Uncertainty Time Series’ Multi-Scale Fractional-Order Association Model

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***Abstract*—This article first systematically classified the uncertainty and provided the multi-scale fractional ordered association model in accordance with the multiple uncertainty time series. From the mathematical point of view, the model used in this thesis extended the integer-order correlation measurement to the fractional-order correlation measurement; elongate the information recognition from point to line, and rolled out the non-process identification to the process identification from the identification point of view. Introduced the multi-scale interaction identification method through the imitation of human beings’ process identification, and achieved the accurate identification form coarse to fine. Example shows that, fractional-order association algorithm can provide much more related information comparing with the integer-order one; the import of the multi-scale interactive iteration greatly enhanced the intelligent of the model and the correlative accuracy.**

***Index Terms*—fractional order, multi-scale, uncertainty, time series, association.**

I. Introduction

A. The Discussion of Uncertainty

Things will be influenced by varied factors from different aspects during the processes of the occurrence, development and the evolution, which made them always in unstable, fuzzy, chaotic states, and we call these the uncertainty of things. It can be interpreted as uncertainty, variability, randomness, contingency or inaccuracy. Uncertainty is the objective inherent nature of things that exists in a variety of social phenomena, natural phenomena and engineering practice. From the logical level, the author divided the uncertainty into the following four categories by their nature and varied generating mode.

1. There are no local laws but only the statistical law, which called the pure uncertainty. E.g. Random information is of pure uncertainty.

2. The uncertainty caused by people’s limited cognitive ability to the objective world is called as the semi-uncertainty. E.g.:grey information is the semi-uncertainty.

3. Description uncertainty means the uncertainty that generated while people interpreting issues. E.g.: the generation of fuzzy information and error information is roots from this kind of uncertainty.

4. The ones that simultaneously possessed with two or three kinds of characteristics of the abovementioned uncertainty are called multiple uncertainties.

B. Multiple Uncertainty Time Series

Time series are a set of chronological observations of a certain indicator. The partial understanding of things objective factors and subjective factors leads the series put up the semi-uncertainty; the fuzzy membership of the description information and the import of the error message also caused series of the description uncertainty. The correlation analysis of the multiple uncertainty time series processed with the semi-uncertainty and description uncertainty can help us to overcome the limitation of the cognitive ability and ultimately find out the internal relations of different objectives. However, small sample size and the lack of information are main factors that restricted our understanding of things. For this reason, this paper analyzed the association model mainly in allusion to the multiple uncertainty time series of small samples and poor information.

II. Overview

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A. Arithmetic Summary of the Serial Correlation

As for the calculation of the multiple uncertainty time series processed with the semi-uncertainty and description uncertainty, the much better solution is the gray relational algorithm[1]. In recent years, gray correlation algorithm obtained a significant development, and many scholars have made great contributions[2-6]. From the relational degree itself, it experienced from the gray relational algorithm of no differential measurement information (such as Tang's correlation, the absolute correlation Ⅱ, the relative correlation, correlation interval Ⅰ, range correlation Ⅱ) to gray relational algorithm with first-order differential metrical information (such as absolute relational degree Ⅰ, slope correlation, and T-type correlation), and then turned to be the gray relational algorithm with second-order differential metrical information (B-type correlation, C-type correlation).

The abovementioned indicates that the introduction of the high order information and the fractional information into the associated metrics of the multiple uncertainty time series is the development trend of related algorithms.

B. Summary of Fractional Differential

Fractional calculus refers to the calculus with order of any real number order. For more than three centuries, many famous scientists did a lot of basic work on fractional calculus; however, fractional calculus really began to grow till the last 30 years. Oldham and Spanier discussed mathematical calculations of the fractional number and their application in areas like physics, engineering, finance, biology, etc. In 1993, Samko made systematic and comprehensive exposition on fractional integral and derivative related properties and their applications. Many researchers have found that, fractional derivative model can more accurately describe the nature of a memory and the genetic material and the distribution process than integer order derivative model[7-9]. The overall and memory characteristic of fractional are widely used in physics, chemistry, materials, fractal theory, image processing[10] and other fields. Currently, the analysis of fractional differential has become a new active researched area that aroused great attention of domestic and foreign scholars, and turned to be the world's leading edge and hot research field.

III. ALGORITHM PRINCIPLE

A. The Import of Fractional Order

Differential operations can enhance the high-frequency and weaken low-frequency of the signals. Fractional differential operation can nonlinearly improve more high-frequency and weaken less low-frequency of the signals with the growth of the order. From the perspective of the information extraction, the order of integer order operations is discrete whereas fractional one is continuous, and can provide more sequence information to help the identification of the sequence.

Each observed value on the time series is the common result of variety of subjective and objective factors and the development of all previous observations; therefore time series is of overall and memory characteristics. Fractional differential operator is intended differential operator with overall and memory characteristics whereas integral order doesn't have this feature. Therefore, from the description of the time series it can conclude that, fractional differential could more accurately describe the memory nature of time series comparing with the integral order one, and was imported to calculate the relevancy of time series[11,12].

B. The Introduction of Multi-scale Interactive Recognition

The essence of series association is recognition. And the development direction of recognition is the simulation of human intelligence that processed coarse-to-fine. Firstly, preliminarily extract identifying information, and then further extract identifying information through the analysis of the recognition results. Human being's recognition is a step-by-step process of identifying objects by extracting identifying information with teleology and more details.

In order to simulate the process of human identification, make the algorithm more intelligent, and to have higher accuracy, this algorithm model introduced the multi-scale. Firstly, roughly recognize on a larger scale and according to the recognition results to select the next identified scale. If all time series are not related, it should increase the scale and continue to identify. If all time series are related, it should narrow the scale and continue to identify. If parts of the time series are related, it should delete the non-associated sequence, then narrow the scale, do precise identification till achieve relevant recognition accuracy.

C. The Nature of Fractional Differential

Fractional differential operator can meet the exchange rate and the overlay standard $D^{v_1}D^{v_2}s(t)=D^{v_2}D^{v_1}s(t)=D^{v_1+v_2}s(t)$. (0, 1) differential order measures the overall situation of the sequence, other differential order results can all be acquired through the iterate integer-order differential on it. First-order differential reflects the slope of the sequence, second-order differential reflects the curvature of the sequence, and they all response to the partial trends of the sequence. To give consideration to the measurements of both global and local trends, non-integral order emphasis on (0, 1) order, integer order taking into account of the first, second and third order, therefor, this paper is only analysis the (0, 4)-order differential related information.

D. Fractional Differential Difference Form

Since time series are discrete, when using the fractional differentials in its associate calculation, the definition pattern of fractional differential algorithms must be change into the difference form. Then, we derive the fractional differential difference formula via Grümwald-Letnikov definition .

Known, v order fractional differential Grümwald-Letnikov definition is

$$ {}_a^G D_t^v s(t) = \lim_{h \to 0} s_t^v(t) $$

$$ = \lim_{\substack{h \to 0 \\ nh \to t-a}} h^{-v} \sum_{r=0}^{n} C_r^{-v} s(t - rh) \quad (1) $$

Where in:

$$ C_r^{-v} = \frac{(-v)(-v+1)...(-v+r-1)}{r!} $$

According to Expression(1), if the persistent period of s(t) is: $t \in [a,t]$, divide [a ,t] into equal parts corresponding to one unit interval h=1, it can be got that:

$$ n = \left[\frac{t-a}{h} \right]^{h=1} = [t-a] $$

Then, v order fractional differential difference expression to unitary signal s(t) can be get:

$$ \frac{d^v s(t)}{dt^v} \approx s(t) + (-v)s(t-1) + \frac{(-v)(-v+1)}{2} s(t-2) + \frac{(-v)(-v+1)(-v+2)}{6} s(t-3) + ... + \frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)} s(t-n) $$

From this differential expression, the difference coefficient of the fractional is:

$$ a_0 = 1, a_1 = -v, a_2 = \frac{(-v)(-v+1)}{2}, $$

$$ a_3 = \frac{(-v)(-v+1)(-v+2)}{6}, ..., $$

$$ a_n = \frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)} \quad (2) $$

IV. ALGORITHM MODEL

Firstly, preprocessing sequences to generate a sequence of a certain measurement scale, and then select the next measure scale in accordance with the analysis of the correlation values, repeat this until you reach the required accuracy.

Assume t sequences: $s_{11}, s_{12}, ..., s_{1m}$, $s_{21}, s_{22}, ..., s_{2m}$, ... , $s_{t1}, s_{t2}, ..., s_{tm}$, where, $s_{11}, s_{12}, ..., s_{1m}$ are the reference sequence.

(1) Pretreatment

Pretreatment Method One: initial value treatment, all the data of series are divided by its first number. The new series shows that, comparing with the multiple of the first value, values at different times in the original series have a common starting point, the new series is of non-dimensional and all data values are greater than 0.

To t sequences: $s_{11}, s_{12}, ..., s_{1m}$, $s_{21}, s_{22}, ..., s_{2m}$, ... , $s_{t1}, s_{t2}, ..., s_{tm}$,the initial value treatment:

$$ x_{ij} = \frac{s_{ij}}{s_{1j}}; i = 1,2,...,t; j = 1,2,...,m \quad (3) $$

Pretreatment Method Two: equalization treatment, all the data of a series are divided by its average value. The new series shows the multiple of the average value of the values at different times in the original series, the new series is of non-dimensional and all data values are greater than 0.

the equalization treatment:

$$ x_{ij} = \frac{s_{ij}}{\frac{1}{m}\sum_{j=1}^{m} s_{ij}}; i = 1,2,...,t; j = 1,2,...,m $$

Pretreatment Method Three: normalization treatment that mainly aimed at the cases that has significant differences between series. The new series is of non-dimensional and all data values are in [0, 1] interval.

The normalization treatment

$$ x_{ij} = \frac{s_{ij} - \min_j s_{ij}}{\max_j s_{ij} - \min_j s_{ij}}; i = 1,2,...,t; j = 1,2,...,m $$

Then get $x_{11}, x_{12}, ..., x_{1m}$, $x_{21}, x_{22}, ..., x_{2m}$, ... , $x_{t1}, x_{t2}, ..., x_{tm}$ 。

Pretreatment methods used in accordance with the actual situation.

(2) In allusion to measurement scale l, then generate t l scale sequence

$x_{11l}, x_{12l}, ..., x_{1nl}$, $x_{1il} = \frac{1}{l}\sum_{j=(i-1)l}^{il} x_{1j}, i = 1,2,...,n$

$x_{21l}, x_{22l}, ..., x_{2nl}$, $x_{2il} = \frac{1}{l}\sum_{j=(i-1)l}^{il} x_{2j}, i = 1,2,...,n$

... ,

$x_{t1l}, x_{t2l}, ..., x_{tnl}$, $x_{til} = \frac{1}{l}\sum_{j=(i-1)l}^{il} x_{tj}, i = 1,2,...,n$

, where $n = \left[\frac{m}{l} \right]$

(3) Calculate the correlation

Assume the degree of association between x_{1l} and x_{jl} under the scale l is

$$ y_{1j} = \frac{1}{n-3}\sum_{i=4}^{n} (x^v{}_{1il} - x^v{}_{jil})^2; $$

$$ r = 1,2,..., p; j = 2,...,t; v \in (0,4) $$

where

$$ x^v{}_{1il} = \sum_{d=1}^{1i} x_{(1i-d+1)} * a_d, x^v{}_{jil} = \sum_{d=1}^{ji} x_{(1j-d+1)} * a_d $$

(4) Interactive iteration

Setting the threshold value of correlation is u_0, let $\int_0^4 y_{1j} dv = u, j = 2,...,t$, if $u \le u_0$, x_{jl} is relevant with x_{1l}; if $u > u_0$, sequence j is not relevant with reference sequence. Determine the next scale according to u.

1. $\max(u) < u_0$,Reduced scale and continue operations.

2. $min(u) \ge u_0$, Increase the scale and continue operations.

3. In addition, delete unrelated sequence, reduced scale and continue operations.

And so forth until you reach the required accuracy.

V ALGORITHM INSTANCE.

Appendix is the list of total amounts of major industrial products from 1978 to 2002, through the calculation of the interaction value between cloth production (x_1) and the production of paper (x_2), yarn (x_3), raw coal (x_4), crude oil (x_5), electricity (x_6), crude steel (x_7), and cement (x_8), identify the main factors that affect the production of cloth. Hereinafter separately use m_2, m_3, m_4, m_5, m_6, m_7, m_8 to represent the production interaction curve of paper, yarn, raw coal, crude oil, electricity, crude steel, cement to cloth.

Firstly, acquired $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8$ through the initial value treatment, and did relational calculus on the generated sequence with the scale of 4, the result is shown in Figure 1.

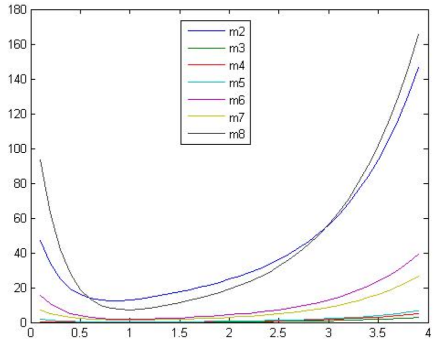

Figure 1. m2,m3,m4,m5,m6,m7,m8 represent the production interaction curve of paper, yarn, raw coal, crude oil, electricity, crude steel, cement to cloth with the scale of 4.

It can be seen from Figure 1 that, if $u_0 = 20$, y_3, y_4, y_5, y_6, y_7 is relevant with y_1, and y_2, y_8 is not relevant with y_1 while the scale is 4.

Removal of m_2 and m_8 that with the larger correlation values, and continue to calculate correlation of the generated scale 2 sequence based on $y_1, y_3, y_4, y_5, y_6, y_7$, the result is shown in Figure 2.

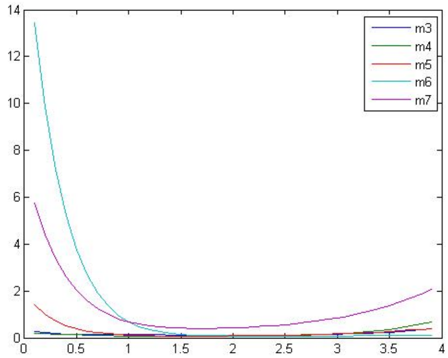

Figure 2. m2,m3,m4,m5,m6,m7,m8 represent the production interaction curve of paper, yarn, raw coal, crude oil, electricity, crude steel, cement to cloth with the scale of 2.

Figure 2 indicts that, y_6 within (0, 1) has larger associated value comparing with other sequences. However, it decreased rapidly within (2, 4) and is lower than others. Comparing with high order differential, low order differential extract more of the low-frequency information and less high-frequency information. As for the time series, low order differential extract more long-term-effect information while high order differential extract more short-term-effect information. The change of the above mentioned m_6 associated curve indicts that changes in electricity production will not have significant impact for the cloth production from a long run, however, short-term output of plant changes can significantly affect cloth production.

As can be seen from Figure 2, if $u_0 = 1$, y_6 is high-order associated with y_1 under scale 2 ,while y_7 is not associated with y_1, y_3, y_4, y_5 are associated with y_1 under scale 2. Remove y_6, y_7, continue to calculate correlation in allusion to y_1, y_3, y_4, y_5, and obtain the results as shown in figure 3.

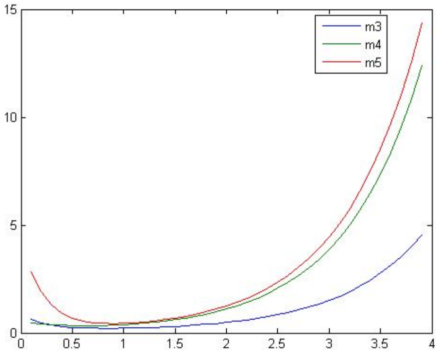

Figure 3.m2,m3,m4,m5,m6,m7,m8 represent the production interaction curve of paper, yarn, raw coal, crude oil, electricity, crude steel, cement to cloth with the scale of 1.

As can be seen from the figure3, if $u_0 = 1$, y_4, y_5 is not associated with y_1 under scale 1, while y_3 is associated with y_1. Then we can conclude that the output of yarn has the maximum correlation with the output of cloth.

We can get the following conclusion according to figure 1, 2, and 3: the output of cloth is associated with the output of yarn for their technique similarity; in a short term, the output of cloth is high-order associated with output of plant, because the production of cloth is effected by the power limitation, therefore, resolve the issue of power shortage in a short time should be the focus of the national development.

VI DISCUSSIONS

From the mathematical point of view, the introduction of fractional order extends the integer-order correlation measurement to the fractional correlation measurement. The fractional correlation associated measurement model elongates the recognition from point to line, acquired more related information, and provides richer support for the interaction decision-making.

From the identification point of view, it extends the non-process identification to the process identification through the simulation of human beings' identification process; the introduction of multi-scale interaction identification method realized the accurate identification coarse-to-fine.

VII. CONCLUSIONS

From the mathematical point of view, the model used in this thesis extended the integer-order correlation measurement to the fractional-order correlation measurement; elongate the information recognition from point to line, and rolled out the non-process identification to the process identification from the identification point of view. Introduced the multi-scale interaction identification method through the imitation of human beings' process identification, and achieved the accurate identification form coarse to fine. Example shows that, fractional-order association algorithm can provide much more related information; the import of the multi-scale interactive iteration greatly enhanced the intelligent of the model and the correlative accuracy.

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