

Stochastic User Equilibrium with Combined Mode in A Degradable Multi-modal Transportation Network

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Abstract—This paper proposes a stochastic user equilibrium assignment model for ascertaining the difference in the travel mode choice behavior in a degradable multi-modal traffic network. In the proposed model, on the basis of the actual travel behavior which often needs to transfer different traffic modals, all the travels are categorized into two classes: pure car trip travelers and combined mode trip(car-subway) travelers. The multi-modal traffic network equilibrium conditions are formulated as an equivalent fixed-point problem, and the proof of its solution existence is provided afterwards. A heuristic solution algorithm MSWA method which is the extension of the classical MSA method is introduced for solving this fixed-point problem. An example is used to illustrate the application of the proposed model and solution algorithm. Moreover, the performance comparison of the MSA and MSWA algorithm is also given to prove the precision and efficiency of MSWA method for solving this problem.

Index Terms—stochastic traffic assignment; degradable transportation network; combined mode; fixed-point problem; MSWA

I. INTRODUCTION

Multi-modal trip distribution and assignment model was introduced by Friesz (1981) and definitely consider the combinatorial problem of modal split and trip assignment for multiple user classes in a multi-modal traffic network. The idea was further developed by Safwat (1988) who proposed a combined model, including trip generation, modal split and trip assignment and Lam and Huang (1992) who incorporated an entropy-

type trip distribution sub-model with the user equilibrium assignment problem. Lots of literature has been focused on the multi-modal transportation network by using mathematical programming (MP) formulations or variational inequality (VI) formulations (Florian and Spiess, 1983; Wong, 1998; Ferrari, 1999; Boyce, 2004; Ho et al., 2006; Sumalee et al., 2011, Wu et al., 2011, Kuang and Xu, 2012). Taking a wide view of these related researches, most of the models are supposed that travelers use only one single traffic mode from start to finish, which consider little about the transfer behavior between different traffic modes.

As the improvement of urban transportation system facility, the combined trips become more and more common in the mordent city. Travelers usually transfer up to one or two times to complete a trip. In a pure private car trip, travelers only make route choice; whereas in the combined mode trip, it needs to decide the route and transfer node at the same time. In recent decades, the combined mode models have been developed in formulation, analysis and computation of multi-modal transportation network. Firesz (1981) presented several approaches to solve this problem, which established a framework for deep research. Wu and Lam (2003) considered the demand uncertainties, proposing an elastic demand network equilibrium model in which transit and walking modes were considered. They adopted the super network conception to build a multi-layer network structure which expressed the travel progress in a clearly and simply way. Lo et al. (2003) used a state augmentation technique to relate the combined trip assignment in which non-linear fare structure for transit ticket is considered for improvement the management. Garía and Marín (2005) discussed this issue for the case of asymmetric cost. They took into account the choice of mode and transfer node with a nested logit distribution traffic demand model. Recently, Zhou et al. (2009) developed two alternative formulations including MP and VI formulations, which not only allowed a systematic and consistent treatment of travel choice over different

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dimensions but also had behavioral richness. Reviews of the state of the art of these models are given in Garía and Marín (2005) and Zhou et al. (2009).

All above studies are assumed that transportation supply side (network link capacity) is perfectly known. In reality, however, the network link capacity may be subject to stochastic variation due to various uncertainties such as vehicle breakdown, accident, adverse weather, etc. (Siu and Lo, 2007; Lam et al., 2008; Chen et al., 2008; Chen et al., 2009). These uncertainties will cause the link capacity degradations which affect the driving condition directly. While traditional Bureau of Public Roads (BPR) function is restricted to the constant link capacity, more recently the studies on the degradation traffic network have been increasingly developed. Clark and Watling (2005) discussed the network travel time reliability under stochastic demand. Their study provided a means for identifying sensitive or vulnerable links and examining the impact on network reliability of changes to link capacities. Siu and Lo (2007) considered the stochastic link capacity degradation as an infrequent part which added to the transportation network in a random manner based on fixed route choice proportions. Shortly thereafter, Siu and Lo (2008) extended this research mentality to a single bottleneck situation in which a departure time scheduling model was deliberated. They proved that the link reliability played an important role in travel choice. Sweet and Chen (2011) used the travel survey data for Chicago to test a regional model of travel time unreliability. Their results showed that unreliability will vary spatially during different time periods, but remain stable across times in the day from the overall view.

In metropolitan areas where a typical kind of combined mode park and ride (P&R) mode, becomes quite common, which is regarded as an economically, financially and sustainable form. P&R mode means that travelers leave their cars at the parking place nearby the public transit stop and continue their trip by public the public transit system. In the academic study, P&R mode can be viewed as a combined trip with two traffic modals, private car and public transit (such as subway). As mentioned above, the urban road network is subject to stochastic variations which will affect the travel time directly, whereas the subway goes in a deterministic network without disturbance. Considering the transfer behavior and network degradation, this study, in turn, focuses on the P&R combined trip assignment problem in the degradable multi-modal transportation network. For concise and generality, two kinds of typical travel mode, car and car-subway, are considered between each O-D pair. We mainly investigate two importation issues: one is that how many travelers choose the pure car mode and combined car-subway mode; the other is that how many travelers will change their travel mode as the road link capacity degradation. Moreover, the application significance is to predict the assignment changes resulting from the different parking lot allocation or relative policies.

The outline of this paper is the following: Section 2 presents the basic notation and formulation of the stochastic user equilibrium (SUE) model with combined modes in a degradable multi-modal transportation network. Section 3 depicts a heuristic solution algorithm. Numerical examples and analysis are given in Section 4. The final section concludes the paper and recommends further research issues.

II. MODEL FORMULATION

A. Multi-modal Transportation Network

The super network conception was first introduced by Sheffi (1985) to represent a multi-modal transportation network, where dotted links were added to interconnect different single-modal networks to represent the transfer link at the same physical locations. Based on this idea, the super multi-modal transportation network can be constructed with many independent single layers which are represented different individual traffic modals. The solid lines on the layer represent the travel route, whereas the dotted lines connecting the different single-modal at the same physical location represent the transfer behavior. The simple and clearly structure is quickly accepted and applied by many studies in transportation academic (Southworth and Peterson, 2000; Benjamins et al., 2002; Carlier et al., 2003; Lo et al., 2003; Wu and Lam, 2003).

In this paper, we consider a two-layer transportation network $G=(N,L)$, where N and L are the sets of nodes and links, respectively. Let M be the set of traffic modals, $M=\{m|a,b\}$, where $m=a$ and $m=b$ represent the car and subway modal. Let A and B be the set of link in the car and subway sub-network, respectively. Let E be the set of transfer link, $e \in E$. As an example, a small network is shown in Figure I. The transportation network comprises the car layer, subway layer, as well as transfer links. There are twelve nodes, seventeen links, where 9-10-11-12 is a section of a subway line. The single-mode network of the generic mode $m \in M$ is represented by a subset of G denoted by $G^m=(N^m,L^m)$, $N^m \subseteq N$, $L^m \subseteq L$.

Denote by f_w^k the traveler flow on route $k \in K_w$ between O-D pair $w \in W$, where K_w is the set of routes between O-D pair w , W is the set of O-D pairs. Let q_w be the traffic demand between O-D pair w and x_l be the traveler flow on link l , $l \in L$.

For description convenience, let W_a and W_b be the set of OD pair in car sub-network and subway sub-network, respectively, W_c be the set of the combined trip OD pair, then $W=W_a \cup W_c$. All related a , b , c hereinafter are the same meaning corresponding these sub-network.

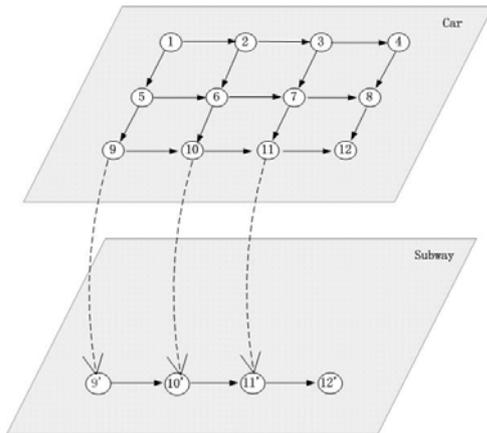


Figure 1. Multi-mode transportation network

For facilitating the presentation, some basic assumptions are listed as follows: All the travelers have the same value of time, so the generalized travel cost can be equaled to the travel time cost; the road transportation network is a stochastic uncertain system which performance may be degradable on several levels; the transfer link is used less than once in a complete trip; as the uncertain road conditions, the travelers perceive their travel time with a perception error which is assumed to follow a Gumbel distribution, so the choice of the travel mode can follow a Logit discrete choice model.

B. Travel Time Function

Usually, the travel time on a path in the pure mode trip can be evaluated as the travel time summation of the correlated link. While the combined mode occurs, the travel time contains not only the in vehicle travel times on the two sub-network (car and subway), but the transfer time together. On the supply side of a degradable multi-modal transportation network, the link travel time on the car sub-network is a random variable as a probability distribution will occur with the degradation of the physical link capacity (Lo et al., 2006). However, as running in an independently space, the subway sub-network is affected little by the uncertain supply which can be regarded as a determinate value depending on the subway run time. Moreover, the transfer time is mostly related with the subway departure frequency after ignoring the walking time for convenient.

• **The travel time of car mode trip**

Due to the uncertain traffic condition, traffic time accordingly becomes unpredictable. We consider this supply uncertainty into the Bureau of Public Roads (BPR) function:

$$t_l(x_l, C_l) = t_l^0 \left[1 + \beta \left(\frac{x_l}{C_l} \right)^n \right], \forall l \in A \quad (1)$$

where $t_l(\square)$ is the link travel time on link l , for the link $l \in A$, t_l^0 and C_l are the free-flow travel time and link capacity, respectively; β and n are correction parameters. Considering the travel time variations in the road network $l \in B$, the link capacity C_l is regarded as a

random variable. Hence link travel time t_l is also a random variable with its mean and variance expressed as

$$E(t_l) = E(t_l^0) + \beta t_l^0 E \left[\left(\frac{x_l}{C_l} \right)^n \right], \forall l \in A \quad (2)$$

$$(\sigma(t_l))^2 = (\sigma(t_l^0))^2 + \beta^2 (t_l^0)^2 \left\{ \sigma \left[\left(\frac{x_l}{C_l} \right)^n \right] \right\}^2, \forall l \in A \quad (3)$$

where $E(\square)$ and $\sigma(\square)$ represent the mean and standard deviation of the random variable t_l .

In order to simplify problem, we assume the random disturbance of link capacity is mutually independent with the link flow. The link free-flow travel time is assumed as a constant, $E(t_l^0) = t_l^0, \sigma(t_l^0) = 0$. Then Eqs.(2)and(3) can be simplified as

$$E(t_l) = t_l^0 + \beta t_l^0 x_l^n E \left[\left(\frac{1}{C_l} \right)^n \right], \forall l \in A \quad (4)$$

$$(\sigma(t_l))^2 = \beta^2 (t_l^0)^2 (x_l)^{2n} \left\{ \sigma \left[\left(\frac{1}{C_l} \right)^n \right] \right\}^2, \forall l \in A \quad (5)$$

We adopt the derivation for the mean and variance of $1/C_l^n$ in Siu and Lo (2008). They consider the lower bound to be a fraction θ of the link design capacity \bar{c}_a . Then the upper and lower bound can be defined as \bar{c}_a and $\theta \bar{c}_a$. Therefore, the mean of $1/C_l^n$ can be derived as follow:

$$E \left(\frac{1}{C_l^n} \right) = \int_{\theta \bar{c}_l}^{\bar{c}_l} \frac{1}{c_l} \cdot \frac{1}{\bar{c}_l - \theta c_l} dc_l = \frac{1 - \theta_l^{1-n}}{\bar{c}_l (1 - \theta_l)(1-n)}, \forall l \in A \quad (6)$$

$$E \left(\frac{1}{C_l^{2n}} \right) = \int_{\theta \bar{c}_l}^{\bar{c}_l} \frac{1}{c_l^{2n}} \cdot \frac{1}{\bar{c}_l - \theta c_l} dc_l = \frac{1 - \theta_l^{1-2n}}{\bar{c}_l^{2n} (1 - \theta_l)(1-2n)}, \forall l \in A \quad (7)$$

Consequently, the variance of $1/C_l^n$ can be derived as:

$$\begin{aligned} \left[\sigma \left(\frac{1}{C_l^n} \right) \right]^2 &= E \left(\frac{1}{C_l^{2n}} \right) - \left[E \left(\frac{1}{C_l^n} \right) \right]^2 \\ &= \frac{1 - \theta_l^{1-2n}}{\bar{c}_l^{2n} (1 - \theta_l)(1-2n)} - \left[\frac{1 - \theta_l^{1-n}}{\bar{c}_l^n (1 - \theta_l)(1-n)} \right]^2, \forall l \in A \end{aligned} \quad (8)$$

where $1/\bar{c}_l^n (1 - \theta_l)$ is the probability density function of the random variable C_l .

We substitute Eqs(6-8) into Eqs(4) and (5), the mean and variance of link travel time t_l can be finally written as :

$$E(t_l) = t_l^0 + \beta t_l^0 x_l^n \frac{1 - \theta_l^{1-n}}{\bar{c}_l (1 - \theta_l)(1-n)}, \forall l \in A \quad (9)$$

$$(\sigma(t_l))^2 = \beta^2 (t_l^0)^2 x_l^{2n} \left\{ \frac{1-\theta_l^{1-2n}}{\bar{c}_l^{2n}(1-\theta_l)(1-2n)} \left[\frac{1-\theta_l^{1-n}}{\bar{c}_l^n(1-\theta_l)(1-n)} \right]^2 \right\}, \forall l \in A \quad (10)$$

Assume that the link travel time is independent with each other, then the total travel time on path k can be expressed as

$$t_w^k = \sum_l \delta_{lk} t_l, \forall l \in A, k \in K_w, w \in W \quad (11)$$

where δ_{lk} is the link-path coincidence variable, $\delta_{lk}=1$ if link l is on path k ; 0 otherwise. Moreover, based on the central limit theorem, the path travel time in the road network follows Normal distributions

$$t_w^k \square N(E(t_w^k), \sigma(t_w^k)), \forall k \in K_w^A, w \in W \quad (12)$$

where the mean and variance of path travel time $E(t_k), \sigma(t_k)$ can be expressed as

$$E(t_w^k) = \sum_l \delta_{lk} E(t_l), \forall l \in A, k \in K_w^A, w \in W \quad (13)$$

$$\sigma(t_w^k) = \sqrt{\sum_l \delta_{lk} (\sigma(t_l))^2}, \forall l \in A, k \in K_w^A, w \in W \quad (14)$$

Above all, we can conclude that: in the pure car trip, the path travel time can be computed as

$$t_w^k = \sum_l \delta_{lk} t_l, \forall l \in A, k \in K_w^A, w \in W \quad (15)$$

where t_w^k follows Normal distributions with the mean and variance of path travel time $E(t_w^k), \sigma(t_w^k)$ as follows:

$$E(t_w^k) = \sum_l \delta_{lk} \left[t_l^0 + \beta t_l^0 x_l^n \frac{1-\theta_l^{1-n}}{\bar{c}_l^n(1-\theta_l)(1-n)} \right], \forall l \in A, k \in K_w^A, w \in W \quad (16)$$

$$\sigma(t_w^k) = \sqrt{\sum_l \delta_{lk} \beta^2 (t_l^0)^2 x_l^{2n} \left\{ \frac{1-\theta_l^{1-2n}}{\bar{c}_l^{2n}(1-\theta_l)(1-2n)} \left[\frac{1-\theta_l^{1-n}}{\bar{c}_l^n(1-\theta_l)(1-n)} \right]^2 \right\}}, \quad (17)$$

$$\forall l \in A, k \in K_w^A, w \in W$$

• **The travel time of combined mode trip**

As explained before, when the combined mode trips occur, the travel time contains the travel cost not only in the car sub-network, but the subway sub-network and the transfer time. Running in the fixed track, the travel time in the subway sub-network can be viewed as a determinate value depending on the subway run time. The transfer time is mostly related with the walking time and the waiting time which can be expressed as as Eq.(18).

$$t_e = t_{walk_e} + t_{wait_e} \quad (18)$$

where $t_e, t_{walk_e}, t_{wait_e}$ represent the transfer time, the transfer walking time, the transfer waiting time, respectively .

Therefore, the travel time on the path k t_w^k in the multi-modal transportation network consider the combined mode can be added as Eq.(19).

$$t_w^k = \sum_{l \in A} \delta_{lk} t_l + \sum_{l \in B} \delta_{lk} t_l + t_e, \forall k \in K_w, w \in W \quad (19)$$

• **The expected minimum travel time**

Position figures and tables at the tops and bottoms of In the real traffic network, travelers do not know their travel time exactly as the road condition variations. Based on the expected utility theory, the expected minimum travel time can be expressed as

$$\hat{t}_w = -\theta \ln \sum_{k \in K_w} \exp(-\theta t_w^k), \forall k \in K_w, w \in W \quad (20)$$

C. Elastic Traffic Demand

On the demand side, a number of existing researches have emphasized the elastic traffic demand case (Watling, 2002; Clark and Watling, 2005; Sumalee et al., 2006; Shao et al., 2006; Lo et al., 2006). A traveler who has originally planned to reach a destination in a specific time period is likely to change the decisions due to the uncertainties. Therefore, we define the traffic demand as a function of the expected minimum travel time:

$$q_w = g_w(\hat{t}_w), \forall w \in W \quad (21)$$

where $g_w(\cdot)$ is the elastic demand function between OD pair w , \hat{t}_w is the expected minimum travel time between OD pair w . Generally, the traffic demand is formulated as an exponential function or linear function as follows

$$g_w(\hat{t}_w) = q_w^{\max} \exp(-\beta \hat{t}_w), \forall w \in W \quad (22)$$

$$g_w(\hat{t}_w) = q_w^{\max} - \beta \hat{t}_w, \forall w \in W$$

D. Abbreviations and Acronyms

The traffic assignment in the multi-modal transportation network is the aggregative result of modal-route combined choice. Travelers always want to choose the route with minimal generalized travel cost. With more and more choices for one route, however, the increasing traffic congestion will raise the generalized travel cost which will causes the route reselection for travelers. Therefore, the traffic assignment in the multi-modal transportation network is an interactional dynamic balancing mechanism.

After given the traffic demand q_w between each OD pair, the logit-based stochastic equilibrium basic constrain condition can be expressed as

$$f_w^k = P_w^k q_w, \forall k \in K_w, w \in W \quad (23)$$

where P_w^k is the path choice proportion on path $k \in K_w$ which can be expressed as

$$P_w^k = \frac{\exp(-\theta t_w^k)}{\sum_{l \in K_w} \exp(-\theta t_w^l)}, \forall k \in K_w, w \in W \quad (24)$$

Another conventional constrain conditions include

$$x_l = \sum_w \sum_k f_w^k \delta_{lk}, \forall l \in L, w \in W \quad (25)$$

$$\sum_k f_w^k = q_w, \forall k \in K_w, w \in W \quad (26)$$

Note that the flow on path k is related with the path choice proportion P_w^k , and the travel time cost is a function of the flow on path k . Then the SUE model maps the flow f_w^k to itself by using the travel time cost as a medium. Consequently, the essence of SUE model is a fixed-point problem about flow.

For convenience, let P , q , f , $t(f, q)$ and $g(\hat{t})$ represent the vectors of $\{P_w^k\}$, $\{q_w\}$, $\{f_w^k\}$, $\{t_w^k\}$ and $\{g_w(t_w)\}$, respectively. The SUE model can be formulated as an equivalent fixed point problem.

$$x = F(x) \quad (27)$$

where $x = \begin{pmatrix} f \\ q \end{pmatrix} \left(\sum_w K_w + R \right) \times 1$, $F(x) = \begin{pmatrix} P(f)q \\ g(t(f, q)) \end{pmatrix} \left(\sum_w K_w + R \right) \times 1$.

Theorem 1. The fixed point problem Eq.(27) has at least one solutions

Proof. Let q_w^{\max} is the maximum demand for O-D pairs W , then $0 \leq f_w^k \leq q_w^{\max}$, $0 \leq g_w(\cdot) \leq q_w^{\max}$. Therefore, the feasible set of x is $\Omega = \left\{ f_w^k, q_w \mid 0 \leq f_w^k \leq q_w^{\max}, 0 \leq q_w \leq q_w^{\max} \right\}$, which is a convex set. As $F(x)$ is a continuous function on interval Ω , we can proof that the fixed point problem Eq.(27) has at least one solutions.

III. SOLUTION ALGORITHM

The Method of Successive Average (MSA) is the typical used method for solving the SUE problem. It contains two main steps: firstly find an initial point satisfies $y^i = F(x^i)$, and then update the iteration point by

$$x^{i+1} = x^i + \alpha^i (y^i - x^i), i=1,2,\dots \quad (28)$$

where $\{\alpha^i\}$ is the sequence of step size, which is predefined to be a simple equal average $\alpha^i = 1/i$. Then the Eq.(28) can be written as

$$x^{i+1} = x^i + \alpha^i (y^i - x^i), \alpha^i = 1/i, i=1,2,\dots \quad (29)$$

Given a stopping tolerance ε , the traditional MSA arithmetic process for solving the Eq(27) can be described as:

Step 1: Calculate $\|y^i - x^i\|$, if $\|y^i - x^i\| \leq \varepsilon$, go to Step 5, otherwise go to Step 2;

Step 2: Set $\alpha^i = 1/i$;

Step 3 : Update $x^{i+1} = x^i + \frac{1}{i}(y^i - x^i)$, then calculate $y^{i+1} = F(x^{i+1})$;

Step 4: Set $i=i+1$, go to Step 1;

Step 5: Stop and output x^i .

The update equation can be rewritten as

$$x^{i+1} = \frac{1}{i} (y^1 + y^2 + \dots + y^i), i=1,2,\dots \quad (30)$$

It can be easily noted that the MSA essentially gives an average of all auxiliary solutions. This feature leads to the convergence rate too fast at the beginning, but too slowly while approaching the optimization which limits the widely application to solve the equivalent fixed-point problem. Realizing that the auxiliary flow pattern is in fact approaching to the solution point when the iteration number is large, Liu et al.(2007) proposed an extension MSA method, called Successive Weighted Averages(MSWA), which included a new step size sequence giving higher weights to the auxiliary flow patterns from the latter. In the MSWA method, x^{i+1} has the following form:

$$x^{i+1} = \frac{1}{M} (1 \cdot y^1 + 2 \cdot y^2 + \dots + i \cdot y^i), M = \sum_{j=1}^i j \quad (31)$$

Then Eq.(28) in MSWA can be expressed as

$$x^{i+1} = \frac{1^d \cdot y^1 + 2^d \cdot y^2 + \dots + i^d \cdot y^i}{1^d + 2^d + \dots + i^d} \quad (32)$$

It can be written as

$$x^{i+1} = x^i + \alpha^i (y^i - x^i), \alpha^i = \frac{i^d}{1^d + 2^d + \dots + i^d}, i=1,2,\dots \quad (33)$$

It can be seen that MWA is the special case for MSWA with $d=0$. Liu et al.(2007) has proved that the convergence speeds of MSWA with $d \geq 1$ are much faster than that of traditional MSA. Therefore, the MSWA solution algorithm is adopted in this paper. The search direction and step size for solving Eq.(27) can be set as

$$y^i - x^i = \begin{pmatrix} f^{i+1} & f^i \\ q^{i+1} & q^i \end{pmatrix} \quad (34)$$

$$a^i = \frac{i^d}{1^d + 2^d + \dots + i^d} \quad (35)$$

where

$$f_w^{k,i+1} = P_w^k q_w^i, \forall k \in K_w, w \in W \quad (36)$$

$$q_w^{i+1} = g_w(t_w^i), \forall w \in W \quad (37)$$

The procedure of the MSWA in our solution can be summarized as follows:

Step 1 Initialization:

- (1) Set $i=1$, $\gamma_0=0$, input the value of d and ε ;

(2) Given the initial parameters, and each distribution function expressions.

(3) Input the initial point $x^i = \begin{pmatrix} f^i \\ q^i \end{pmatrix}$, calculate $\{v_l^i\}$,

the travel time $\{t_w^{k,i}\}$, expected minimum travel time $\{\hat{t}_w^i\}$. y^i can be obtained by

$$y^i = F(x^i) = \begin{pmatrix} P(f^i)q^i \\ q^i \end{pmatrix}$$

where $q^i = \max(0, g(t(f^i, q^i)))$.

Step 2 Update:

- (1) Calculate $\|y^i - x^i\|$, if $\|y^i - x^i\| \leq \varepsilon$, go to (5), otherwise go to (2);
- (2) Set $\chi^i = i^d$, $\gamma^i = \gamma^{i-1} + \chi^i$, $\alpha^i = \chi^i / \gamma^i$;
- (3) Calculate $x^{i+1} = x^i + \alpha^i (y^i - x^i)$ and $y^{i+1} = F(x^{i+1})$;
- (4) Set $i = i + 1$, go to (1);
- (5) Stop and output x^i .

IV. NUMERICAL EXAMPLE

To illustrate this SUE model and its solution qualities, a multi-modal transportation network with two traffic modal (car and subway) is developed as shown in Fig.1 with two OD pairs including (1,12) and (6,12). The subway line runs through 9', 10', 11', 12' with the exclusive rail links corresponding to the physical nodes in the road network, each point of which allows the transfer behavior.

The in-vehicle time in the car sub-network can be modeled by a BPR type of function as Eq.(1) with $\beta=0.15$, $n=4$ where t_l^0 and C_l are defined in Table I. The link of (6,7) and (11,12) are the bottleneck link in the road sub-network.

TABLE I.
FREE-FLOW TRAVEL TIME AND LINK CAPACITY IN THE ROAD SUB-NETWORK

Link#	t_l^0	C_l	Link#	t_l^0	C_l
(1,2)	15	1000	(7,8)	18	1000
(2,3)	16	800	(5,9)	20	1000
(3,4)	12	1000	(6,10)	18	1000
(1,5)	17	800	(7,11)	22	500
(2,6)	18	500	(8,12)	16	1000
(3,7)	20	1000			

For the subway sub-network, the running line is in an exclusive right-of-way; the in-vehicle travel time is

depended on the travel time of the subway link which is shown in Tab.2. Transfer time is also shown in Table II.

TABLE II.
PARAMETERS FOR THE SUBWAY LINK AND THE TRANSFER LINK

Link#	t_l	Link#	t_{walk}	t_{wait}	t_e
(9',10')	10	(9-9')	2.75	2.75	5.50
(10',11')	12	(10-10')	2.75	3.00	5.75
(11',12')	13	(11-11')	3	2.75	5.75

Other parameters can be valued as follows: $\theta=0.01$, $d=1$, $\varepsilon=0.01$. The traffic demand is formulated as a linear function as Eq.(38) with $q_{1-12}^{max}=1500$, $q_{6-12}^{max}=1200$.

$$g_w(\hat{t}_w) = q_w^{max} - 900\hat{t}_w, \forall w \in W \tag{38}$$

Five times experiments are run in the test network for different road conditions including 15% degradable road network, 30% degradable road network, and 45% degradable road network, and 20% expansion road network. For example, 15% degradable road network means the bottleneck link (6,7) and (11,12) diminish their 15% link capability; 20% expansion road network means the bottleneck link (6,7) and (11,12) expand their 20% link capability. The equilibrium results are shown respectively in Table III using MSWA.

The pure car trip flow declines at once along with link capacity degradation while the combined trip flow drops slowly. The farther travelers are from the destination, the more possibility to change the travel mode when the road link capacity becomes degradable. Furthermore, we can note that when the road supply satisfies most of the travel demand, the expansion for the road network has not much effect on the traffic mode choice guidance.

TABLE III.
EQUILIBRIUM RESULTS IN DIFFERENT LEVEL ROAD CONDITIONS

Rout/transfer segments	Travel mode	Normal	-15%	-30%	-45%	+20%
1-12	a	709	686	653	609	728
1-12	b	808	831	862	906	789
1-9-12	b	96	100	106	115	93
1-10-12	b	268	279	293	313	259
1-11-12	b	444	452	463	478	437
6-12	a	548	521	482	427	570
6-12	b	661	687	725	779	639
6-10-12	b	256	272	295	329	242
6-11-12	b	405	415	430	450	397

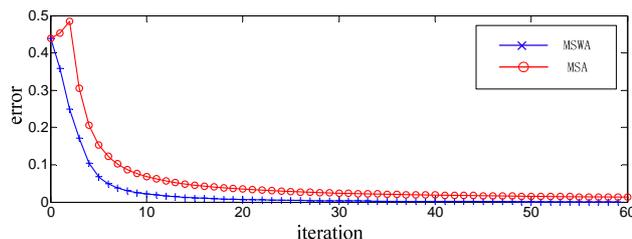


Figure 2. Convergence performance comparisons

To show advantages of MSWA algorithm, a comparison experiment is made using MSA in the same network. The experimental results show that the iteration number required by MSA is much greater than MSWA. MSA convergence needs 1213 iterations to reach the convergence precision, while MSWA only needs 60 iterations for the same network. Figure II shows the convergence of the MSWA and MSA algorithm in the first 60 iterations. Though MSA algorithm converges fast at the beginning, its convergence speed becomes extremely slow afterwards due to the small step size. Overall, MSWA can achieve much better than MSA for solving the SUE model with faster convergence speed and higher accuracy.

V. CONCLUSION

The goal of this paper is to develop an approach to model the stochastic user equilibrium problem with combined mode in a degradable multi-modal transportation network. The main motivation is to investigate the change of travel mode choice while the traffic system is uncertain. Two typical travel modes are considered including pure car travel mode and combined car-subway travel mode. An equivalent fixed-point problem is presented for this problem where the link capacity is a random variable. The proof of the existence of a solution is also provided.

As the combined trip becomes more and more common in the mordent city, the study of travel mode choice is increasingly needed. We first built a multi-modal transportation network on the basis of super network conception. The multi-modal super transportation network is divided into multi sub-network, in which each layer represents one single traffic modal. Solid lines on the sub-network layer express the physical running routes while the dotted lines between different sub-networks express the transfer routes. Therefore, this transportation network structure explicitly describes the travel behavior including the in-vehicle and transfer behavior. Then the traffic time accordingly includes in-vehicle and transfer time.

The model explicitly takes into account the choices of travel route, traffic modal at the same time, in a Logit disperse choice model. Due to the uncertain road traffic condition, the road link capacity is regarded as a random variable which affects the pure car trip travel time directly. Meanwhile, the travel time in the subway sub-network is rarely influenced by the external factor which is a determinate value. Transfer time is simplified into two parts: walking time and waiting time.

The model has greatly flexibility not only to research the traffic assignment model in the multi-modal transportation network, but to discuss the policy impact in the applications. An improved MSA solution algorithm, MSWA, is introduced for solving the fixed-point problem. Numerical results have shown that travelers will change their travel mode choice while the link capacity is degradable. When road traffic condition is severely serious, parts of travelers will choose the combined travel mode or cancel this trip. For the slightly degradation, the farther travelers are from the destination, the more possibility to change their travel mode when the road link capacity becomes degradable. While the road supply satisfies most of the travel demand, the expansion in the road network has not much effect on the traffic mode choice guidance. In addition, the performance comparison of the MSA and MSWA is given finally, which is illustrated the accuracy and efficiency of the proposed model and algorithm.

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