The Two-parameter Analyzing Model and Statistical Distribution Law of the Random Signal in Optical Sensor

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***Abstract*—In this paper, on the basis of giving a reference amplitude value, the two-parameter analyzing model of the random signal is established by introducing two basic conceptions---the amplitude and the width. Based on which, the statistical distribution law of the random signal group's characteristic parameters is studied. The experimental results show that the amplitude and the width counting statistical distributions of the random pulse signal group and subsets are obviously asymmetric. Besides, the distribution functions of the whole signal group and its characteristic subsets match well with the lognormal distribution while the natural number as the independent variable. That is to say, the statistical self-similarity character of the random signals' internal structure is definitely shown by applying the two-parameter analyzing model to deal with them.**

***Index Terms*—random signal; statistical self-similarity; two-parameter analyzing model; lognormal distribution**

I. Introduction

The particle counter, based on the light scattering, is the important testing equipment of modern ultra-clean environment with high detection efficiency and sensitivity. For a long time, the study of laser airborne particle counter mostly focuses on the aspect of Mie scattering theory of sphericity particle[1,2], design of photo-electricity sensor[3,4], etc.

The high-precision experimental results and the mathematical model show that the particle size distribution of a finely divided system is found to meet the lognormal format [5]. This is an important aspect of nanoparticle science[5], soil science[6], environmental science[7], biology and medical science[8]. As known, in the system of the laser airborne particle counter, the voltage signals generated by the aerosol are discrete and random pulse signals, while the system determines the particle size according to the value of voltage amplitude. So, we should investigate this kind of signals.

The pulse signal generated by the aerosol has two basic parameters, including the width τ and the amplitude $\Delta V = V - V_c$, where V_c is the reference voltage. Taking the observation time series as the base, the amplitude and the width of the random signal are random. Besides, a large number of samples' statistical analysis indicates that the randomness appears the steady law which is the reflection of the particles' size structure and the measurement system's character. And, it is the foundation of the inverse algorithm for particles' size parameter. In particular, it is the important manifestation of the basic laws of the information transmission characteristics of the random signal. However, the common signal analyzing methods, such as Fourier frequency analyzing method, can not give the amplitude and the width counting statistical distributions of the random signal. So, we need to establish a new analyzing model of the random signal, which can show the general character of the random signals and give their statistical distribution function.

In this study, the independent characteristic parameters---the amplitude and the width can be introduced as two basic conceptions. Then, giving a reference amplitude value, a two-parameter analyzing model of the random signal is established. In further, the statistical distribution law of the characteristic parameters of the random signal group is also studied. As a result, this study will provide some reference for improving the resolution of the particle counter and expanding its measurable range.

II. The Concepts Of The Random Signal And The Random Pulse

It is well known that the output signal of photo-electricity in airborne particle counter corresponds to a time series $f(t)$. In this paper, a sub-sample of the output signal can be seen in Fig.1. For a certain measurement cycle T, when the time precision Δt satisfies $\Delta t \ll T$, $f(t)$ can be considered as the continuous waveform. If $f(t)$ doesn't overlap in any cycle, it can be named as the random signal. However, the signal in actually experimental measurement could not be continuous, which can be attributed to the

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experimental results have a certain time accuracy Δt. In other words, $f(t)$ must be written as $f(k\cdot\Delta t)$ which is a natural number sequence. Therefore, the measurement result $f(t)$ should be regarded as the combinations of the histogram sequences $\tilde{f}(k)$.

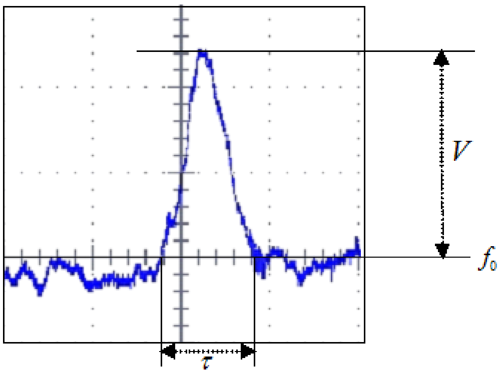


Figure.1 Sketch map of the pulse structure

Taking a reference value f_0 to process the experimental results, a subset $V(t)=f(t)-f_0>0$ can be got, which means a new structural character is formed. Obviously, the number of the subset's element will be different in the different f_0. Based on this, the elements, which have continuous time sequence in subset $V(t)$, compose a pulse structure whose amplitude and width are defined as V and τ respectively, as shown in Fig.1. It is worthy to note that the width parameter τ is the integer times of the time resolution Δt, and the minimum width τ_m is $2\Delta t$. Meanwhile, the amplitude parameter V is the integer times of the amplitude resolution ΔV, and the minimum amplitude V_m is ΔV.

On the basis of the above analysis, a new characteristic subset $A(V,\tau)$ composed by random pulse signal sequences can be used to describe the random signal group $f(t)$ which is dealt with f_0. Owing to the universality of $f(t)$, this signal processing method has universal significance, where Δt and ΔV represent the characteristics of the measurement devices, while V and τ are the characteristic parameters of the pulse signal. Based on these, the subset $A(V,\tau)$ can be statistical analyzed, as a result of which a new signal analyzing method can be established as follows.

III. The Foundation Of The Two-Parameter Analyzing Model For The Random Signal

In fact, it is a kind of the mathematically descriptive transformation from the group $\{\tilde{f}(k)\}$ to the subset $\{A(V,\tau)\}$. Once the parameter group f_0, $\Delta\tau$ and ΔV are given, the physical parameters of the signal subset $\{A(V,\tau)\}$ can be described by the natural numbers. That is, we can use a set of natural numbers (l_τ, l_V) to establish the statistical mapping form of the signal subset $\{A(V,\tau)\}$:

$$\begin{aligned} l_\tau &= \left\lceil \frac{\tau_i}{\Delta\tau} - 1 \right\rceil \quad l_\tau \in \left(1, 2, \cdots, L_\tau = \frac{\tau_M}{\Delta\tau} - 1\right) \\ l_V &= \left\lceil \frac{V_i}{\Delta V} \right\rceil \qquad l_V \in \left(1, 2, \cdots, L_V = \frac{V_M}{\Delta V}\right) \end{aligned} \tag{1}$$

Then, the geometric format of elements (l_τ, l_V) of the pulse signal A_i is the two-dimensional rectangular map T_i, as shown in Fig. 2. That is, Eq. (1) is equivalent to take the two-dimensional atlas $\{T_i\}$ as a special mapping form of $\{f(t)\}$, as a result of which it is called the two-parameter model of the random signal $f(t)$.

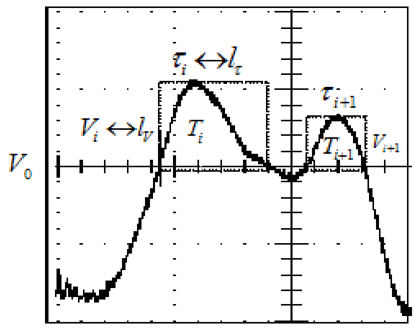


Figure.2 Sketch map of the random signal's two-parameter model

According to the size sequence of the observation character values l_τ (or l_V), the characteristic subset and the statistically descriptive form of $\{A(V,\tau)\}$ can be established. Let N denote the total number of $\{A(V,\tau)\}$, while $\Delta N(l_\tau)$ and $\Delta N(l_V)$ denote the pulse numbers of $\{A(V,\tau|\tau\in l_\tau\Delta\tau)\}$ and $\{A(V|V\in l_V\Delta V,\tau)\}$ respectively, the pulse counting distribution functions of characteristic subset can be expressed as:

$$\begin{aligned} p(l_\tau) &\equiv \frac{\Delta N(l_\tau)}{N} = \frac{N_{l_\tau}}{N} > 0 \\ q(l_V) &\equiv \frac{\Delta N(l_V)}{N} = \frac{N_{l_V}}{N} > 0 \end{aligned} \tag{2}$$

where distribution functions $p(l_\tau)$ and $q(l_V)$ are called for the one-dimensional characteristic counting distribution functions. Considering the knowledge of the statistics, the distribution functions $p\left(l_\tau\right)$ and $q\left(l_V\right)$ will tend to steady values when the sample's number of subsets satisfies $1 \ll \Delta N(l_\tau), \Delta N(l_V) \ll N$ in actual measurement. In principle, the uniform channel mode can be established by adjusting the measurement accuracy $\Delta\tau$ and ΔV in their definition domains for an assured random signal group $\left\{ f\left(t\right)\right\}$. And then, the total channel number of the different parameter becomes the same, which means $L_\tau = L_V$. Commonly, when the two parameters both choose the uniform channel mode, it has $p\left(l\right) \neq q\left(l\right)$.

Because the characteristic subset $\left\{A\left(V \middle| V \in l_V \Delta V, \tau\right)\right\}$ has different widths, it should have more refined parameter distribution structure. That is, for a pulse signal group with certain amplitude value V , the dependence of the counting distribution on the width τ can be given, which implies that the counting distribution functions of the two-dimensional basic characteristic parameters for the pulse subset $\left\{A\left(V, \tau\right)\right\}$ could be expressed as:

$$p_{l_V}\left(l_\tau\right) \equiv \frac{\Delta N_{l_V}(l_\tau)}{N_{l_V}} > 0 \tag{3}$$

$$q_{l_\tau}\left(l_V\right) \equiv \frac{\Delta N_{l_\tau}(l_V)}{N_{l_\tau}} > 0 \tag{4}$$

where $\Delta N_{l_\tau}(l_V)$ denotes the number of the pulses with the amplitude $l_V \Delta V$ when the width measured value is $l_\tau \Delta\tau$, while $\Delta N_{l_V}(l_\tau)$ denotes the number of the pulses with the width $l_\tau \Delta\tau$ when the amplitude measured value is $l_V \Delta V$. Obviously, for a given $\left\{A\left(V, \tau\right)\right\}$, there is $\Delta N_{l_\tau}(l_V) = \Delta N_{l_V}(l_\tau)$.

Obviously, up to now, the two-parameter model of the random pulse signal is established, and the statistical counting distribution expressions are also given.

IV. The Experimental Measurement On The Characteristic Parameters' Statistical Distribution Of The Random Pulse Signal Group

In the following study, the experiment on the characteristic parameters' statistical distribution of the random pulse signal group will be proceeded. By using the laser airborne particle counting system, we will use two-parameter analyzing model to deal with the measurement results of the random signal generated by the aerosol firstly. And then, in order to study the applicability of the two-parameter analyzing model, the most common random signals generated by the measuring process will be measured.

A. Experimental System

The laser airborne particle counting system, whose sampling rate is 28.3L/min and is shown in Fig.3, is applied as the experimental measuring equipment. In this equipment, the output signals of optical sensor (photomultiplier tube used in experiment) are digitized by the high-speed data acquisition card PCI-9812. And then, they are inputted to the computer to implement the multi-channel counting statistics.

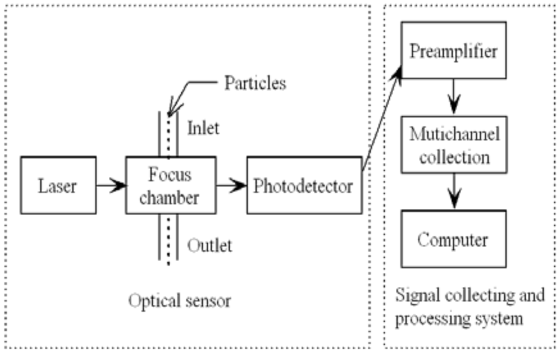


Figure. 3 Schematic diagram of the measuring system of the optical particle counter

The main technical parameters are as follow: the maximum signal sampling rate f_e is $20MHz$ (corresponding the time-precision $\Delta\tau$ is $0.05\mu s$), the average width of pulse signal $\overline{\tau}$ is $5\mu s$; the range of voltage is 5V, the division precision ΔV of 2048 counting channels is $5V/2048 \sim 2.44mV$.

B. The Measurement of the Statistical Distributions of the Aerosol

By using the above device, taking the serial width range $\tau_{l_\tau} \sim \tau_{l_\tau} + \Delta\tau$, the scattering pulse signal generated by the aerosol is measured. Then, the two-dimensional counting distributions $q_{l_\tau}\left(l_V\right)$ as amplitude characteristic parameter of the random pulse signal can be described by the discrete points in Fig.4.

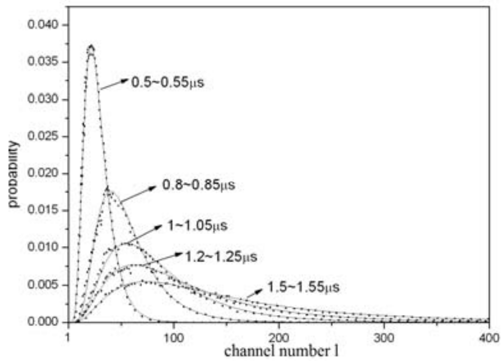


Fig.4 Scattering signal amplitude distributions under different signal width

As shown, the function $q_{l_\tau}\left(l_V\right)$ has center asymmetry obviously. Besides, the nonlinearly similar characteristic between the different function sequences is quite well. After the nonlinear scaling transformation, the scattering amplitude counting distributions of the different pulse width interval can be coincident, which can be seen in Fig. 5. At the same time, the transformation parameters are shown in Tab. 1. That is, the amplitude distribution functions corresponding to the different widths meet:

$$q_{l_{\tau_i}}(l_V) \square q_{l_{\tau_j}}\left[\left(b_{ij} l_V\right)^{\alpha_{ij}}\right] \quad (5)$$

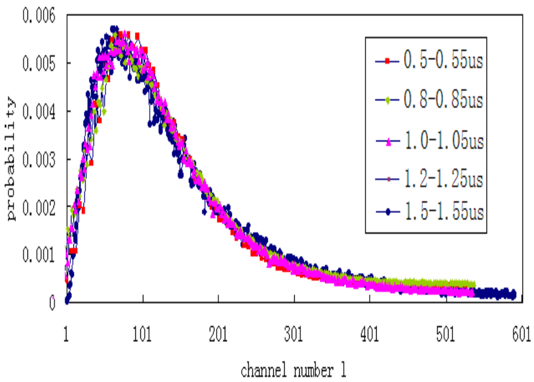


Fig.5 Nonlinearly similar characteristics between the different functions $q_{l_\tau}(l_V)$

TABLE.1
NONLINEAR TRANSFORMATION PARAMETERS OF THE DIFFERENT FUNCTIONS $q_{l_\tau}(l_V)$

signal width range	b	α
$0.5-0.55\mu s$	3.6	0.56
$0.8-0.85\mu s$	3.3	0.68
$1.0-1.05\mu s$	2.9	0.76
$1.2-1.25\mu s$	1.9	0.86
$1.5-1.55\mu s$	1.0	1.00

The common statistical function, which satisfies the nonlinear transformation relation in Eq. (5), is the lognormal distribution function:

$$p(x)=\frac{1}{\sqrt{2\pi}\sigma_{\ln x}x}e^{-\frac{(\ln x-\mu_{\ln x})^2}{2\sigma_{\ln x}^2}} \quad (6)$$

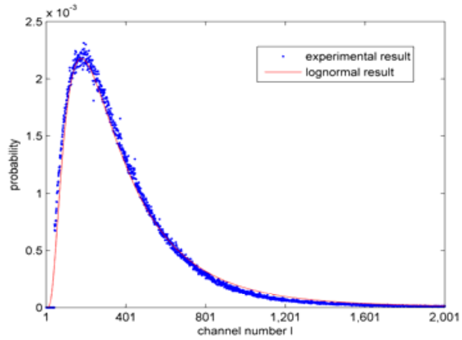
So, we use Eq. (6) as the characteristic parameter's counting distribution of the random pulse signals, but the definition domains of their independent variables are natural number. As expected, the fitting results are displayed by the sequence of solid lines in Fig.4 (its statistical parameters' values can be seen in Peng Gang et al. [9]). The continuous function, corresponding to the solid lines in Fig. 5, can be described by:

$$q_{l_\tau}(l_V)\equiv\frac{1}{\Omega\sigma_{\ln V}l_V}e^{-\frac{(\ln l_V-\mu_{\ln V})^2}{2\sigma_{\ln V}^2}} \quad (7)$$

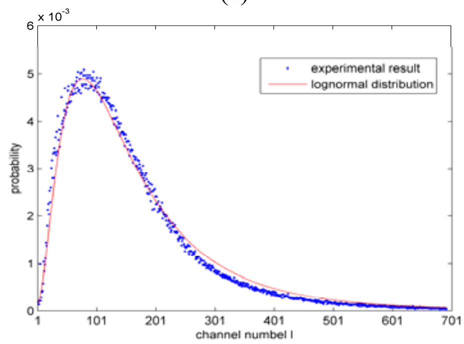
where $\mu_{\ln V}=\overline{\ln l_V}$ and $\sigma_{\ln V}=\sqrt{\overline{\left(\ln l_V-\overline{\ln l_V}\right)^2}}$ are the limits on the condition of $L_M\to\infty$, Ω is the normalization coefficient which relates to the specific value range of the signal group and the total channel number.

According to Fig. 4, the calculation results obtained by Eq.(7) match well with the experimental results.

Using the same device, the one-dimensional statistical distributions of the amplitude and the width of the pulse signal group generated by the aerosol are measured, and the results are displayed in Fig.6. The discrete points in Fig.6 represent the experimental measurement values, while the continuous curves represent the results calculated by the log-normal distribution functions with natural number as the independent variable, namely $q(l_V)$ and $p(l_\tau)$.



(a)



(b)

Fig.6 The amplitude and the width counting distributions of the pulse signal group (a) the amplitude (b) the width

According to the above researches, we use two-parameter analyzing model to deal with the measurement results of the random signal generated by the aerosol, the results show that the amplitude and the width counting distributions all match well with the log-normal distribution function while the natural number as the independent variable.

C. The Applicability of the Two-parameter Analyzing Model

In order to study the applicability of two-parameter analyzing model and the general character of the random signals, the most common random signals, generated by the measuring process, will be measured. Selecting a reference voltage f_0, the background signal of measuring device could be treated as random pulse signal. And then, the width counting distributions $p_{l_V}(l_\tau)$ of the random pulse signal are measured in the different amplitude range $V_{l_V} \square V_{l_V}+\Delta V$, and they are shown as the discrete points in Fig.7.

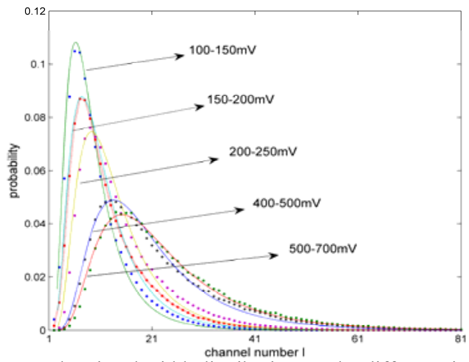


Figure.7 Pulse signal width distributions under different signal amplitude

It is obvious that the experimental curves $p_{l_V}(l_\tau)$ have the center asymmetry. Besides, they have good nonlinear similar characteristics between the different functions. The curves in Fig. 8 are also fitted by the lognormal distribution function with the natural numbers as the independent variables, while the specific parameters are given by Tab. 2.

TABLE.2

STATISTIC PARAMETER VALUES OF FITTING CURVES

Pulse amplitude range $/mV$	Fitting data	
	Mean of the channel number's natural $\mu_{\ln\tau}$	Standard deviation of the variable's natural $\sigma_{\ln\tau}$
100-150	2.00	0.595
150-200	2.21	0.595
200-250	2.42	0.555
400-500	2.84	0.555
500-700	2.96	0.555

Figs.4-7 show that, $q(l_V)$, $p(l_\tau)$, $q_{l_\tau}(l_V)$ and $p_{l_V}(l_\tau)$ all reveal the log-normal distribution with the natural number as independent variables, when using the two-parameter analyzing model to process the random signal. In other words, the experimental results prove that the distribution curves of the amplitude and width have the statistical self-similarity character between the different subsets and the signal group.

V.ANALYSIS AND DISCUSSION

In this study, a new two-parameter signal analyzing model, which is different from the traditional processing method, is used. Compared with the traditional Fourier frequency analyzing method, this model has the following features: 1) the continuous signal is separated to become many discrete signals through the given reference voltage value f_0; 2) the definition domain of the statistical function is a series of natural numbers, which correspond to measuring process directly; 3) this method maintains the basic parameters(V and τ) of the signal, so, it analyzes the signal itself directly.

The two-parameter model can extract the amplitude and the width distributions of the pulse signal simultaneously. In the current particle size measuring system, only the amplitude is used, while the information of the width is not applied. In fact, the value of the width is directly related to the mass of the particle, which implies that we can adopt the width to obtain the particle's mass information.

At the same time, it usually determines the particle size directly by the value of voltage amplitude in the traditional particle's size measurement method. However, due to the non-uniformity of the light intensity in photosensitive area and the irregular shape of the particles, the resolution of the measurement results by this method is often lower. According to the Ref.[10], if the characteristic parameters' counting distribution functions $q(l_V)$ and $p(l_\tau)$ of the random pulse signal group have log-normal distribution form, then the corresponding parameters in the definition domain meet the relation:

$$\frac{V}{v_0} = \left(\frac{\tau}{\tau_0} e^{\frac{\sigma_{\ln\tau}}{\sigma_{\ln V}}\mu_{\ln V}-\mu_{\ln\tau}}\right)^{\frac{\sigma_{\ln V}}{\sigma_{\ln\tau}}} \quad V\in\left(V_0,V_M\right) \quad (10)$$

where $\alpha = \sigma_{\ln V}/\sigma_{\ln\tau}$ is not a natural number as $\sigma_{\ln V} \neq \sigma_{\ln\tau}$.

Owing to the particle size distribution $p(D)$ and its scattering signal amplitude distribution both obey the lognormal distribution [1], then relation $\frac{V}{v_0} = \left(\frac{D}{d_0}k_D\right)^{\alpha_D}$ can be got from Eq. (10). Therefore, the size distribution can be obtained via the particles' scattering pulse amplitude distribution in the practical particle measurement, once the two parameters k_D and α_D are ascertained. Because the number of calibration factors for this method is less than the number of calibration factors for traditional method, so this method can be applied to improve the resolution of particle counter and expand the measurable range, which can promote the further development of particle measurement technology. In particular, it has great theoretical value and broad application prospects.

VI.CONCLUSIONS

In this paper, the signal reference voltage value f_0 is used to deal with the random signal, which produces the two-parameter (amplitude and width) analyzing model to describe the structure of the random pulse signal group. By using the new analyzing model, the characteristic parameters' counting distribution functions $q(l_V)$ and $p(l_\tau)$ of the pulse signal group and distribution functions $q_{l_\tau}(l_V)$ and $p_{l_V}(l_\tau)$ of the different signal characteristic parameter subsets take on well statistical self-similarity character. Besides, they stably obey the log-normal distribution while the natural number is used as the independent variable.

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