among them, $name_i$ won't be changed after its exposure.

C. Definitions of Transition in Index π-nets

Transition T_π describes the event that has been triggered in the system. In this article, T_π mainly focus on the output of a channel and the α operation. These two functions are the key factors to define T_π.

Definition of T_π is defined as follows: transition $\zeta(T) = \{fun, newnm\}$, where fun is a function identifier like $fun = \overline{a}y$, showing that the transition outputs a y trough channel a, or proceed the α operation during the output process. For instance, $fun = \overline{a}y(u/y)$ means replace the initiative name y with u while the channel is outputting a name. It's notable that u and y here are not real internal names, both of them have the characteristic of former parameters only to notice the system of the α operation. And the specific name of operands is given in $newnm$. $newnm$ represents a set whose size depends on the set σ in the equation $\zeta(F_\pi) = \{\lambda, \sigma, \tau\}$ when $F_s = T \times S$. The elements in both $newnm$ and σ must be the same both in quantity and order.

The channel $name_j$ and $name_k$ requiring the α operation are what we get when

$$\zeta(F_{(t,s)}) = \{3, \{name_i, name_j, name_k\}, \{name_j, name_k\}\} \quad (5)$$

therefore the order of elements in the set *newnm* in transition is settled, here:

$$\zeta(T) = \{\bar{a}y(u/y), \{name'_j, name'_k\}\}\ . \quad (6)$$

An action of the outputting and α operation will occur if the transition is triggered at relevant state of places. Expressions of this action in the π-calculus are:

$$\bar{a}name_j(name'_j / name_j) \quad and$$

$$\bar{a}name_k(name'_k / name_k) \quad (7)$$

III. Transition Regulations for Index Π-nets

A. State Sets

Let M_π the state set of index π-nets whose elements amount equals the number of places, and each member of M_π is a set. M_π reflects the current state of index π-nets. In particular, let $M^0{}_\pi$ be the initiative state. The specific form of M_π is as follows:

$$M_\pi = \{\alpha_{s_1}, ..., \alpha_{s_n}\} \quad (8)$$

Where $\alpha_{s_1}, ..., \alpha_{s_n}$ corresponds with the second element of $\zeta(s_1), ..., \zeta(s_n)$.

Along with the occurrence of transitions, the system state are changed as well, that is to say, the state changes to a new one as the transition sequence happens. And the procedure will repeat if a new transition can be inspired by the new state, otherwise the state will be the final state of the system.

B. Conditions to Inspire a Transition

This section is an elaboration of the transition regulations, including inspiration conditions of the transition and the alteration of tokens in its predecessor and successor places after a transition.

The inspiration conditions of a transition are given:

Let the current state be M^0_π, and $M^0_\pi = \{\alpha_{s_1}, ..., \alpha_{s_n}\}$, ${}^\bullet t$ is the predecessor place of *t*, then there is ${}^\bullet t \subseteq \{s_1, ..., s_n\}$. Similarly, when $t^\bullet$ is the successor place of *t*, we have $t^\bullet \subseteq \{s_1, ..., s_n\}$. To be convenient for explanation, assume that ${}^\bullet t = \{s_i, ..., s_m\}$ and the member amount of ${}^\bullet t$ is no more than n. Now let the amount values k, then there are *k* arcs and each arc has its own metric. Transition is inspired since certain interrelation between the metric and the functions is satisfied described in the place. Considering that the inspiration is only connecting with the predecessor place, we'll leave the change of successor places discussed next section.

Let $F_{(s_i,t)}, ..., F_{(s_m,t)}$ be the directed arc between $s_i, ..., s_m$ and transition t. Try not confuse the reader, we use $F_{(s_i,t)} \to \sigma$ to replace the second member σ in $\zeta(F_{(s_i,t)})$. Analogously, $s_i \to \alpha$ is the replacement of the second element α in $\zeta(s_i)$.

In the following two sets Λ and Ω are given:

$$\Lambda = (s_i \to \alpha) \oplus ... \oplus (s_m \to \alpha)\ ;$$
$$\Omega = (F_{(s_i,t)} \to \sigma) \oplus ... \oplus (F_{(s_n,t)} \to \sigma) \quad (9)$$

Here Λ expresses the minimum set which can be dealt with under the current state, Ω is the representation of token requirements related to current t on all arcs. So the relationship is offered on the basis of index π-nets as follows:

We claim that the occurrence of t is authorized by current system state M_π when $\Omega \subseteq \Lambda$. Meanwhile, M_π changes to M'_π. Although the conditions to inspire a transition differ from that of petri nets in essence, this paper still takes expressions in petri nets, which is written as $M_\pi[t > M'_\pi$

C. Effects on the Successor Places

First of all, we consider what happen to $t^\bullet$ after transition t took place.

Let $t^\bullet = \{s_j, ..., s_w\}$, the homologous directed arc in consistent with $t^\bullet$ pointing from the transition to the successor plate is $F_{(t,s_j)}, ..., F_{(t,s_w)}$. When a transition *t* is inspired, the successor place accepts the new token indicated by the metric on the arc. Some of the tokens are inherited from the predecessor place of the *t*, some have their names changed during the process under the α operation. What makes these two kinds of tokens different are the metrics attach to the arc and the *t*.

Then we take the transformation into account of tokens without changing their names after the transition. According to the definitions of π-nets, the token added strictly to $s_j \to \alpha$ of s_j is in:

$$(F_{(t,s_j)} \to \sigma) - (F_{(s_j,t)} \to \tau). \quad (10)$$

With this correspond, $|(F_{(t,s_j)} \to \sigma) - (F_{(s_j,t)} \to \tau)|$ is the variation of $s_j \to \lambda$.

The following is what happened to the transitions whose name changed by α operation. Since the operation is just an action taking place on transitions and arcs, the changed token is still saved in $s_j \to \alpha$ when it is going to inject to a new token. The new name is obtained from $t \to newnm$ during the operation which operates as $F_{(t,s_j)} \to \tau$, here we use $(t \to newnm)/(F_{(t,s_j)} \to \tau)$ to represent the operation process.

Then, we get:

$$(s'_j - s_j) \to \alpha = ((F_{(t,s_j)} \to \sigma) - (F_{(s_j,t)} \to \tau))$$
$$+ ((t \to newnm) / (F_{(t,s_j)} \to \tau)) \quad (11)$$

$$(s'_j - s_j) \to \lambda = |(F_{(t,s_j)} \to \sigma)| \quad (12)$$

Still this paper refers to the notation in petri nets which means the alternation of the successor place after the transition is expressed by $s'_j - s_j$. And the transition regulation equations won't be mentioned here because they can be concluded if we add the impact of successor places to the transition regulation.

D. Index π-nets Modeling

As shown in Fig. 1, the exposing procedure is given, where the tokens of restricted channel are stored in place s_i and place s_j contains tokens of the free channel. Some channels represented by tokens in set s_j are used to transport the restricted channels requiring to be exposed in s_i.

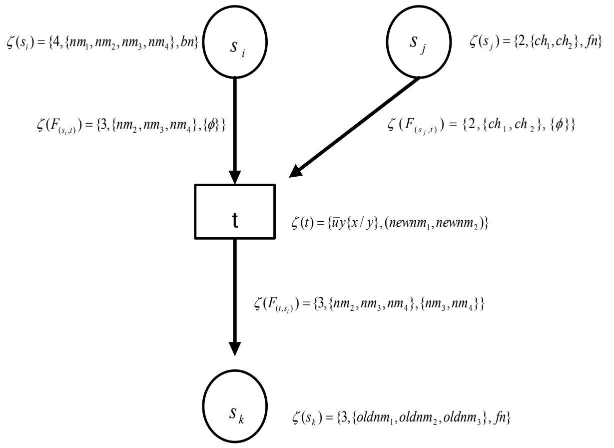

Figure 1. An internal restricted channel exposed to external system in Index π-nets

$$\zeta(F_{(s_i,t)}) = \{3, \{nm_2, nm_3, nm_4\}, \{\varphi\}\} \tag{13}$$

Equation (13) shows that the arc checks whether there exists the token which is needed to inspire a transition in s_i. To make the procedure clear, let $|{}^{\bullet}t| = 1$, then there is $((F_{(s_i,t)} \to \sigma) \oplus ... \oplus (F_{(s_n,t)} \to \sigma)) \subseteq ((s_i \to \alpha) \oplus ... \oplus (s_m \to \alpha))$, and the current M_π authorizes the occurrence of t which has the requirement of an α operation $\zeta(t) = \{\overline{u}y, (newnm_1, newnm_2)\}$. We can obtain the real name of a channel from s_j, that is in：

$$\overline{chan_1} y\{newnm_1 / y\}, \overline{chan_2} y\{newnm_2 / y\}. \tag{14}$$

The replacement is carrying on when passing through the arc $\zeta(F_{(t,s_i)}) = \{3, \{nm_2, nm_3, nm_4\}, \{nm_3, nm_4\}\}$.

Conclusions are as follows by taking all these factors into account above:

$$\overline{chan_1} nm_3\{newnm_1 / nm_3\}, \overline{chan_2} nm_4\{newnm_2 / nm_4\}. \tag{15}$$

After the transition, we have (16):

$$\zeta(s_k') = \{6, \{oldnm_1, oldnm_2, oldnm_3, nm_2, newnm_1 / nm_3, newnm_2 / nm_4\}, fn\} \tag{16}$$

Equation (16) puts another three tokens in s_k and nm_2 got directly from s_i, the left two arguments are added after the α operation.

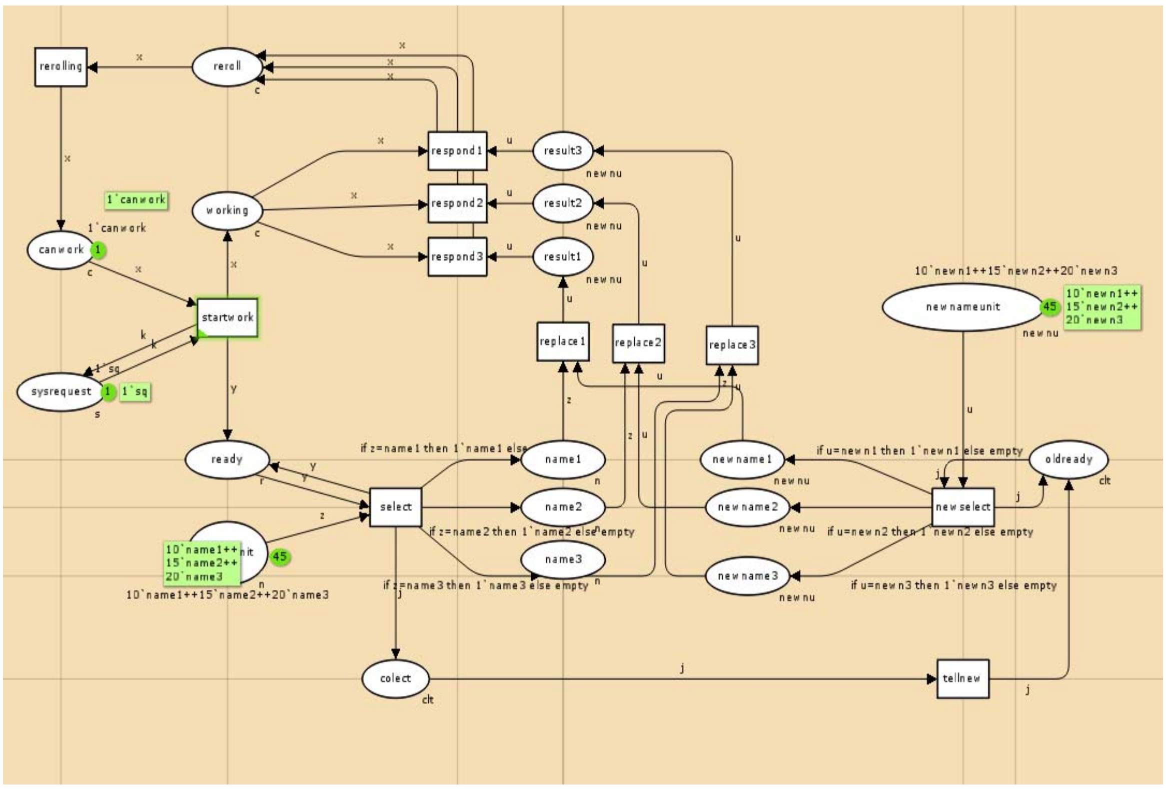

Figure 2. Modeling index π-nets by CPN tools.

E. Bisimulation of Index π-nets and Petri nets

Instead of changing the structure of petri net, this paper defines index π-nets based on making some mappings to the elements of the classical petri nets, so these two kinds of nets are structural congruence.

For any given M stands for state in petri nets and M_π for state in index π-nets, $M \sim M_\pi$. If there is a t which makes $M[t > M'$, then there must be $M_\pi[t_\pi > M'_\pi$ and $M' \sim M'_\pi$, hence the relationship between petri nets and index π-nets is bisimulation.

IV. Testify Index Π-nets by Utilizing CPN Tools

A. Index π-nets Modeling by Utilizing CPN Tools

Colored Petri Nets (CP-nets or CPNs) is a language for the modeling and analysis of distributed systems and others systems in which concurrency, communication, resource sharing and other kinds of synchronization plays a crucial role [9]. CPN tools, which are powerful tools to modeling the Colored petri nets, are adopted in this article to modeling index π-nets. There are two reasons for adopting, first of all, the structural congruence feature of these two nets facilitate the action simulation of index π-nets approximately; secondly, the system state can be intuitive to see while running, it is advantageous to index π-nets to observe the state directly at every moment.

Fig. 2 shows that there are three interfaces when utilizing the CPN tools.

Fig. 3 displays that the *nameunit* of the model, which is a set of internal restricted channel that required to be exposed external, containing 3 kinds of restricted channels: *name1,name2* and *name3*, of whom the quantities are *10,15* and *20*.

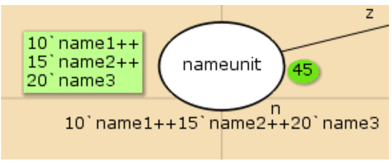

Figure 3. nameunit

Fig. 4 shows the model of *sysrequest* in CPN tools.

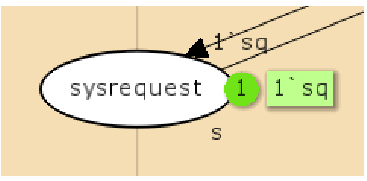

Figure 4. Sysrequest

Fig. 5 shows the *newnameunit* which is a set of new names, that is to say names replacing the initiative names of the exposed channels, where the quantities of *newn1, newn2* and *newn3* are the same with that of *name1, name2* and *name3.*

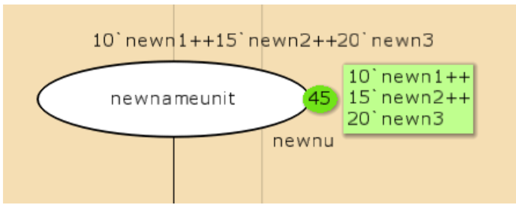

Figure 5. Sysrequest

Besides, it is necessary to make an explanation to the newly built colored set and the variables.

```
▼colset c = unit with canwork;
▼colset s = unit with sq;
▼colset r= unit with ready;
▼colset n = with name1 | name2 |name3;
▼colset clt = unit with colect;
▼colset newnu= with newn1 | newn2 | newn3;
▼var x: c;
▼var y: r;
▼var z: n;
▼var u: newnu;
▼var j: clt;
▼var k: s;
```

Figure 6. Declarations of variables in CPN tools.

B. Utilizing CPN Tools to Verify Index π-nets

Now, it is possible to verify the model of index π-nets by utilizing CPN tools. The first transition *startwork* is triggered.

From Fig. 7 we can see that after triggering transition *startwork*, new tokens are generated by place *ready*, which triggers the transition *select*.

At this moment, *name3* is being exposed. In order to get *newname3*, the transition *newselect* is triggered constantly. And after 5 times, there are new tokens generated in *newname3*. Results are shown in Fig. 8.

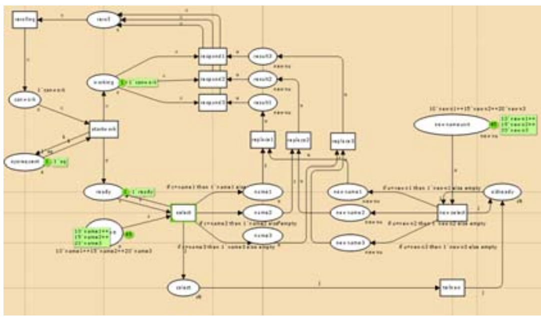

Figure 7. Transition *sartwork* is triggered

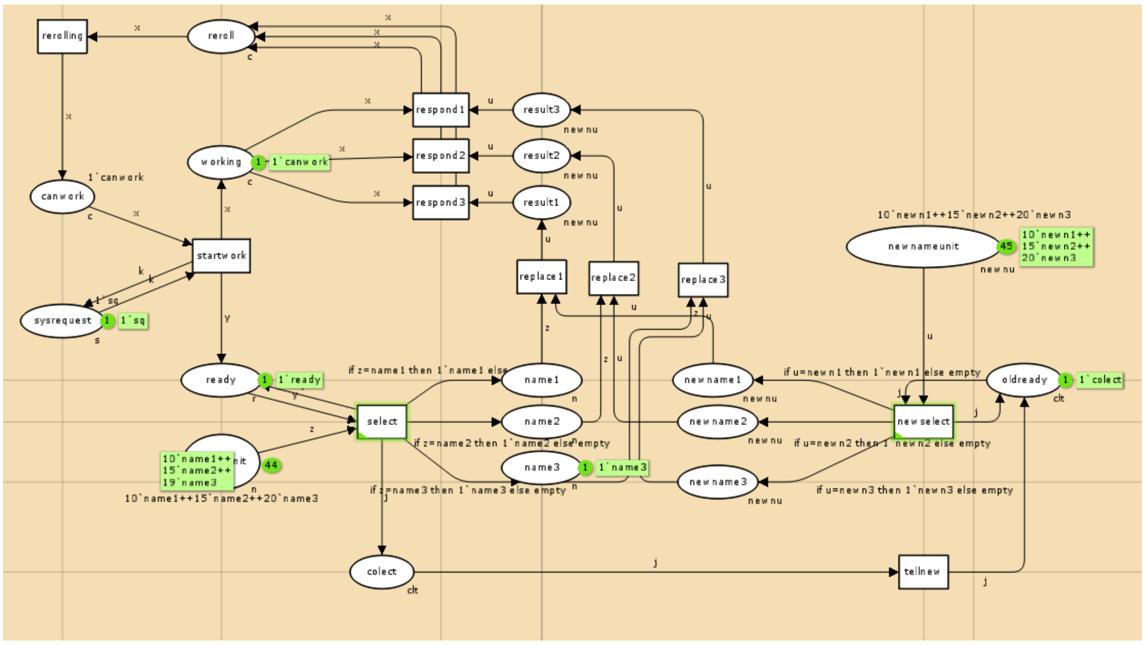

Figure 8. Transition select is triggered.

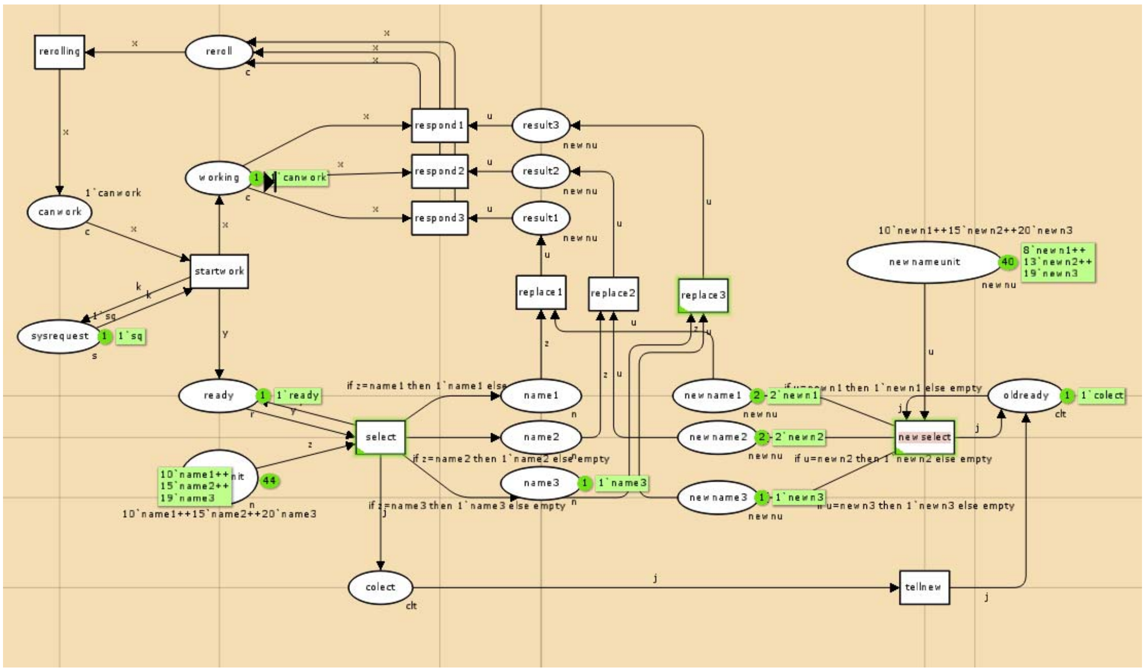

Figure 9. Transition newselect is triggered for 5 times.

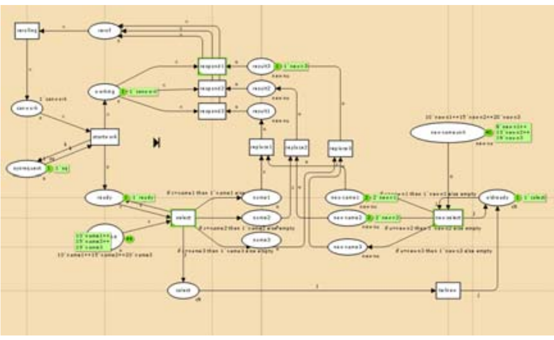

Figure 10. Transition *replace* is triggered.

Then we activate replace3 to let the system proceed with the α operation. Results are shown in figure 4.9.

In the figure above we can see that there is a token named *newname3* generated in *result3* which means the operation is done. Then the system rolls back to its initiative state.

Figure 4.10 reflects that there are new tokens appearing in place *canwork*, which suggests that the system rolls back to its initiative state to wait for new system demands.

In this section, we use CPN tools to simulate index π-nets and observe the working process by activating transition sequence. It is obviously that index π-nets accomplish to replace the name of an internal channel while exposing it to external system and it is quite consistent with the characteristic we mentioned in section 3. The structural congruence between the Colored petri nets and index π-nets is testified by simulating index π-nets based on petri nets.

V. Conclusion and Future Research

In this paper, index π-net is proposed by joining the semantic of index π-calculus into the classical petri nets. And the definitions of places, arcs and places are given. The conditions to activate a transition and the effects on its successor place are given as well. The formal models of index π-nets are introduced and its feature of bisimulation is expounded. In the end, we build and verify the model of index π-nets by utilizing CPN tools.

The index π-nets we built in this article is an expansion of the classical petri nets which discussed how to add new restrains to let the classical petri nets have more semantic features.

The future work of index π-nets will be focused on two aspects:

a. The combination of label places and other functional modules;

b. The bisimulation proof of the combination among basic modules.

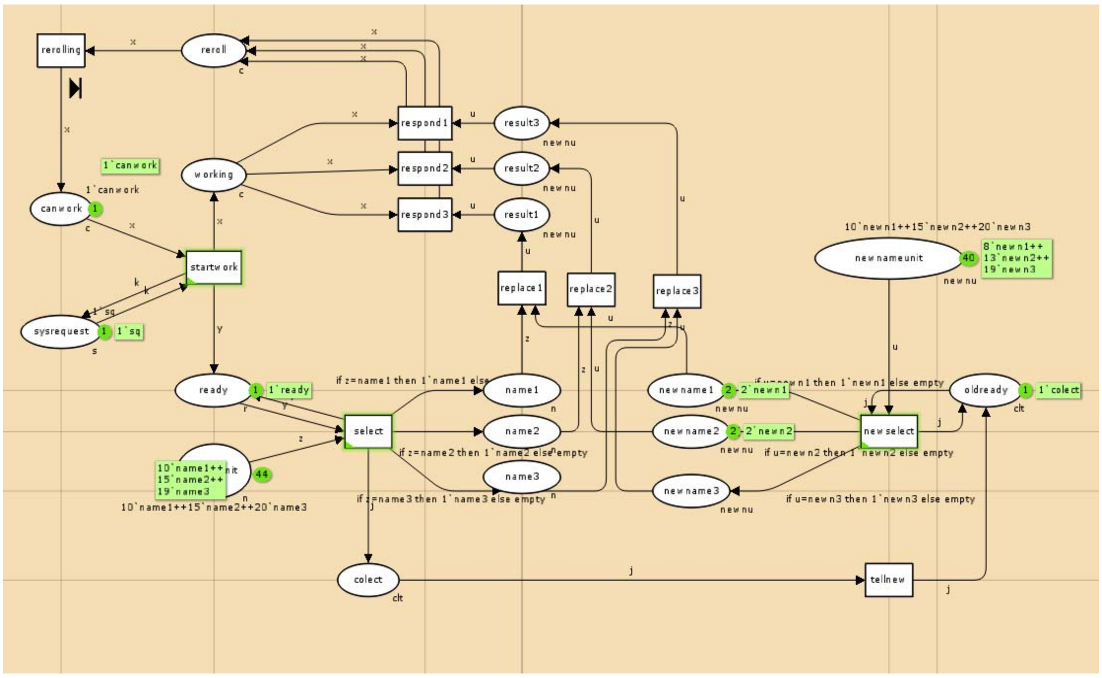

Figure 11. Trigger the transition reroll to rollback to initiative state.

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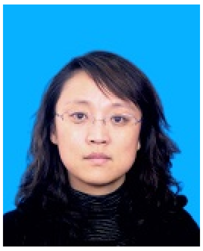

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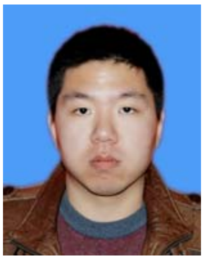

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