Adaptive Trajectory Tracking Control of a High Altitude Unmanned Airship

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***Abstract*—Nonlinear dynamic model of a high-altitude unmanned airship, expressed by generalized coordinate, was built. A nonlinear compensation was introduced into the control loop to linearize and decouple the nonlinear system globally. In view of the imprecisely known inertia parameters of the airship, an adaptive law was proposed based on the feedback linearization to realize asymptotic tracking of any continuous time-varying desired trajectory from an arbitrary initial condition. The stability of the closed-loop control system was proved via the use of Lyapunov stability theory. Finally, numerical simulation results demonstrate the validity and effectiveness of the proposed adaptive control law.**

***Index Terms*—adaptive control, feedback linearization, trajectory tracking, high-altitude unmanned airships**

I. INTRODUCTION

High-altitude unmanned airships, which have a wide application prospect in communication, surveillance and investigation, are capable of hovering for a long time. According to the task demands, desired trajectory is designated. Modeling, control method and verification test of high altitude unmanned airships are the focus of the domestic and international studies [1]~[10].

Trajectory tracking, based on adaptive feedback linearization, is designed to solve the control problem on imprecisely known inertia parameters of a high altitude unmanned airship. This paper is organized as follows: nonlinear dynamic model of a conventional airship is built, expressed by generalized coordinate in section II. In section III the feedback linearization control law is designed. Adaptive feedback linearization control law and estimation law of inertia parameters are designed, and stability is proved in section IV. The effectiveness of tracking desired continuous time-varying trajectory is validated via simulation without wind disturbance in section V. Finally, conclusion and future work are summarized in section VI.

Manuscript received September 16, 2011; revised September 30, 2011; accepted December 1, 2011.
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II. KINETIC MODEL OF THE AIRSHIP

A. Defination of Coordinate

This paper studies an ellipsoid full-actuated high altitude unmanned airship which is symmetrical with respect to the vertical axis, and its tail fin with the cross elevator and rudder, is bisymmetric. The gondola is equipped with a pair of differential propellers under the body.

The earth reference frame is denoted by $O_e x_e y_e z_e$, and the body reference frame $Oxyz$ whose origin is located at the center of volume, as shown in Fig.1.

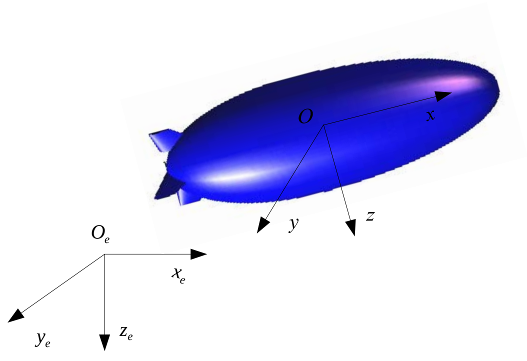

Figure 1. Definition of coordinate

B. Dynamics Fundamental Equations of Airship

Several basic assumptions are needed:

A1. The volume center coincides with the gross center of buoyancy.

A2. The airship forms a rigid body such that elastic effects can be ignored.

A3. The shape and the whole mass are constant in hovering.

In view of the symmetry, the center of mass is located under the center of volume in longitudinal profile, and products of inertia satisfy $I_{xy} = I_{yz} = 0$, The dynamic equations of the airship can be formulated as follows[1]~[10]:

doi:10.4304/jcp.7.11.2781-2787

$$\bar{\boldsymbol{M}}\dot{\boldsymbol{V}} = \bar{\boldsymbol{N}} + \bar{\boldsymbol{G}} + \bar{\boldsymbol{B}}\boldsymbol{u} \tag{1}$$

where $\boldsymbol{V} \triangleq [p,q,r,u,v,w]^{\mathrm{T}}$, $[u,v,w]^{\mathrm{T}}$ denotes linear velocity vector, and $[p,q,r]^{\mathrm{T}}$ angular velocity vector of the airship.

$$\bar{\boldsymbol{M}} = \begin{bmatrix} I_x & 0 & -I_{xz} & 0 & -mz_c & 0 \\ 0 & I_y+\rho\nabla k_3 & 0 & mz_c & 0 & 0 \\ -I_{xz} & 0 & I_z+\rho\nabla k_3 & 0 & 0 & 0 \\ 0 & mz_c & 0 & m+\rho\nabla k_1 & 0 & 0 \\ -mz_c & 0 & 0 & 0 & m+\rho\nabla k_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & m+\rho\nabla k_2 \end{bmatrix}$$

where I_x, I_y, I_z, I_{xz} are inertia parameters, k_1, k_2, k_3 are inertial factors of the airship, ∇ is the volume of the airship, ρ is atmospheric density of the flying height, z_c is the position coordinates of the center of mass, and m is the whole mass of the airship.

$$\bar{\boldsymbol{N}} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \end{bmatrix}^{\mathrm{T}}$$

where

$$\begin{aligned} a_1 &= -\left(I_z - I_y\right)qr + I_{xz}pq + mz_c\left(ur - wp\right) + \\ &\quad QC_{L2}\sin\beta\sin\left(|\beta|\right), \\ a_2 &= -\left(I_x - I_z\right)pr - I_{xz}\left(p^2 - r^2\right) - mz_c\left(wq - vr\right) - \\ &\quad Q[C_{M1}\cos\left(\alpha/2\right)\sin\left(2\alpha\right) + \\ &\quad C_{M2}\sin\left(2\alpha\right) + C_{M3}\sin\alpha\sin\left(|\alpha|\right)], \\ a_3 &= -\left(I_y - I_x\right)pq - I_{xz}qr + Q[C_{N1}\cos\left(\beta/2\right)\sin\left(2\beta\right) + \\ &\quad C_{N2}\sin\left(2\beta\right) + C_{N3}\sin\beta\sin\left(|\beta|\right)], \\ a_4 &= -\left(m + \rho\nabla k_1\right)\left(wq - vr\right) - mz_c pr - \\ &\quad Q\left[C_{X1}\cos^2\alpha\cos^2\beta + C_{X2}\sin\left(2\alpha\right)\sin\left(\alpha/2\right)\right], \\ a_5 &= -\left(m + \rho\nabla k_2\right)\left(ur - wp\right) - mz_c qr - \\ &\quad Q[C_{Y1}\cos\left(\beta/2\right)\sin\left(2\beta\right) + C_{Y2}\sin\left(2\beta\right) + \\ &\quad C_{Y3}\sin\beta\sin\left(|\beta|\right)], \\ a_6 &= -\left(m + \rho\nabla k_2\right)\left(vp - uq\right) + mz_c\left(p^2 + q^2\right) - \\ &\quad Q[C_{Z1}\cos\left(\alpha/2\right)\sin\left(2\alpha\right) + C_{Z2}\sin\left(2\alpha\right) + \\ &\quad C_{Z3}\sin\alpha\sin\left(|\alpha|\right)]. \end{aligned}$$

where $\alpha = \arctan(w/u)$ and $\beta = \arctan(v\cos\alpha/u)$ are the flow angle when wind speed is zero, $Q = \rho V^2/2$ is dynamic pressure, V is the flow speed from a distance, $C_{Li}, C_{Mi}, C_{Ni}, C_{Xi}, C_{Yi}, C_{Zi}, i = 1,2,3$ are the aerodynamic coefficients[7].

$$\bar{\boldsymbol{G}} = \begin{bmatrix} -z_c mg\cos\theta\sin\phi \\ -z_c mg\sin\theta \\ 0 \\ \left(B_f - mg\right)\sin\theta \\ -\left(B_f - mg\right)\cos\theta\sin\phi \\ -\left(B_f - mg\right)\cos\theta\cos\phi \end{bmatrix}$$

where g is gravity acceleration, B_f is buoyancy acted on the airship, θ, ψ, ϕ are attitudes.

$$\bar{\boldsymbol{B}} = \begin{bmatrix} c\xi & c\xi & 0 & 0 & 0 & 0 \\ s\xi & -s\xi & 0 & 0 & 0 & -2QC_{M4} \\ 0 & 0 & 1 & 1 & -2QC_{N4} & 0 \\ -z_p s\xi & -z_p s\xi & y_p & -y_p & 0 & 0 \\ z_p c\xi & z_p c\xi & -x_p & -x_p & 2QC_{Y4} & 0 \\ h_1 & h_2 & 0 & 0 & 0 & -2QC_{Z4} \end{bmatrix}$$

where $s\xi \triangleq \sin\xi$, $c\xi \triangleq \cos\xi$, $h_1 = x_p s\xi - y_p c\xi$, $h_2 = x_p s\xi + y_p c\xi$, (x_p, y_p, z_p) and $(x_p, -y_p, z_p)$ are position coordinates of left and right propellers about the body reference frame, ξ is an angle toward outside of propellers, C_{M4}, C_{N4}, C_{Y4}, C_{Z4} are the aerodynamic coefficients[7].

$$\boldsymbol{u} = \begin{bmatrix} F_1 c\zeta_1 & F_2 c\zeta_2 & F_1 s\zeta_1 & F_2 s\zeta_2 & \delta_{RUD} & \delta_{ELV} \end{bmatrix}^{\mathrm{T}}$$

Six control variables of the airship are thrust F_1 and F_2, turning angles ζ_1, ζ_2 about axis y, rudder angle δ_{RUD} and elevator angle δ_{ELV}, respectively.

C. Dynamics Model Expessed by Generalized Coordinate

Define generalized coordinate

$$\boldsymbol{\mu} \triangleq \left[\theta, \psi, \phi, x_g, y_g, z_g\right]^{\mathrm{T}}$$

where (x_g, y_g, z_g) is the coordinate of the center of volume about the earth reference frame. Based on the fundamental kinematics we have

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{O}_{3\times3} \\ \boldsymbol{O}_{3\times3} & {}^{\mathrm{b}}\boldsymbol{S}_{\mathrm{e}} \end{bmatrix}\dot{\boldsymbol{\mu}} \triangleq \boldsymbol{S}\dot{\boldsymbol{\mu}} \tag{2}$$

where ${}^{\mathrm{b}}\boldsymbol{S}_{\mathrm{e}}$ is homogenous transformation matrix from the earth reference frame to the body reference frame of the airship.

$$\boldsymbol{S}_{11} = \begin{bmatrix} 0 & -s\mu_1 & 1 \\ c\mu_3 & c\mu_1 s\mu_3 & 0 \\ -s\mu_3 & c\mu_1 c\mu_3 & 0 \end{bmatrix}$$

$$ {}^{\mathrm{b}}\boldsymbol{S}_{\mathrm{e}} = \begin{bmatrix} c\mu_1 c\mu_2 & c\mu_1 s\mu_2 & -s\mu_1 \\ f_1 & f_2 & c\mu_1 s\mu_3 \\ f_3 & f_4 & c\mu_1 c\mu_3 \end{bmatrix} $$

$s\mu_i \triangleq \sin\mu_i, c\mu_i \triangleq \cos\mu_i \quad i = 1,2,3,4,5,6$,

$f_1 \triangleq s\mu_1 c\mu_2 s\mu_3 - s\mu_2 c\mu_3,\ f_2 \triangleq s\mu_1 s\mu_2 s\mu_3 + c\mu_2 c\mu_3,$

$f_3 \triangleq s\mu_1 c\mu_2 c\mu_3 + s\mu_2 s\mu_3,\ f_4 \triangleq s\mu_1 s\mu_2 c\mu_3 - c\mu_2 s\mu_3.$

Differentiating equation (2) yields

$$ \dot{\boldsymbol{V}} = \dot{\boldsymbol{S}}\dot{\boldsymbol{\mu}} + \boldsymbol{S}\ddot{\boldsymbol{\mu}} \tag{3} $$

Multiply both sides of (3) by $\bar{\boldsymbol{M}}$, and we have

$$ \bar{\boldsymbol{M}}\dot{\boldsymbol{V}} = \bar{\boldsymbol{M}}\dot{\boldsymbol{S}}\dot{\boldsymbol{\mu}} + \bar{\boldsymbol{M}}\boldsymbol{S}\ddot{\boldsymbol{\mu}} \tag{4} $$

Combining (1) with (4) obtains

$$ \boldsymbol{M}(\boldsymbol{\mu})\ddot{\boldsymbol{\mu}} + \boldsymbol{N}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \boldsymbol{G}(\boldsymbol{\mu}) = \boldsymbol{B}(\boldsymbol{\mu})\boldsymbol{u} \tag{5} $$

where $\boldsymbol{M}(\boldsymbol{\mu}) = \bar{\boldsymbol{M}}\boldsymbol{S}, \boldsymbol{N}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) = \bar{\boldsymbol{M}}\dot{\boldsymbol{S}}\dot{\boldsymbol{\mu}} - \bar{\boldsymbol{N}}, \boldsymbol{G}(\boldsymbol{\mu}) = -\bar{\boldsymbol{G}}$, $\boldsymbol{B}(\boldsymbol{\mu}) = \bar{\boldsymbol{B}}$. Because $\bar{\boldsymbol{M}}$ and $\boldsymbol{S}$ are invertible, then $\boldsymbol{M}(\boldsymbol{\mu})$ is invertible too. Since $|\boldsymbol{B}(\boldsymbol{\mu})| \neq 0$, $\boldsymbol{B}(\boldsymbol{\mu})$ is invertible. According to (5), we can derive

$$ \boldsymbol{M}(\boldsymbol{\mu})\ddot{\boldsymbol{\mu}} + \boldsymbol{N}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \boldsymbol{G}(\boldsymbol{\mu}) = \boldsymbol{\tau} \tag{6} $$

where $\boldsymbol{\tau} \triangleq \boldsymbol{B}(\boldsymbol{\mu})\boldsymbol{u}$.

III. Feedback Linearization Control Design

A. Control Objective

In view of inertia parameter uncertainty, design feedback linearization and adaptive control law[11] to realize asymptotic tracking of any desired trajectory from an arbitrary initial condition. Let $\boldsymbol{\mu}_{\mathrm{d}}(t)$ denote an arbitrary twice differentiable time-varying trajectory, with $\dot{\boldsymbol{\mu}}_{\mathrm{d}}(t)$ and $\ddot{\boldsymbol{\mu}}_{\mathrm{d}}(t)$ are bounded.

B. Control Law

We choose

$$ \boldsymbol{\tau} = \boldsymbol{N}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \boldsymbol{G}(\boldsymbol{\mu}) + \boldsymbol{M}(\boldsymbol{\mu})\boldsymbol{r} \tag{7} $$

where $\boldsymbol{r}$ will be designed later. Substituting (7) into (6) yields

$$ \boldsymbol{M}(\boldsymbol{\mu})\ddot{\boldsymbol{\mu}} + \boldsymbol{N}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \boldsymbol{G}(\boldsymbol{\mu}) = \boldsymbol{N}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \boldsymbol{G}(\boldsymbol{\mu}) + \boldsymbol{M}(\boldsymbol{\mu})\boldsymbol{r} $$

Then we get

$$ \boldsymbol{M}(\boldsymbol{\mu})\ddot{\boldsymbol{\mu}} = \boldsymbol{M}(\boldsymbol{\mu})\boldsymbol{r} $$

which is equivalent to a decoupling linear time-invariant system $\ddot{\boldsymbol{\mu}} = \boldsymbol{r}$. When $\boldsymbol{\mu}_{\mathrm{d}}(t)$ is given, $\dot{\boldsymbol{\mu}}_{\mathrm{d}}(t)$ and $\ddot{\boldsymbol{\mu}}_{\mathrm{d}}(t)$ are known. Let error be $\boldsymbol{e} = \boldsymbol{\mu}_{\mathrm{d}} - \boldsymbol{\mu}$, and

$$ \begin{aligned} \boldsymbol{r} &= \ddot{\boldsymbol{\mu}}_{\mathrm{d}} + \boldsymbol{K}_{\mathrm{d}}(\dot{\boldsymbol{\mu}}_{\mathrm{d}} - \dot{\boldsymbol{\mu}}) + \boldsymbol{K}_{\mathrm{p}}(\boldsymbol{\mu}_{\mathrm{d}} - \boldsymbol{\mu}) \\ &= \ddot{\boldsymbol{\mu}}_{\mathrm{d}} + \boldsymbol{K}_{\mathrm{d}}\dot{\boldsymbol{e}} + \boldsymbol{K}_{\mathrm{p}}\boldsymbol{e} \end{aligned} \tag{8} $$

where $\boldsymbol{K}_{\mathrm{d}}$ and $\boldsymbol{K}_{\mathrm{p}}$ are positive definite matrices, then (8) can be rewritten as

$$ \ddot{\boldsymbol{e}} + \boldsymbol{K}_{\mathrm{d}}\dot{\boldsymbol{e}} + \boldsymbol{K}_{\mathrm{p}}\boldsymbol{e} = 0 \tag{9} $$

Thus, $(\boldsymbol{e},\dot{\boldsymbol{e}}) = (0,0)$ is exponentially stable. For any initial condition $(\boldsymbol{\mu}_0,\dot{\boldsymbol{\mu}}_0)$, there exists $(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) \to (\boldsymbol{\mu}_{\mathrm{d}},\dot{\boldsymbol{\mu}}_{\mathrm{d}})$. Substituting (8) into (7)yields the expression of the feedback linearization control law

$$ \boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{\mu})(\ddot{\boldsymbol{\mu}}_{\mathrm{d}} + \boldsymbol{K}_{\mathrm{d}}\dot{\boldsymbol{e}} + \boldsymbol{K}_{\mathrm{p}}\boldsymbol{e}) + \boldsymbol{N}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \boldsymbol{G}(\boldsymbol{\mu}) \tag{10} $$

To this end, the actual control input can be calculated as

$$ \boldsymbol{u} = \boldsymbol{B}^{-1}(\boldsymbol{\mu})\left[\boldsymbol{M}(\boldsymbol{\mu})(\ddot{\boldsymbol{\mu}}_{\mathrm{d}} + \boldsymbol{K}_{\mathrm{d}}\dot{\boldsymbol{e}} + \boldsymbol{K}_{\mathrm{p}}\boldsymbol{e}) + \boldsymbol{N}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \boldsymbol{G}(\boldsymbol{\mu})\right] $$

IV. Adaptive Control Law Design

A. Adaptive Control Law

Denote the imprecisely known inertia parameter vector as $\boldsymbol{\eta} = [I_x, I_y, I_z, I_{xz}, mz_c]^{\mathrm{T}}$ and the estimated one as $\hat{\boldsymbol{\eta}} = [\hat{I}_x, \hat{I}_y, \hat{I}_z, \hat{I}_{xz}, m\hat{z}_c]^{\mathrm{T}}$. The feedback linearization control law is modified as

$$ \boldsymbol{\tau} = \hat{\boldsymbol{M}}(\boldsymbol{\mu})(\ddot{\boldsymbol{\mu}}_{\mathrm{d}} + \boldsymbol{K}_{\mathrm{d}}\dot{\boldsymbol{e}} + \boldsymbol{K}_{\mathrm{p}}\boldsymbol{e}) + \hat{\boldsymbol{N}}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \hat{\boldsymbol{G}}(\boldsymbol{\mu}) \tag{11} $$

The actual control input can be obtained:

$$ \boldsymbol{u} = \boldsymbol{B}^{-1}(\boldsymbol{\mu})[\hat{\boldsymbol{M}}(\boldsymbol{\mu})(\ddot{\boldsymbol{\mu}}_{\mathrm{d}} + \boldsymbol{K}_{\mathrm{d}}\dot{\boldsymbol{e}} + \boldsymbol{K}_{\mathrm{p}}\boldsymbol{e}) + \hat{\boldsymbol{N}}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \hat{\boldsymbol{G}}(\boldsymbol{\mu})] $$

where $\hat{\boldsymbol{M}}, \hat{\boldsymbol{N}}, \hat{\boldsymbol{G}}$ are the estimated matrices of $\boldsymbol{M}, \boldsymbol{N}, \boldsymbol{G}$ of $\hat{\boldsymbol{\eta}}$. Substituting (11) into (6) yields

$$ \begin{aligned} &\boldsymbol{M}(\boldsymbol{\mu})\ddot{\boldsymbol{\mu}} + \boldsymbol{N}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \boldsymbol{G}(\boldsymbol{\mu}) \\ &= \hat{\boldsymbol{M}}(\boldsymbol{\mu})(\ddot{\boldsymbol{\mu}}_{\mathrm{d}} + \boldsymbol{K}_{\mathrm{d}}\dot{\boldsymbol{e}} + \boldsymbol{K}_{\mathrm{p}}\boldsymbol{e}) + \hat{\boldsymbol{N}}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \hat{\boldsymbol{G}}(\boldsymbol{\mu}) \end{aligned} \tag{12} $$

Since $\ddot{\boldsymbol{\mu}}_{\mathrm{d}} = \ddot{\boldsymbol{e}} + \ddot{\boldsymbol{\mu}}$ and (12) can be formulated as a linear function about the dynamics parameter vector:

$$ \begin{aligned} &\hat{\boldsymbol{M}}(\boldsymbol{\mu})(\ddot{\boldsymbol{e}} + \boldsymbol{K}_{\mathrm{d}}\dot{\boldsymbol{e}} + \boldsymbol{K}_{\mathrm{p}}\boldsymbol{e})) \\ &= \tilde{\boldsymbol{M}}(\boldsymbol{\mu},\boldsymbol{\eta} - \hat{\boldsymbol{\eta}})\ddot{\boldsymbol{\mu}} + \tilde{\boldsymbol{N}}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}},\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}) + \tilde{\boldsymbol{G}}(\boldsymbol{\mu},\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}) \\ &= \hat{\boldsymbol{M}}(\boldsymbol{\mu},\ddot{\boldsymbol{\mu}},\tilde{\boldsymbol{\eta}}) + \hat{\boldsymbol{N}}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}},\tilde{\boldsymbol{\eta}}) + \tilde{\boldsymbol{G}}(\boldsymbol{\mu},\tilde{\boldsymbol{\eta}}) \\ &= \left[\hat{\hat{\boldsymbol{M}}}(\boldsymbol{\mu},\ddot{\boldsymbol{\mu}}) + \hat{\hat{\boldsymbol{N}}}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}}) + \hat{\boldsymbol{G}}(\boldsymbol{\mu})\right]\tilde{\boldsymbol{\eta}} \\ &\triangleq \boldsymbol{Y}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}},\ddot{\boldsymbol{\mu}})\tilde{\boldsymbol{\eta}} \end{aligned} \tag{13} $$

where $\tilde{\boldsymbol{M}} \triangleq \boldsymbol{M} - \hat{\boldsymbol{M}}, \tilde{\boldsymbol{N}} \triangleq \boldsymbol{N} - \hat{\boldsymbol{N}}, \tilde{\boldsymbol{G}} \triangleq \boldsymbol{G} - \hat{\boldsymbol{G}}, \tilde{\boldsymbol{\eta}} \triangleq \boldsymbol{\eta} - \hat{\boldsymbol{\eta}}$. Usually we have $\hat{\boldsymbol{\eta}} \neq \boldsymbol{\eta}$, and thus $\tilde{\boldsymbol{M}} \not\equiv 0$, $\tilde{\boldsymbol{N}} \not\equiv 0$, $\tilde{\boldsymbol{G}} \not\equiv 0$. Since (13) can't be changed to linear constant system like (9), a real-time estimator to the parameters needs to be designed to realize $\boldsymbol{e} \to 0, \dot{\boldsymbol{e}} \to 0$.

Assume the estimated parameter $\hat{\boldsymbol{\eta}}$ renders $\hat{\boldsymbol{M}}(\boldsymbol{\mu})$ invertible, so the following the closed-loop system can be obtained

$$\ddot{\boldsymbol{e}}+\boldsymbol{K}_{\mathrm{d}}\dot{\boldsymbol{e}}+\boldsymbol{K}_{\mathrm{p}}\boldsymbol{e}=\hat{\boldsymbol{M}}^{-1}(\boldsymbol{\mu})\boldsymbol{Y}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}},\ddot{\boldsymbol{\mu}})\tilde{\boldsymbol{\eta}} \triangleq \boldsymbol{\Phi}\tilde{\boldsymbol{\eta}} \qquad (14)$$

Let $\boldsymbol{x}=\left[\boldsymbol{e}^{\mathrm{T}},\dot{\boldsymbol{e}}^{\mathrm{T}}\right]^{\mathrm{T}}$, and then (14) can be written as

$$\dot{\boldsymbol{x}}=\begin{bmatrix}0 & \boldsymbol{I}\\ -\boldsymbol{K}_{\mathrm{p}} & -\boldsymbol{K}_{\mathrm{d}}\end{bmatrix}\boldsymbol{x}+\begin{bmatrix}0\\ \boldsymbol{I}\end{bmatrix}\boldsymbol{\Phi}\tilde{\boldsymbol{\eta}} \triangleq \boldsymbol{H}\boldsymbol{x}+\boldsymbol{D}\boldsymbol{\Phi}\tilde{\boldsymbol{\eta}} \qquad (15)$$

In view of the positive definite matrices $\boldsymbol{K}_{\mathrm{d}}$ and $\boldsymbol{K}_{\mathrm{p}}$ $\boldsymbol{H}$ is Hurwitz, and there exists a positive definite matrix $\boldsymbol{Q}$ such that

$$\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}+\boldsymbol{R}\boldsymbol{H}=-\boldsymbol{Q}$$

Which has an unique positive definite matrix solution $\boldsymbol{R}$, Choose parameter estimator

$$\dot{\hat{\boldsymbol{\eta}}}=\boldsymbol{\Gamma}^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{x} \quad (\boldsymbol{\Gamma}>0) \qquad (16)$$

The parameter estimator (16) and control law (11) constitute the adaptive feedback linearization trajectory tracking control design of the airship.

B. Stability Proof

Since the unmanned airship operates in the vicinity of the cruise altitude 20km, inertial parameters $\boldsymbol{\eta}$ can be regarded as a constant vector, and the parameter estimation law (16) can be written as

$$\dot{\tilde{\boldsymbol{\eta}}}=-\boldsymbol{\Gamma}^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{x} \qquad (17)$$

The state vector is defined as

$$\begin{bmatrix}\boldsymbol{x}\\ \tilde{\boldsymbol{\eta}}\end{bmatrix}=\begin{bmatrix}\boldsymbol{e}\\ \dot{\boldsymbol{e}}\\ \tilde{\boldsymbol{\eta}}\end{bmatrix}$$

Select the candidate Lyapunov function as

$$L(t)=\boldsymbol{x}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{x}+\tilde{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{\eta}}$$

The derivative of trajectory can be calculated along the closed-loop system (15) and (17):

$$\begin{aligned}\dot{L}(t)&=\dot{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{x}+\boldsymbol{x}^{\mathrm{T}}\boldsymbol{R}\dot{\boldsymbol{x}}+\dot{\tilde{\boldsymbol{\eta}}}^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{\eta}}+\tilde{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{\Gamma}\dot{\tilde{\boldsymbol{\eta}}}\\ &=\boldsymbol{x}^{\mathrm{T}}\left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}+\boldsymbol{R}\boldsymbol{H}\right)\boldsymbol{x}+2\tilde{\boldsymbol{\eta}}^{\mathrm{T}}\left[\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{x}+\boldsymbol{\Gamma}\dot{\tilde{\boldsymbol{\eta}}}\right] \quad (18)\\ &=-\boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x}\le 0\end{aligned}$$

Therefore, the state of closed-loop system $[\boldsymbol{x}^{\mathrm{T}},\tilde{\boldsymbol{\eta}}^{\mathrm{T}}]^{\mathrm{T}}$ is bounded.

From the definition of the Lyapunov function $L(t)$ and (18), one has

$$0\le L(\infty)\le L(t)\le L(t_0)<\infty, \quad \forall t\ge t_0\ge 0$$

and

$$0\le\lambda_m(\boldsymbol{R})\|\boldsymbol{x}\|_2^2+\lambda_m(\boldsymbol{\Gamma})\|\tilde{\boldsymbol{\eta}}\|_2^2\le L(t)\le L(t_0)<\infty$$

$\boldsymbol{x}$, $\tilde{\boldsymbol{\eta}}$ are bounded, and from (18) we get

$$\dot{L}(t)\le-\lambda_m(\boldsymbol{Q})\|\boldsymbol{x}\|_2^2$$

Thus,

$$\begin{aligned}\int_{t_0}^{\infty}\|\boldsymbol{x}\|_2^2&\le-\frac{1}{\lambda_m(\boldsymbol{Q})}\int_{t_0}^{\infty}\dot{L}(t)dt=\frac{1}{\lambda_m(\boldsymbol{Q})}\left[L(t_0)-L(\infty)\right]\\ &\le\frac{1}{\lambda_m(\boldsymbol{Q})}L(t_0)<\infty,\forall t\ge t_0\ge 0\end{aligned}$$

so $\boldsymbol{x}$ is square integrable.

Because $\boldsymbol{x}$, $\tilde{\boldsymbol{\eta}}$, $\boldsymbol{\mu}_{\mathrm{d}}$, $\dot{\boldsymbol{\mu}}_{\mathrm{d}}$, $\ddot{\boldsymbol{\mu}}_{\mathrm{d}}$, $\boldsymbol{\eta}$ are bounded, from control law expression (11) $\boldsymbol{\mu}$, $\dot{\boldsymbol{\mu}}$, $\boldsymbol{\tau}$ are bounded, The boundedness of $\ddot{\boldsymbol{\mu}}$ yields the boundedness of

$$\dot{\boldsymbol{x}}=\begin{bmatrix}\dot{\boldsymbol{e}}\\ \ddot{\boldsymbol{e}}\end{bmatrix}=\begin{bmatrix}\dot{\boldsymbol{\mu}}_{\mathrm{d}}-\dot{\boldsymbol{\mu}}\\ \ddot{\boldsymbol{\mu}}_{\mathrm{d}}-\ddot{\boldsymbol{\mu}}\end{bmatrix}$$

Since $\boldsymbol{x}$ is square integrable and $\dot{\boldsymbol{x}}$ is bounded, $\boldsymbol{x}$ is uniformly continuous. Barbalat lemma guarantees $\boldsymbol{x}\to 0(t\to\infty)$, i.e. $\boldsymbol{e}\to 0,\dot{\boldsymbol{e}}\to 0$, $\boldsymbol{\mu}\to\boldsymbol{\mu}_{\mathrm{d}}$, $\dot{\boldsymbol{\mu}}\to\dot{\boldsymbol{\mu}}_{\mathrm{d}}$.

V. SIMULATION

A. Parameter Values

The elements of matrix $\boldsymbol{Y}(\boldsymbol{\mu},\dot{\boldsymbol{\mu}},\ddot{\boldsymbol{\mu}})_{6\times 5}$ can be obtained from (13) , where

$$y_{11}=\ddot{\mu}_3-\ddot{\mu}_2 s\mu_1-\dot{\mu}_1\dot{\mu}_2 c\mu_1,$$

$$y_{12}=\left(\dot{\mu}_1 c\mu_3+\dot{\mu}_2 c\mu_1 s\mu_3\right)\left(\dot{\mu}_1 s\mu_3-\dot{\mu}_2 c\mu_1 c\mu_3\right),$$

$$y_{14}=\ddot{\mu}_1 s\mu_3-\ddot{\mu}_2 c\mu_1 c\mu_3+\dot{\mu}_2^2 s\mu_1 c\mu_1 s\mu_3+2\dot{\mu}_1\dot{\mu}_2 s\mu_1 c\mu_3,$$

$$\begin{aligned}y_{15}=&\ gc\mu_1 s\mu_3+\ddot{\mu}_4\left(s\mu_2 c\mu_3-s\mu_1 c\mu_2 s\mu_3\right)-\\ &\ddot{\mu}_5\left(c\mu_2 c\mu_3+s\mu_1 s\mu_2 s\mu_3\right)-\ddot{\mu}_6 c\mu_1 s\mu_3,\end{aligned}$$

$$\begin{aligned}y_{21}=&-\dot{\mu}_1\dot{\mu}_3 s\mu_3+\dot{\mu}_2\dot{\mu}_3 c\mu_1 c\mu_3+\\ &\dot{\mu}_1\dot{\mu}_2 s\mu_1 s\mu_3-\dot{\mu}_2^2 s\mu_1 c\mu_1 c\mu_3,\end{aligned}$$

$$\begin{aligned}y_{22}=&\ \dot{\mu}_2\left(\dot{\mu}_3 c\mu_1 c\mu_3-\dot{\mu}_1 s\mu_1 s\mu_3\right)+\ddot{\mu}_1 c\mu_3-\\ &\dot{\mu}_1\dot{\mu}_3 s\mu_3+\ddot{\mu}_2 c\mu_1 s\mu_3,\end{aligned}$$

$$\begin{aligned}y_{23}=&\ \dot{\mu}_1\dot{\mu}_3 s\mu_3-\dot{\mu}_2\dot{\mu}_3 c\mu_1 c\mu_3-\\ &\dot{\mu}_1\dot{\mu}_2 s\mu_1 s\mu_3+\dot{\mu}_2^2 s\mu_1\mu_1 c\mu_3,\end{aligned}$$

$$\begin{aligned}y_{24}=&-\dot{\mu}_1^2+\dot{\mu}_2^2+\dot{\mu}_3^2-\left(\dot{\mu}_2 c\mu_1\right)^2+\left(\dot{\mu}_1 c\mu_3\right)^2-\\ &2\dot{\mu}_2\dot{\mu}_3 s\mu_1-\left(\dot{\mu}_2 c\mu_1 c\mu_3\right)^2+2\dot{\mu}_1\dot{\mu}_2 c\mu_1 c\mu_3 s\mu_3,\end{aligned}$$

$$y_{25}=gs\mu_1+\ddot{\mu}_4 c\mu_1 c\mu_2+\ddot{\mu}_5 c\mu_1 s\mu_2-\ddot{\mu}_6 s\mu_1,$$

$$y_{31}=\left(\dot{\mu}_2 s\mu_1-\dot{\mu}_3\right)\left(\dot{\mu}_1 c\mu_3+\dot{\mu}_2 c\mu_1 s\mu_3\right),$$

$$y_{32}=\left(\dot{\mu}_3-\dot{\mu}_2 s\mu_1\right)\left(\dot{\mu}_1 c\mu_3+\dot{\mu}_2 c\mu_1 s\mu_3\right),$$

$$y_{33}=\ddot{\mu}_2 c\mu_1 c\mu_3-\dot{\mu}_2\left(\dot{\mu}_1 s\mu_1 c\mu_3+\dot{\mu}_3 c\mu_1 s\mu_3\right)-$$
$$\ddot{\mu}_1 s\mu_3-\dot{\mu}_1\dot{\mu}_3 c\mu_3,$$

$$y_{34}=-\ddot{\mu}_3+\ddot{\mu}_2 s\mu_1+\dot{\mu}_1\dot{\mu}_2 c\mu_1-$$
$$\left(\dot{\mu}_1 c\mu_3+\dot{\mu}_2 c\mu_1 s\mu_3\right)\left(\dot{\mu}_1 s\mu_3-\dot{\mu}_2 c\mu_1 c\mu_3\right),$$

$$y_{35}=0,$$

$$y_{41}=y_{42}=y_{43}=y_{44}=0,$$

$$y_{45}=-\dot{\mu}_2^2 s\mu_1 c\mu_1 c\mu_3+2\dot{\mu}_2\dot{\mu}_3 c\mu_1 c\mu_3+$$
$$\ddot{\mu}_1 c\gamma_3-2\dot{\mu}_1\dot{\mu}_3 s\mu_3+\ddot{\mu}_2 c\mu_1 s\mu_3,$$

$$y_{51}=y_{52}=y_{53}=y_{54}=0,$$

$$y_{55}=\dot{\mu}_1\dot{\mu}_2 c\mu_1\left(c\mu_3\right)^2-\dot{\mu}_1^2 c\mu_3 s\mu_3-\dot{\mu}_1\dot{\mu}_2 c\mu_1\left(s\mu_3\right)^2+$$
$$\dot{\mu}_2\dot{\mu}_1 c\mu_1+\dot{\mu}_2^2\left(c\mu_1\right)^2 c\mu_3 s\mu_3+\ddot{\mu}_2 s\mu_1-\ddot{\mu}_3,$$

$$y_{61}=y_{62}=y_{63}=y_{64}=0,$$

$$y_{65}=-\left(\dot{\mu}_3-\dot{\mu}_2 s\mu_1\right)^2-\left(\dot{\mu}_1 c\mu_3+\dot{\mu}_2 c\mu_1 s\mu_3\right)^2.$$

In order to validate the feedback linearization adaptive control algorithm, the airship simulation model parameters[7] are shown in table I and table II. The parameter values in table II are dimensionless.

TABLE I.

PARAMETER VALUES OF THE MODEL

Parameter	Value	Unit	Parameter	Value	Unit
m	55749.7	kg	ρ	0.072157	kg/m^3
D	736311	m^3	I_x	5x10^7	kg·m^2
I_y	2.9x10^8	kg·m^2	I_z	2.9x10^8	kg·m^2
I_{xz}	-6x10^4	kg·m^2	z_c	15	m

TABLE II.

PARAMETER VALUES OF THE MODEL

Param-eter	Value	Param-eter	Value	Param-eter	Value
k_1	0.1054	k_2	0.8259	k_3	1773.2
C_{X1}	227.8	C_{X2}	2307.1	C_{L1}	24059
C_{L2}	8080	C_{Y1}	2307.1	C_{Y2}	3037.6
C_{Y3}	9932.7	C_{Y4}	657.3	C_{Z1}	2307.1
C_{Z2}	3037.6	C_{Z3}	9730.7	C_{Z4}	657.3
C_{M1}	384515.9	C_{M2}	356916.5	C_{M3}	373391
C_{M4}	77238.5	C_{N1}	384515.9	C_{N2}	356916
C_{N3}	373391	C_{N4}	77238.5		

B. Simulation

A helix is chosen as the desired trajectory to be followed and expressed as

$$\begin{cases} x_{\mathrm{d}}=500\sin\left(0.01t\right) \\ y_{\mathrm{d}}=500\cos\left(0.01t\right) \\ z_{\mathrm{d}}=-20000-0.01t \end{cases}$$

Referred to [10], the desired attitude can be calculated through the airship dynamic model and the corresponding Frenet –based kinematic description:

$$\begin{cases} \theta_{\mathrm{d}}=0.1974\mathrm{rad} \\ \psi_{\mathrm{d}}=\arctan 2\left(-\sin\left(0.01t\right),\cos\left(0.01t\right)\right)\mathrm{rad} \\ \phi_{\mathrm{d}}=0\mathrm{rad} \end{cases}$$

The initial condition and control parameters in simulation are summarized as follows:

$$\boldsymbol{\mu}_0=\left[0,0,0,0,505,-20150\right]^{\mathrm{T}},\ \hat{I}_{x0}=1.5\times10^7\,\mathrm{kg\cdot m^2},$$

$$\hat{I}_{y0}=\hat{I}_{z0}=1\times10^8\,\mathrm{kg\cdot m^2},\ \hat{I}_{xz0}=-4\times10^4\,\mathrm{kg\cdot m^2},$$

$$m\hat{z}_{c0}=4.4\times10^5\,\mathrm{kg\cdot m},\ B_f=mg,\ \boldsymbol{K}_{\mathrm{d}}=2\boldsymbol{I}_{6\times6},$$

$$\boldsymbol{K}_{\mathrm{p}}=\boldsymbol{I}_{6\times6},\ \boldsymbol{\Gamma}=\mathrm{diag}(10^{-4},10^{-7},10^{-7},5\times10^{-4},10^{-4}),$$

$$\boldsymbol{Q}=\boldsymbol{I}_{12\times12}.$$

It is noted that the solution matrix $\boldsymbol{R}$ can be sovled by the Lyapunov equation, and control law expressions are obtained by (16) and (11) , and the simulation result can be therefore performed.

Without considering wind disturbance the system cruise simulation results are shown in Fig. 2 ~ Fig. 4. The trajectory tracking errors are asymptotically stable, as depicted in Fig. 2. The attitudes track the desired ones quickly and accurately as depicted in Fig. 3. As shown in Fig. 4, all of the inertial parameters estimations are stabile, although they do not converge to the true values, which is consistent with the theoretic analysis. It is found from the values of the longitudinal coordinate in Fig. 4 that the inertia parameters are very large so that the variation of the estimated values is quite subtle.

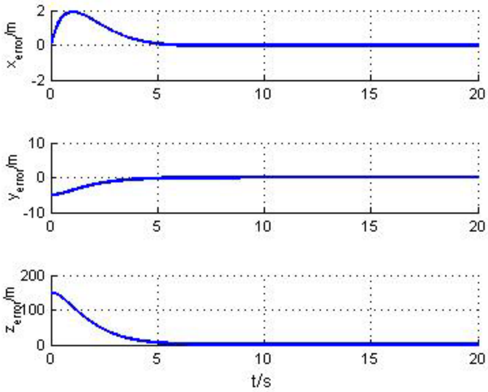

Figure 2. Trajectory tracking errors

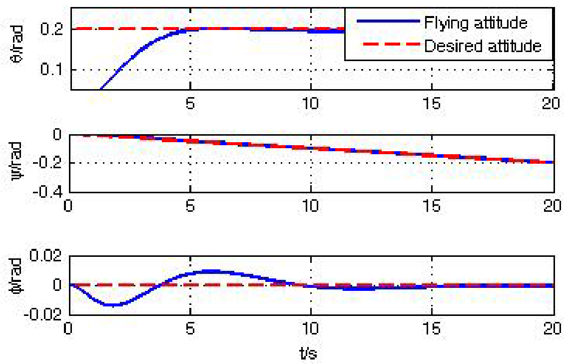

Figure 3. Euler attitudes

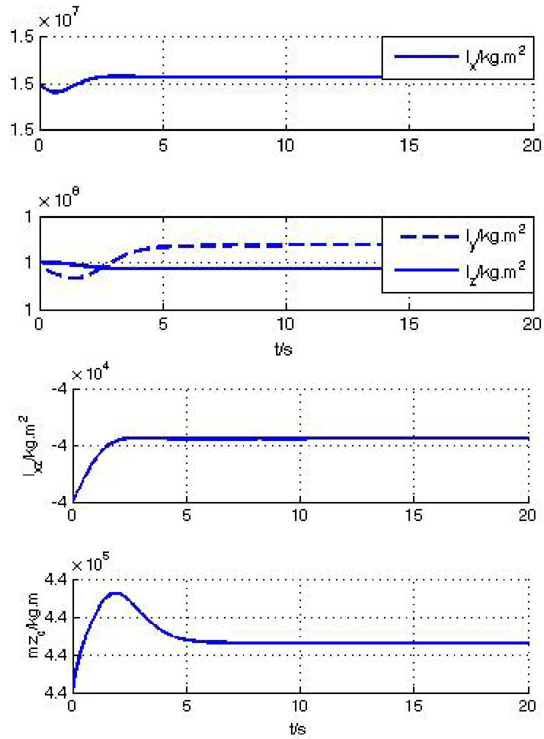

Figure 4. Estimated inertia values

VI. Conclusion

An adaptive feedback linearization control method is proposed, with online inertial parameter compensation, which renders asymptotic tracking of any given continuous time-varying trajectory from any initial conditions. The closed-loop stability is proved, although the estimated values of the inertial parameters don't converge to their true values. The simulation result is coincident with the theoretical analysis. The method is only suitable for a full-actuated airship with the inertial parameters imprecisely known. The trajectory tracking control method for the under-actuated airship [12] [13] under the control constraint [14], as well as the actuator saturation should be studied further.

Acknowledgment

The authors would like to thank Professor Zhe WU of Beihang University. This work was supported in part under a grant from Advanced Aircraft Design Center of Beihang University.

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