

# A Study on the Control Methods Based on 3-DOF Helicopter Model

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**Abstract**—The 3 degree of freedom (3-DOF) helicopter system is a typical higher-order times, instability, multi-variable, nonlinear and strong coupling control system. This paper presents the analysis of the mathematical model and the basic principle of PID control and fuzzy PID control, which are based on 3-DOF helicopter systems. In allusion to balance control of 3-DOF helicopter system, PID controller and fuzzy PID controller are designed respectively to control the model. The MATLAB simulation results demonstrate the control effects of both controllers achieve the requirements in this system, and fuzzy self-tuning PID shows more advantages.

**Index Terms**—helicopter system, Fuzzy PID, matlab, pitching axis, lateral axis, revolving shafts

## I. INTRODUCTION

The 3 degree of freedom (3-DOF) helicopter model is a typical multi-input and multi-output system with high-order, which contains the properties of strong channel coupling and high nonlinearity. Since the motion equations of its three axes in elevation, pitch, and travel are always used to simulate the dynamic characteristics of real helicopter, 3-DOF helicopter model becomes an efficient tool for teaching and researching in control method [1], [2].

Many efforts have been demonstrated it is difficult to illustrate the dynamic characteristic of the helicopter, because of its extremely complex and particular flying state. Normally, its dynamic characteristics will be correspondingly varied with flying altitude and flying state, all of which are nonlinear and multivariable coupling. Consequently, it is probably impossible to achieve formulations of the helicopter as well as its accurate models. It seems that the helicopter models used in engineering tend to be processed simply. In this dissertation, a 3-DOF helicopter model as the object of study, so as to researching the attitude control and simplifying the complexity of research [3].

According to the dynamic trait of pitch axis, elevation axis and travel axis, the system mathematic model was established, the model offered basic for the PID control and theoretical support for the controller improvement. Finally, the fuzzy control was used to control the system. The control results of the simulation experiments proved that the using of the fuzzy control worked better than the pure PID controller. Compared with the traditional PID control, it had a remarkable improvement. Both the PID controller and the fuzzy controller had their respective advantages. Based on their excellent merits, PID and fuzzy controller were combined together as one controller which fit together with switch and weighted value. It optimizes the flying gesture of the helicopter [4], [5].

## II. SYSTEM MODELING

A 3-DOF helicopter control system introduced in this paper, is inherently unstable with MIMO, nonlinear and high degree of coupling. This system consists of some basic components, such as a pedestal, an equalizer bar, counterbalance and blades. Based on pedestal for fulcrum, the equalizer bar can be driven to pitch and turn. Counterbalance and blades are separately installed on both ends of the equalizer bar. Accordingly, the equalizer bar due to the lift force which the blades produced to pitch (based on pedestal for fulcrum). Under the similar condition, equalizer bar turns by velocity different of two blades pitching (based on pedestal for axis). In order to measure data of revolving shafts and pitch axis of equalizer bar and lateral axis of propellers, encoders are installed respectively on revolving shafts and pitching axis of equalizer bar and lateral axis of propellers. Two blades are driven by two Direct Current Brushless Motors, blades are proposed power simultaneously. Hence, models are established for three axes (degree of freedom) as the characteristics of the system [6].

### A. Pitching Axis

Structure of pitching axis as Fig.1,

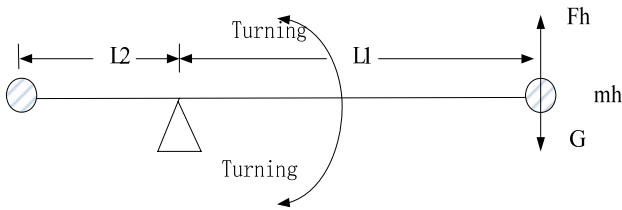


Figure 1 Structure of pitching axis

Torque of pitching axis is generated by two propeller-motors producing the direct lift F1 and F2. Helicopter increases when lift Fh is greater than gravity G; conversely it drops. Assuming that helicopter is hung in the air and the degree of pitch is zero; equations can be given as follows:

$$\begin{aligned} J_e \ddot{\epsilon} &= l_1 F_h - l_1 G = l_1 (F_1 + F_2) - l_1 G \\ J_e \ddot{\epsilon} &= K_c l_1 (V_1 + V_2) - T_g = K_c l_1 V_s - T_g \end{aligned} \quad (1)$$

In equation (1),  $J_e$  is the rotational inertia of the pitching axis;  $V_1$  and  $V_2$  are voltages of two motors;  $K_c$  is the lift constant for blades motor;  $l_1$  is the distance from fulcrum to motors;  $l_2$  is the distance from fulcrum to counterbalance;  $T_g$  is the effective gravity torque produced by gravity of pitching axis;  $m_h$  and  $m_b$  are respectively quality of blades and counterbalance of helicopters;  $\ddot{\epsilon}$  is the rotation acceleration of pitching axis.

If gravity disturbance torque is ignored, the transfer function of this part of the system as following equation:

$$\frac{\epsilon(s)}{V_s} = \frac{K_c l_1}{J_e s^2} \quad (2)$$

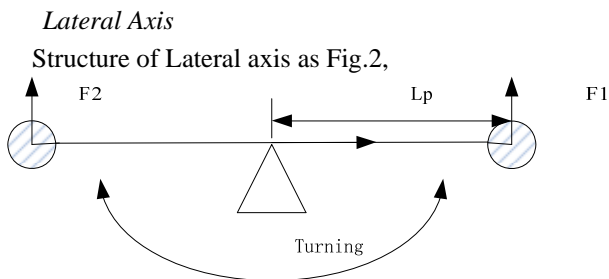


Figure 2 Structure of lateral axis

From above image, lateral axis controlled by lift produced by two blades. If the lift created by F1 is greater than that which is created by F2, blades will tilt, and then there will be a lateral force to make helicopter turn around the base, i.e.

We can get equation as follows:

$$J_p \ddot{p} = F_1 l_p - F_2 l_p = K_c l_p (V_1 - V_2) = K_c l_p V_d \quad (3)$$

In equation (3),  $J_p$  is the rotational inertia of the lateral axis, the distance from fulcrum to motor, and P the rotation acceleration of the lateral axis.

The transfer function of this part of the system can be given by formula (4) as follows:

$$\frac{p(s)}{V_d} = \frac{K_c l_p}{J_p s^2} \quad (4)$$

B. Revolving Shafts

Structure of revolving shafts as Fig.3,

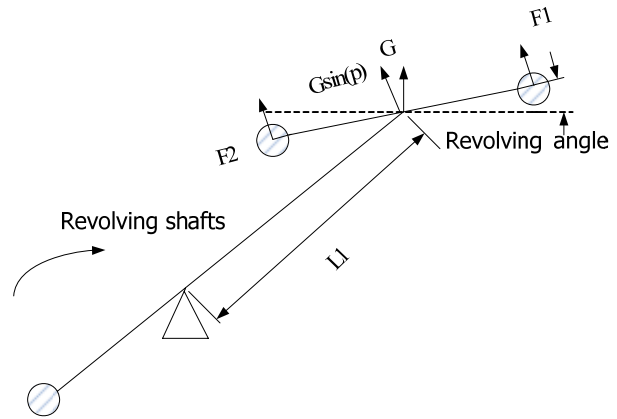


Figure 3 Structure of revolving shafts

Its dynamics equation is as follows:

$$J_r \dot{r} = -G \sin(p) l_1 \quad (5)$$

In equation (5), r is the rotating speed, rad/sec as one unit. Sin (p) is the sine value of lateral angle p. Generally the angle is small, approximately considered as  $\sin(p) \approx p$  here. The transfer function of this part of the system can be got by formula (6) as follows:

$$\frac{r(s)}{P(s)} = -\frac{G l_1}{J_t S} \quad (6)$$

III. CONTROLLER DESIGN

Helicopter control requires the ability to produce moments and forces on the vehicle to produce equilibrium and to change the helicopter's velocity, position and orientation [7]. The problem of helicopter control has received much attention and especially during the last two decade [8]-[10]. The usage of the traditional PI, PD controllers is not satisfied because the helicopter parameters are very dependent on the operating point. These controllers only work well in a very small area around that set point. Also, when dealing with multivariable systems, one of the major concerns are the cross-couplings of he system [11].

A. PID Controller Design

Structure of PID controller as Fig.4,

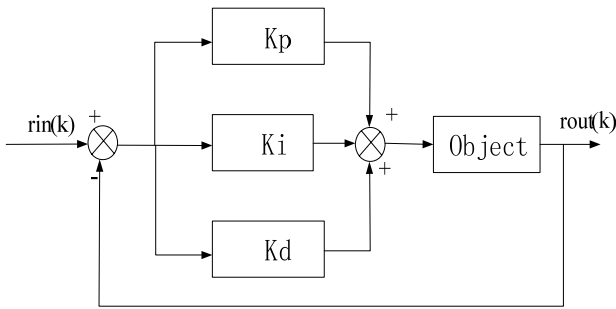


Figure 4 Structure of PID controller

PID controller is a linear controller, the control rules as follows [12]:

$$u(t) = k_p (error(t) + K_i \int_0^t error(t)dt + K_d \frac{derror(t)}{dt}) \quad (7)$$

Where, error(t) is the deviation composed by given value rin(t) and actual output value yout(t), Kp, Ki, Kd, respectively known as proportional coefficient, integral coefficient and differential coefficient. The effect of the three links of PID control is obvious:

- a) Proportional link: the deviation signal is reflected proportionally, and the controller would produce effect to reduce deviation if deviation produced;
- b) Integrating link: mainly used to eliminate static errors, and improve indiscrimination degree of the system;
- c) Differential modulus, to reflect the rates of deviation signal change, and to lead in an effective early modify signal in system before error signal becomes too large, to speed up the responsiveness of the system, and to reduce the setting time.

Then, three parameters operate convenient in practical control [13].

1) Controller of Pitching Axis

According to equation (7), a PD controller is designed as follows:

$$V_s = K_{ep} (\varepsilon - \varepsilon_c) + k_{ed} \dot{\varepsilon} \quad (8)$$

In equation (8),  $\varepsilon$  represents actual angle of pitch, and  $\varepsilon_c$  represents the pitch angle of expectation.

The transfer function of this part of the system can be given as follows:

$$\frac{\varepsilon(s)}{\varepsilon_c(s)} = \frac{-k_c k_{ep} l_1 / J_e}{s^2 - k_c k_{ep} l_1 / J_e s - k_c k_{ep} l_1 / J_e} \quad (9)$$

In equation (9), the denominator can be expressed as

follows:

$$s^2 + 2\xi\omega_0 + \omega_0^2 \quad (10)$$

Peak time as  $t_p = \frac{\pi}{\omega_0 \sqrt{1-\xi^2}}$ , by choosing expected peak time  $t_p$  and damping ratio, to determine the  $k_{ep}$  and  $k_{ed}$  to satisfy expectations response.

2) Controller of Lateral Axis

The rotation speed of the helicopter can be controlled by changing the size of slant angle of lateral axis. According to equation (7), we design a PD controller as follows,

$$V_d = K_{pp} (p - p_c) + k_{pd} \dot{p} \quad (11)$$

The transfer function of this part of the system can be got as follows,

$$\frac{p(s)}{p_c(s)} = \frac{-k_c k_{pp} l_p / J_e}{s^2 - k_c k_{pd} l_p / J_p s - k_c k_{pp} l_p / J_p} \quad (12)$$

Then, as the controller of pitching axis, we can determine the  $k_{pp}$  and  $k_{pd}$  by choosing expected peak time and damping ratio to satisfy expectations response.

3) Controller of revolving shafts

According to equation (7), we design a controller to get expected Lateral Angle,

$$P_c = K_{rp} (r - r_c) + k_{ri} \int (r - r_c) \quad (13)$$

The transfer function of this part of the system can be get as follows:

$$\frac{r(s)}{r_c(s)} = \frac{-k_{rp} G l_1 s + k_{ri} G l_1 / J_t}{s^2 - k_{rp} G l_1 / J_t s - k_{ri} G l_1 / J_t} \quad (14)$$

This controller can also be designed according to the second-order controller, by choosing expected peak time  $t_p$  and damping ratio, to determine the  $k_{rp}$  and  $k_{rd}$  to satisfy expectations response.

B. FUZZY-PID Controller Design

This fuzzy self-adaptive PID controller used the signals of the error e and the error varying ec as inputs, modifying PID parameters at any time through the fuzzy control rules, and then puts control signal into actuators to adjust controlled object [14]. The structure of FUZZY-PID controller is shown in Fig.5.

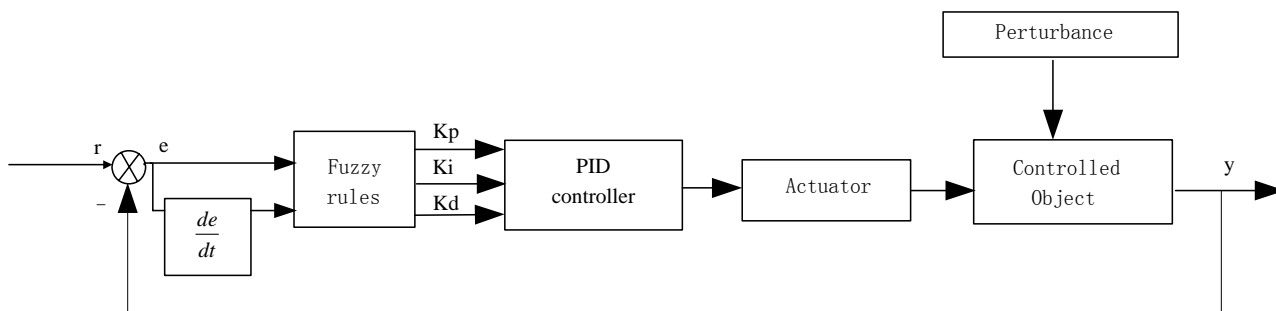


Figure 5 Structure of fuzzy self-adaptive PID controller

In view of the different error  $e$  and error rate, different PID control parameters of  $K_i$ ,  $K_d$ ,  $K_p$  are demanded, and we set rules as follows.

1) When  $|e|$  is big, for the system to have good tracking performance, we should take large  $K_p$  and small  $K_d$ ; meanwhile, in order to avoid large overshoot in system response, we should set a limit to the integral effect, usually we take  $K_i = 0$ .

2) When  $|e|$  and  $|ec|$  are median size, for the system to have a small overshoot, we should take small  $K_p$ . In this case,  $K_d$  is bigger impact on the system, we should take a small value,  $K_i$  should be proper.

3) When  $|e|$  is small, for the system to have a good stability, we should take big  $K_i$  and  $K_d$ ; meanwhile, in order to avoid appearing oscillation within the set value, and considering anti-interference performance of the system, when  $|ec|$  is large,  $K_d$  should be taken a smaller value, and vice versa[15].

Three-dimensional diagram of rules of  $K_p, K_i, K_d$  as Fig. 6, Fig. 7 and Fig. 8.

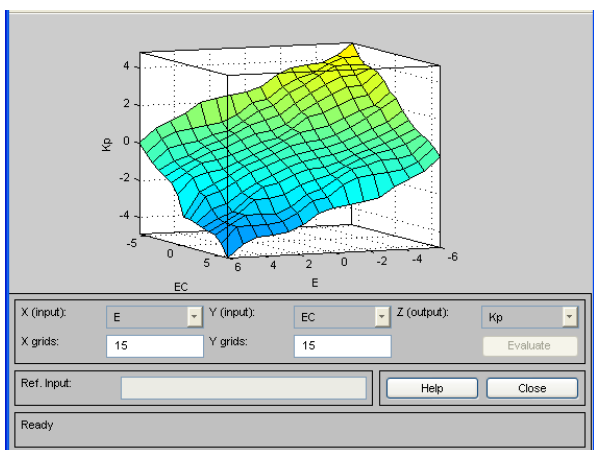


Figure 6 Three-dimensional diagram of rules of  $K_p$

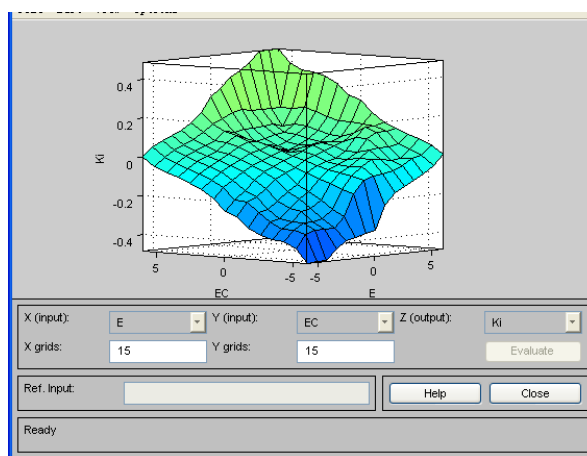


Figure 7 Three-dimensional diagram of rules of  $K_i$

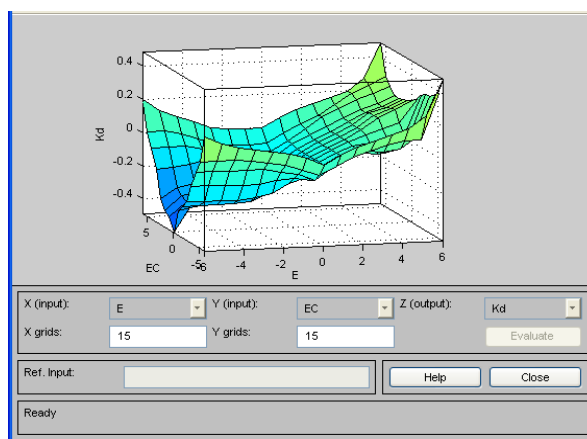


Figure 8 Three-dimensional diagram of rules of  $K_d$

The fuzzy self-adaptive PID controller in this paper, all of the linguistic values of inputs and outputs are divided into seven linguistic values: NB, NM, NS, O, PS, PM and PB.  $K_i$ ,  $K_d$ ,  $K_p$  parameters of the controller process on-line self-tuning, its structure consists of two parts, the routine PID controller section and fuzzy reasoning calibration parameters section. In on-line operation process, control system does online self-adjusting and fuzzy self-adjusting for  $K_i$ ,  $K_d$ ,  $K_p$ , through processing, look-up table and calculation about results of fuzzy logic rules. Computational formula as equation (15),

$$\begin{cases} Kp = Kp^* + \{e_i, e_{c_j}\}q_p \\ Ki = Ki^* + \{e_i, e_{c_j}\}q_i \\ Kd = Kd^* + \{e_i, e_{c_j}\}q_d \end{cases} \quad (15)$$

In equation (15),  $Kp$ ,  $Ki$  and  $Kd$  are initial-value of three control parameters of self-adaptive fuzzy PID controller,  $Kp, Ki, Kd$  are the adjusted parameters of PID,  $q_p, q_i$  and  $q_d$  are corrective factors of PID,  $\{e_i, e_{c_j}\}$  is error  $e$  and error rate  $ec$  corresponding to output values of the fuzzy control rules table. In process of establishing the fuzzy rule base, we should consider that input and output variables of the fuzzy controller are accurate quantity, and fuzzy reasoning is the quantity of fuzzy signal [15].

In order to adapt to more systems, membership functions of this paper selected smoothing continuous Z type membership function at negative boundary, and smoothing continuous S membership function at positive boundary, and we choose triangular type membership function which has higher delicacy in the middle part. The universe of error  $e$ , error rate  $ec$  and the output  $P$  is  $\{-6 \dots 6\}$ , and the universe of the output  $I$  and  $D$  is  $\{-0.6 \dots 0.6\}$ . Membership function diagram as Fig. 9, Fig. 10 and Fig.11.

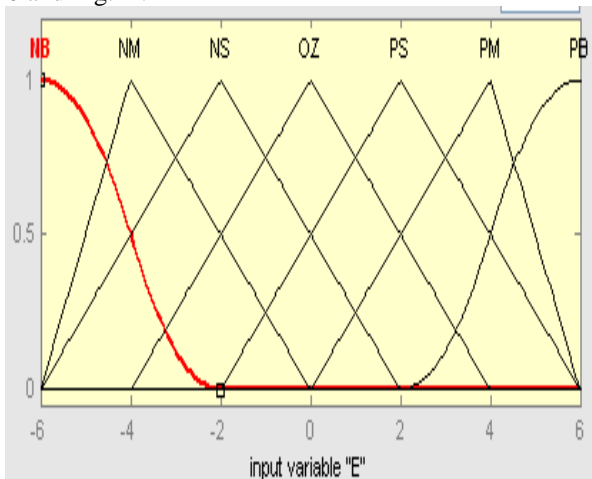


Figure 9 Membership function diagram of E

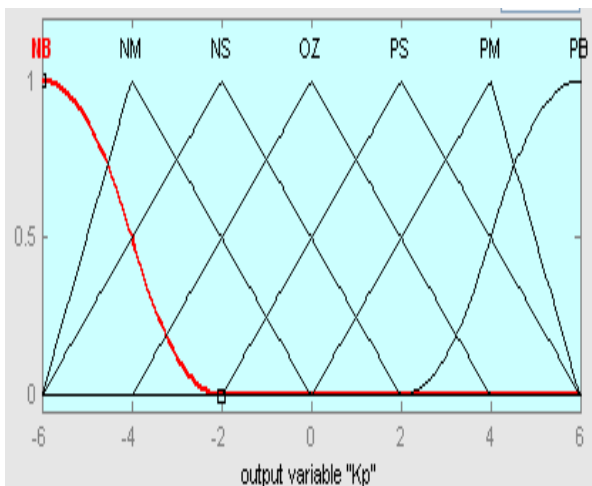


Figure 10 Membership function diagram of Kp

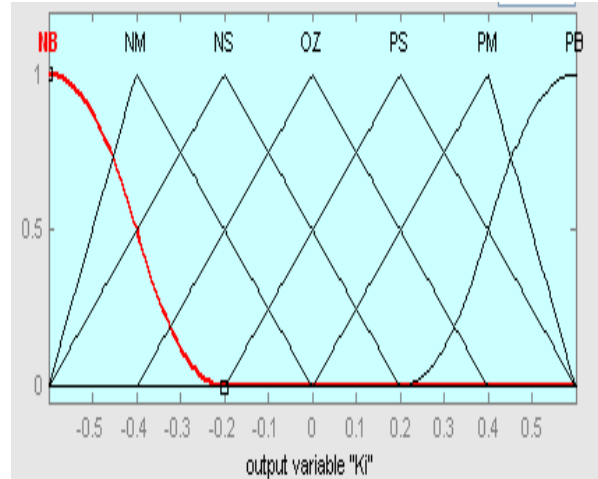


Figure 11 Membership function diagram of Ki

And in this paper, rules of Fuzzy PID controller as Table I

TABLE I

CONTROL RULES OF  $\Delta K_p, \Delta K_i, \Delta K_d$

Control rules of  $\Delta K_p$ :

$\Delta K_p$	$ec$						
$e$	$e$						
	NB	NM	NS	ZO	PS	PM	PB
NB	PB	PB	PM	PM	PS	ZO	ZO
NM	PB	PB	PM	PS	PS	ZO	NS
NS	PM	PM	PM	PS	ZO	NS	NS
ZO	PM	PM	PS	ZO	NS	NM	NM
PS	PS	PS	ZO	NS	NS	NM	NM
PM	PS	ZO	NS	NM	NM	NM	NB
PB	ZO	ZO	NM	NM	NM	NB	NB

Control rules of  $\Delta K_i$  and  $\Delta K_d$ :

	NB	NB	NM	NM	NS	ZO	ZO
NB	NB	NB	NM	NS	NS	ZO	ZO
NM	NB	NB	NM	NS	NS	ZO	ZO
NS	NM	NM	NS	NS	ZO	PS	PS
ZO	NM	NM	NS	ZO	PS	PM	PM
PS	NM	NS	ZO	PS	PS	PM	PB
PM	ZO	ZO	PS	PS	PM	PB	PB
PB	ZO	ZO	PS	PM	PM	PB	PB
	NB	PS	NS	NB	NB	NB	NM
NB	PS	NS	NB	NM	NM	NS	ZO
NS	ZO	NS	NM	NM	NS	NS	ZO
ZO	ZO	NS	NS	NS	NS	NS	ZO
PS	ZO	ZO	ZO	ZO	ZO	ZO	ZO
PM	PB	NS	PS	PS	PS	PS	PB
PB	PB	PM	PM	PM	PS	PS	PB

Using fuzzy toolbox in MATLAB, Mandani reasoning methods is adopted to taking And method as

Min, and Or method as Max according based on fuzzy control rules and fuzzy membership functions. Implication method is Min, Aggregation method is Max and considering defuzzification method as centroid. According to all linguistic values in rule tables, its forms can be used (i.e. If...then) in order to complete fuzzy reasoning section, and prepare simulation in next step in simulink, And relations are used between outputs and inputs to edit rules [16].

FIS Editor in matlab of Fuzzy PID controller as Fig.12.

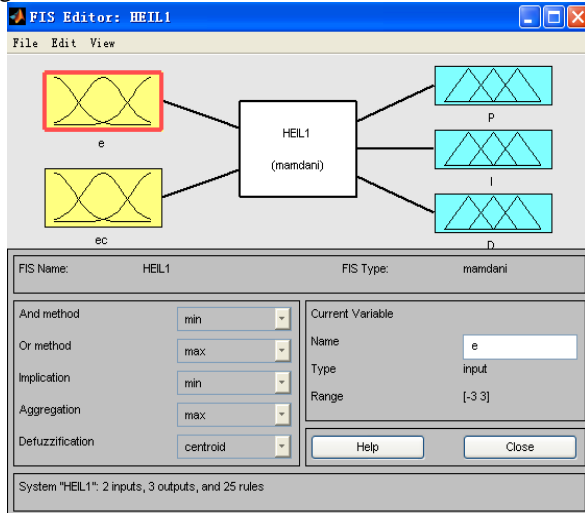


Figure 12 FIS Editor

IV. THE SIMULATION RESULTS AND ANALYSIS

According to transfer function of controlled object and relationship of every link which are got in section 2 , we build the controlled object model in matlab/simulink as Fig.13.

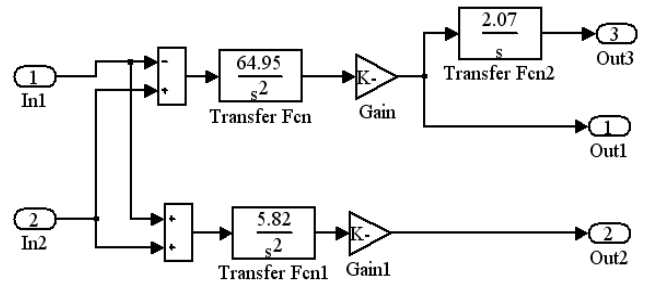


Figure 13 Structure of controlled object model

According to Section 3, we establish subsystem of PID controller, subsystem of fuzzy PID controller and full figure of system as Fig. 14, Fig. 15 and Fig.16.

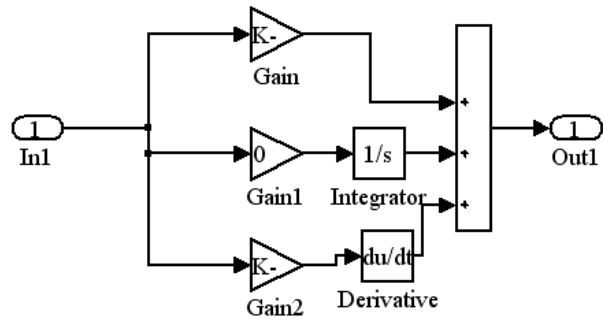


Figure 14 Structure of PID controller

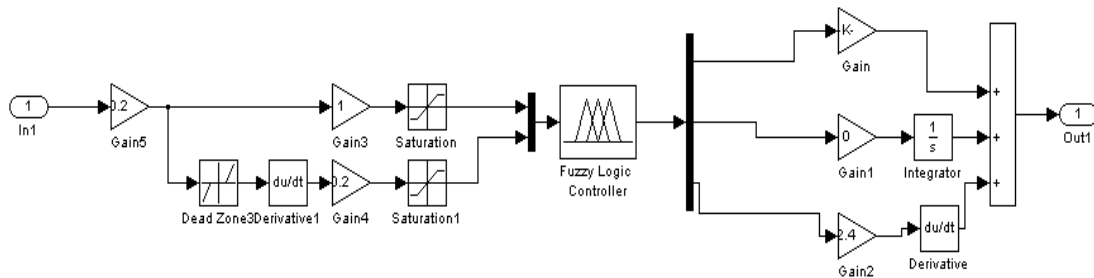


Figure 15 Structure of fuzzy PID controller subsystem

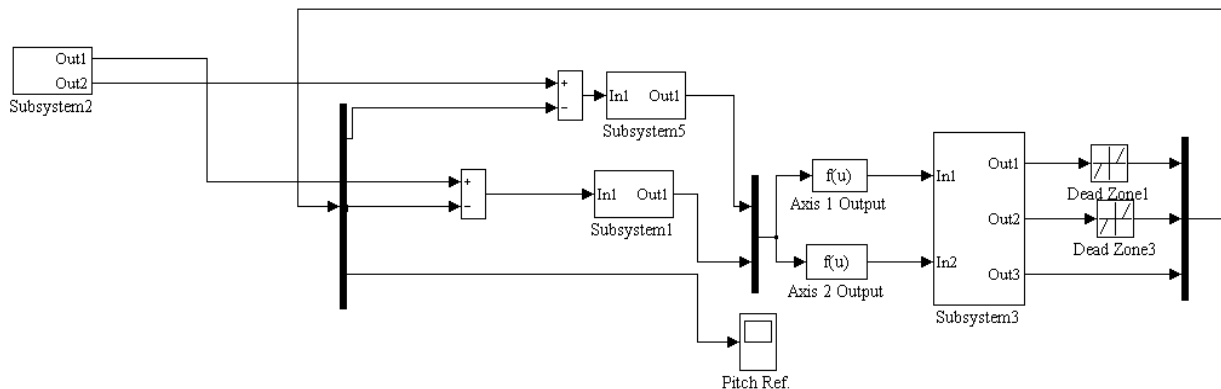


Figure 16 Full Figure of system

The set value of input module is: input 1 is set as setting value of the pitch, and input 2 is set as setting value of the revolving shafts. The expected control effect is: the helicopter raise at first falls at 10 seconds, rises again at 20 seconds, and falls back to the initial lift at 30 seconds. It starts to revolve from 40 seconds, counterclockwise rotation at first, and clockwise rotation at 50 seconds [17].

According to the intended target, in simulation process, we set input 1 initial as the unit step, and the set value reduced to 0 at 10 seconds, added to 2 at 20 seconds, and reduced to unit step at 30 seconds; we set input 2 initial as 0, and it reduced to -1 at 40 seconds, added to 0.5 at 50 seconds, then kept uniform. We add a pulse disturbance angle of pitch at the 42 seconds to simulate collision disturbance of the environment. Response curve of simulation figures without delay link shown in Fig.17 and Fig.18.

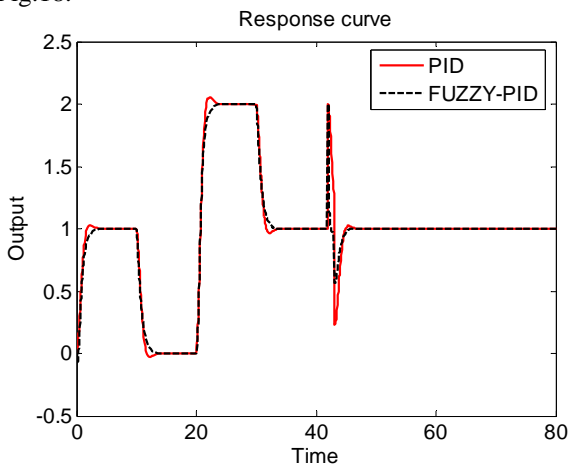


Figure 17 Response curve of angle of pitch without delay

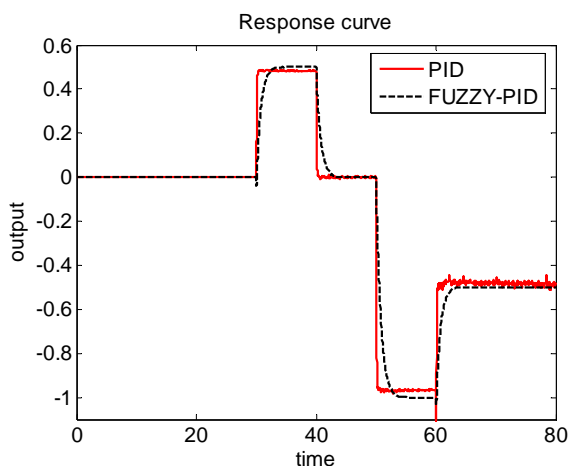


Figure 18 Response curve of lateral angle without delay

Through the simulation curve, we can get two groups of system performance data of control modes, shown in Table II.

TABLE II

SYSTEM PERFORMANCE DATA OF CONTROL MODES				
	Setting time	Overshoot	Static errors of pitching angle	Static errors of lateral angle
PID	3.395	4.8%	0	3%
Fuzzy-PID	2.682	0	0	0

Add a delay link about 0.02s into model, then, response curve of angle of pitch with delay and response curve of lateral angle with delay shown in Fig.19 and Fig.20.

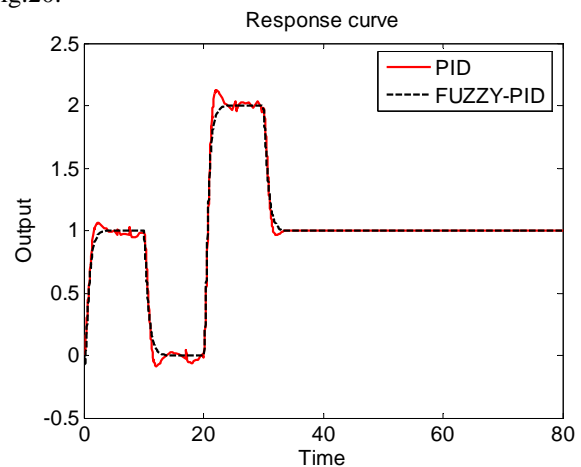


Figure 19 Response curve of angle of pitch with delay

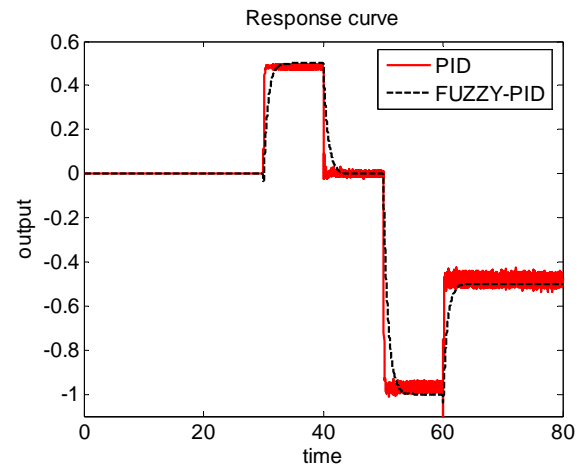


Figure 20 Response curve of lateral angle with delay

It can be seen from above tables that both of PID control method and fuzzy PID control method can achieve the purpose of remaining the system stable. However, PID control had disadvantages in the rapidity and accuracy. Also, PID control appeared overshoot, even appeared static error at response curve of lateral angle. Furthermore, system performance of PID control method was significantly worse after adding the time delay process, especially smaller

irregular shocks appeared. Correspondingly, system of fuzzy PID control method is not influenced almost.

V. CONCLUSION

The paper focus on analyzing 3-dof helicopter model, and designing the PID controller and the PID parameters self-adjusting controller, which depend on fuzzy inference functions to achieve.

Simulation results demonstrate that both of the controllers can be applied into 3-dof helicopter system to keep stable. Admittedly, the accuracy improved and the speed met the requirements of 3-dof helicopter model. In addition, the fuzzy PID control method has better adaptability for parameters change and environmental change, so it is applicable to environment with a delay and other actual existence but working trouble. With comparison, fuzzy PID controller could probably develop the characteristics of stronger robustness, better dynamic response, more quickly regulating time and smaller overshoot of fuzzy control. As a result, fuzzy PID controller performances better than PID controller dose.

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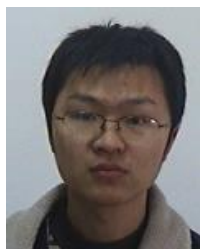
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