

# Research on the Grey Assessment System of Dam Failure Risk

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**Abstract**—The security of the dam is a vital problem for the reasonable use of the water resources. It is important and meaningful to research methods of protecting the safety of dam .So the dam safety assessment is related with the national economy and the people's livelihood, and it is necessary to build a system to evaluate the security risk of dam. The dam failure disaster risk is regarded as main researching object in this thesis. The theories and methods, such as knowledge of dam engineering, risk analysis, analytic hierarchy processing, grey theory and so on, are introduced into the thesis. This research performs relatively detailed study on methods of comprehensive risk assessment, synthesis assessment structure system, method of measuring assessment index of the dam, and development of the grey assessment system of Dam Failure Risk.

**Index Terms**—Dam Failure, Risk Analysis, Analytic Hierarchy Processing, Grey Theory, Assessment System

## I. INTRODUCTION

The grey theory was advanced by Prof. Deng Ju Long in 1982, which deals with decisions characterized by incomplete information and explores system behavior using relational analysis and model construction<sup>[1,2]</sup>. With a view to investigate the inherent law of complex systems, the theory establishes models describing its dynamic variation characteristics based on its own history data. In the field of control technologies, the tint of a color is often used as a metaphor to describe the amount of known information. Black denotes that nothing is known regarding the system internal structure, parameters and characteristics. White refers to the completeness of the information known about a system. Between white and black is grey, which represents an incomplete understanding of system characteristics and structure. In Grey theory, random variables are regarded as grey numbers, and a stochastic process is referred to as a grey process. A grey system is defined as a system containing information presented as grey numbers; and a grey decision is defined as a decision made within a grey system. Fields covered by Grey theory include systems analysis, data processing, modeling, prediction, as well as

decision making and control.

Water is the fundamental natural resource and strategic economic resource. It is impossible the basal existence of people and the sustainable development of the social economy without water. So taking action to protect the security of water resources has increasingly become the major studied issue for all the countries worldwide. Dams have been built to make full use of water resources in many countries. The risks caused by breakage of the dams could be very serious, which have been constructed for a number of purposes such as provision of drinking and irrigation water as well as generation of electric power. Therefore the security of the dam is a vital problem in life, society and environment. It is of necessity to build the system to analyze and evaluate the security risk and safe state of dam. Dam failure risk assessment system can help evaluate the dam safety condition, locate the weak link of the dam, find the key factors affecting the dam safety, direct the management of dam security and guarantee the dam safety. In this way, the security of the water resources is thus assured by used the grey assessment system of dam failure risk.

Based on prototype observations, the mathematical and mechanical methods are conventionally used for evaluating the dam failure risk. These classical methods are very important to monitor dam safety. However, these methods lay usually no strong emphasis on the learning of experiences and expert knowledge. In the risk assessment of dam failure, it is very difficult to effectively quantify the risk with only collecting and counting data, so it is significative to use the principles of the grey theory to give some heuristics for the risk assessment system of dam failure. This paper proposed grey assessment method. The quantitative change and qualitative change of dam safety property were integrated into a matter-element. Based on the grey assessment module, the contradictory problem for dam safety assessment can be solved. A grey assessment system of Dam Failure Risk was developed with the method. In practice, the proposed system has been used for evaluating the dam failure risk successfully. The application showed that the bionics model is feasible and the proposed key technologies were effective. The system can supply technical support for improving the level of dam safety management, extending normal run time of dam and avoiding dam failure.

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## II. THEORY AND METHODOLOGY

### A. Grey Theory

Grey theory is mainly utilized to study systems that model uncertainty, analyze relations between systems, establish models, and make forecasts and decisions [3,4,5]. The grey theory emphasizes investigating the inherent law, which is specialized in quantitative analysis of complex systems with incomplete information.

The Grey-forecasting Model (GM) is the core of Grey Theory, the grey model represented by GM (n, h) predicts its own trend by a dynamic different equation established on the history data. Where n represents the rank of the different equation and h represents the number of the variables. The model GM is often used in practical trend analysis and condition prediction.

Supposing the history data series is represented by  $X_{(0)} = [x_{(0)}(1), x_{(0)}(2), \dots, x_{(0)}(n)]$ , and the model GM (1, 1) is as follow:

$$\frac{X^{(1)}(t)}{t} + X^{(1)}(t) = u \quad (1)$$

The Grey Theory builds a model, which must apply one order Accumulated Generating Operation (AGO) to the primitive series in order to provide the middle message of building a model and to weaken the tendency of variation.

So the accumulated generating operation (AGO) result of the history data series is as follow:

$$X^{(1)}(n) = \sum_{i=1}^n X^{(0)}(i) \quad (2)$$

After AGO any non-negative series is translated into an increasing series of which the randomness is reduced while the orderliness is strengthened.

The vector  $a = [a, u]^T$ , composed of the coefficient  $a$  and the parameter  $u$ , can be solved by the least-mean-square (LMS) algorithm.

$$a = [a, u]^T = (B^T B)^{-1} B^T X_{(0)}^{\Lambda} \quad (3)$$

where  $X_{(0)}^{\Lambda} = X_{(0)}^T$

$$B = \begin{bmatrix} -\frac{1}{2}[x^{(1)}(1) + x^{(1)}(2)] & 1 \\ -\frac{1}{2}[x^{(1)}(2) + x^{(1)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(1)}(n-1) + x^{(1)}(n)] & 1 \end{bmatrix}$$

Then the solution of GM (1, 1) is as following:

$$x^{\Lambda(1)}(k+1) = (x^{(0)}(1) - \frac{u}{a})e^{-ak} + \frac{u}{a} \quad (4)$$

The predictive equation of the history series is:

$$\begin{aligned} x^{\Lambda(0)}(k+1) &= x^{\Lambda(1)}(k+1) - x^{\Lambda(1)}(k) \\ &= (x^{(0)}(1) - \frac{u}{a})(1 - e^{-a})e^{-ak} \end{aligned} \quad (5)$$

In intelligent diagnosis applications the predictive vectors from the GM (1, 1) is usually put into the recognition module. According to the grey theory, the relation degree evolves from the relation coefficient. The relation coefficient of the two series  $X_i$  and  $X_j$ , is represented by  $\xi_{ij}(k)$ , where k represents the sampling points.

$$a_{ij} = |X_j(k) - X_i(k)| \quad k \in \{1, 2, \dots, N\} \quad (6)$$

$$a_{\min} = \min_j \min_k a_{ij}(k) \quad a_{\max} = \max_j \max_k a_{ij}(k) \quad (7)$$

$\xi_{ij}(k)$  is defined as:

$$\xi_{ij}(k) = \frac{a_{\min} + a_{\max} \cdot m}{a_{ij}(k) + a_{\max} \cdot m} \quad k \in \{1, 2, \dots, N\} \quad (8)$$

where m is a constant with the range from 0 to 1. The relation degree of the two series  $X_i$  and  $X_j$  are as follow:

$$\gamma_{ij} = \sum_{k=1}^N \xi_{ij}(k) + \sum_{k=2}^{N-1} \xi_{ij}(k) \quad (9)$$

The relation degree represented by  $\gamma_{ij}$  shows the comparability of the series  $X_i$  and  $X_j$ . It is often applied to grey cluster in practice.

### B. Analytic Hierarchy Process

The Analytic Hierarchy Process uses paired comparisons to derive a scale of relative importance for alternatives [6,7]. We investigate the effect of uncertainty in judgment on the stability of the rank order of alternatives. The uncertainty experienced by decision makers in making comparisons, which is measured by associating with each judgment and interval of numerical values.

*Construct a pair-wise comparison matrix:* To construct pair-wise comparison matrix using a scale of relative importance. The judgments are entered using the fundamental scale of the analytic hierarchy process. An attribute compared with it is always assigned the value 1, so the main diagonal entries of the pair-wise comparison matrix are all 1. The numbers 3, 5, 7, and 9 correspond to the verbal judgments 'moderate importance', 'strong importance', 'very strong importance', and 'absolute importance' (with 2, 4, 6, and 8 for compromise between the previous values). Assuming M attributes, their pair-wise comparisons yield a square matrix  $A_{M \times M}$  as follow

$$A_{M \times M} = \begin{matrix} & A_1 & A_2 & \cdots & A_M \\ A_1 & \begin{bmatrix} 1 & a_{12} & \cdots & a_{1M} \\ a_{21} & 1 & \cdots & a_{2M} \\ \vdots & \vdots & \cdots & \vdots \\ a_{M1} & a_{M2} & \cdots & 1 \end{bmatrix} \end{matrix} \quad (10)$$

where  $a_{ji} = 1/a_{ij}$  denotes the relative importance of attribute  $j$  over attribute  $i$ .

*Find the relative normalized weight:* The relative normalized weight ( $w_j$ ) of each attribute is found by (11) calculating the geometric mean (GM) of the  $i$ th row and (12) normalizing the geometric means of rows in the comparison matrix. This can be represented as:

$$GM_j = \frac{\sum_{i=1}^M A_{ij}}{\sum_{k=1}^M A_{kj}}, (i, j = 1, 2, \dots, M) \quad (11)$$

and

$$w_j = GM_j / \sum_{j=1}^M GM_j \quad (12)$$

It is simplicity and ease to find out the maximum eigenvalue and to reduce the inconsistency in judgments. So the method of AHP is used for finding out the relative normalized weights of the attributes

*Calculate matrices:* To Calculate matrices  $A_3$  and  $A_4$  such as  $A_3 = A_1 \times A_2$  and  $A_4 = A_3 / A_2$ , where  $A_2 = [w_1, w_2, \dots, w_j]^T$ . The maximum eigenvalue  $\lambda_{\max}$  can be found out (i.e. the average of matrix  $A_4$ ).

*Calculate the consistency index:* To calculate the consistency index  $CI = (\lambda_{\max} - M) / (M - 1)$ . The smaller the value of  $CI$ , the smaller is the deviation from the consistency. The consistency in the judgments of relative importance of attributes reflects the knowledge of the analyst (i.e. decision maker).

The number of attributes can obtain the random index ( $RI$ ), which can be used in decision-making.

*Calculate the consistency ratio:* To calculate the consistency ratio  $CR = CI / RI$ . Usually, a  $CR$  of 0.1 or less is considered as acceptable and reflects an informed judgment that could be attributed to the knowledge of the analyst about the problem under study.

### C. Grey Assessment Method of Dam Failure Risk

This research aims to develop an assessment system of dam failure risk with the theories and methods, such as risk analysis, analytic hierarchy processing, and grey theory. By the principle of selecting factors, correlation theory and criteria, the primary factors and sub-factors of the dam's outburst are selected, and to build the multistage indexes system of the dam-break disaster risk analysis. The standards for judging and the suitable subordinate functions can be determined, so the assessment index

system is established. According to the factors affected the dam risk are complicated and the standards for judging are fuzzy, the factors are classified in different types and different layers through the method of risk analysis. In the assessment of dam risk, different factors reflect different status and importance make different contribution to the risk assessment that is each factor has its own weight. Weights on the factors are determined through analytic hierarchy process. Then each factor risk degree will be got and the final result can be gain by the method of Grey Assessment of dam-break disaster risk.

*Build the multistage indexes system:* The first step of carrying out the assessment of the dam risk is to set up the performance evaluation indexes and decision factors. By the principle of selecting factors, correlation theory and criteria, to select the primary factors and sub-factors of the dam's outburst, and to build the multistage indexes system of the dam-break disaster risk analysis.

Under abominable work environment, the safety status of dam changes dynamically, which emerges in the quantitative and qualitative change manners. Therefore, quantitative and qualitative changes need be considered comprehensively in the process of dam safety evaluation. In extension evaluation method, the professional knowledge in dam field is combined with extension theory<sup>[8,9]</sup>. The quantitative change and qualitative change of dam safety status are integrated into a matter-element. According to analysis the risk sources and corresponding risk characteristics of the dam-break disaster are determined under uncertainty, and the multilevel risk tree model of dam-break disaster is constructed as Fig.1.

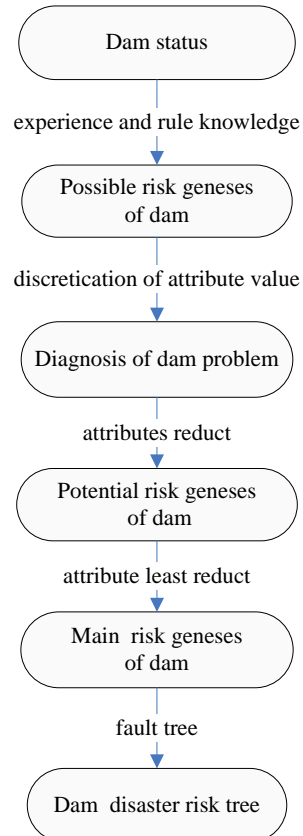


Figure 1. Diagnosis model of dam-break disaster risk

*Establish the weight matrix:* Suppose that there are  $n$  factors to describe the status of the dam. Then, we can define a finite set,  $U = \{u_i, i = 1, 2, \dots, n\}$  as an evaluation factors set. Thus, the classification of evaluation factors can be described as  $U = [u_1, u_2, \dots, u_n]$ .

The linguistic evaluation variable set  $V$ ,  $V = (v_1, v_2, \dots, v_m)$  is used for describing the danger of object performance under the assessment. It is determined based on whether the problem is normal or dangerous. For instance, the dam safety status is decomposed into three grades,  $V = (\text{normal, little abnormal, dangerous})$ . The voting matrix can facilitate evaluating each factor or factor item for object performance in terms of assessment linguistic variable matrix  $V$ . It can be described by a form, of which each expert has one. The multi-variable weight  $W$  is confirmed by AHP, the weight matrix can be established  $A$ , and  $w$  can be shown as follow, respectively:

$$A = \begin{matrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{matrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \quad (13)$$

*Calculate the whitenization weight function:* The determination of whitenization weight functions of grey is a key link from the qualitative analysis to the quantitative modeling in the process of grey assessment. The whitenization weight functions are obtained as follows:

$$f_1(r) = \begin{cases} r/5 & 0 \leq r \leq 5 \\ 1 & r \geq 5 \end{cases} \quad (14)$$

$$f_2(r) = \begin{cases} r/4 & 0 \leq r < 4 \\ 1 & 4 \leq r \leq 5 \end{cases} \quad (15)$$

$$f_3(r) = \begin{cases} r/3 & 0 \leq r < 3 \\ (5-r)/2 & 3 \leq r \leq 5 \end{cases} \quad (16)$$

$$f_4(r) = \begin{cases} r/2 & 0 \leq r < 2 \\ (5-r)/3 & 2 \leq r \leq 5 \end{cases} \quad (17)$$

$$f_5(r) = \begin{cases} 1 & 0 \leq r < 1 \\ (5-r)/4 & 1 \leq r \leq 5 \end{cases} \quad (18)$$

The relation degree is denoted as:

$$\gamma_{ij} = \frac{\sum_{k=1}^p f_l(r_{ij})}{\sum_{l=1}^n \sum_{k=1}^p f_l(r_{ij})} \quad (19)$$

where,  $n$  is the amount of grey clusters,  $p$  is the amount of estimators,  $f_l(r_{ij})$  is the whitenization weight functions.

Then the grey assessment matrix  $B$  is as follow:

$$B = \begin{matrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{matrix} \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1m} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nm} \end{bmatrix} \quad (20)$$

*Evaluate the risk grade:* The result of the grey assessment  $C$  is denoted as:

$$C = A \cdot B \quad (21)$$

The risk grade is confirmed based on the maximum membership grade principle.

### III. DEVELOPMENT OF THE ASSESSMENT SYSTEM

#### A. Requirement Analysis of the System

According to the life process of dam and the usual thought of dam safety diagnosis, the Fig.2 describes the model of dam failure risk assessment.

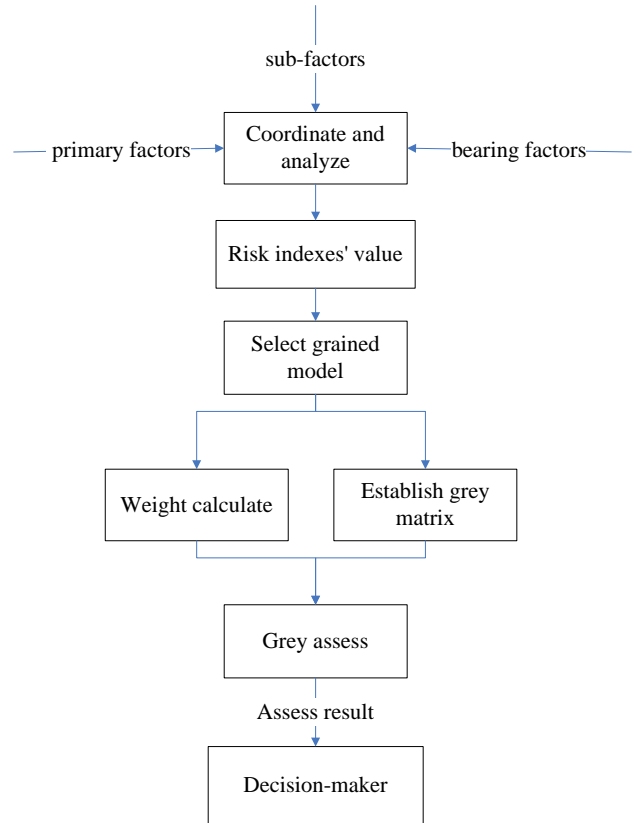


Figure 2. Model of dam failure risk assessment

The information on external environment and structural characteristics of dam is acquired in real time by supervisor. Next, observations are analyzed and abnormal symptoms are founded by the theories and methods integrating mathematics, mechanics and so on. With the advanced technologies for data processing, the accurate and reliable data mined from a mass of observation data sources are inputted into assessment system. Finally,

based on the systemic and whole viewpoint, the status of dam risk is evaluated dynamically.

So the assessment system should possess three functions. First, the user can put the risk data into the system, which can be saved, amended and deleted. Second, the data should be managed in the system, and the assessment result can be gain correctly. Third, the result of assessment can be shown to the user intuitually.

### B. Main Modules of the System

According to the requirement analysis of the assessment system, the system should be consisted of integration control module, risk indexes input module, weight calculation module, grey assessment module, assessment output module and project database (Fig.3).

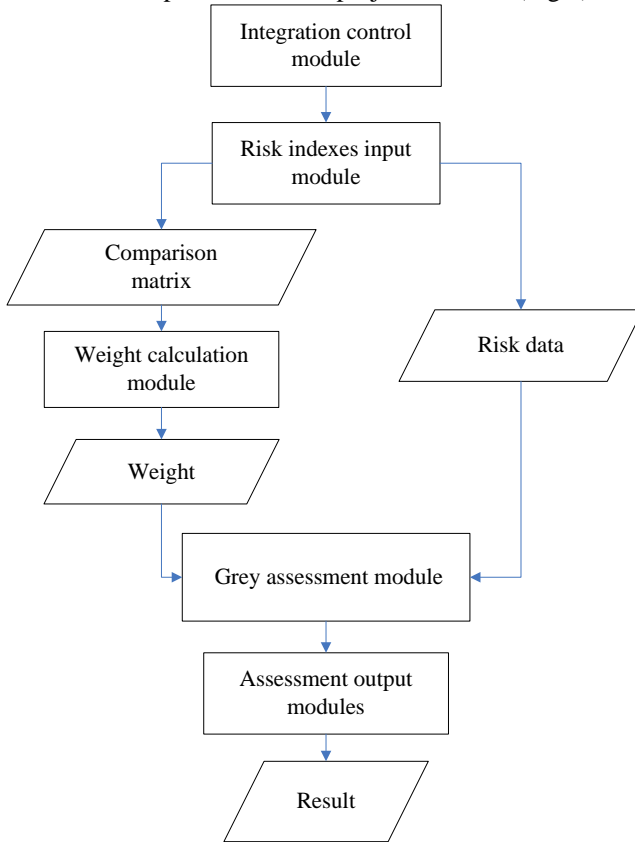


Figure 3. Main modules of the system

*Integration control module:* The integration control module is a multilevel control menu, which can harmonize risk indexes input module, weight calculation module, grey assessment module, assessment output module and project database.

*Risk indexes input module:* In the Risk indexes input module, we can input the parameters of the risk indexes by level and select the grained module. In this module, the multilevel risk tree model of dam-break disaster will be put into the system.

*Weight calculation module:* The weight calculation module can calculate the risk indexes' weights well and truly, and the weight matrix can be established.

*Grey assessment module:* The grey assessment module is used for analyzing and calculating the data of

the dam, and evaluating the dam failure risk, which is the most important module in the assessment system.

*Assessment output module:* The assessment output is used for showing the module assessment result. The result is shown in the graphics, so the result is clear at a glance.

*Project database module:* The project database is used for storing the large number of project archives and observing data.

## IV. IMPLEMENTATION OF THE MAIN MODULES

There are two important modules in the dam failure assessment system, just as weight calculation module and grey assessment module.

### A. Development Environment of the System

According to the analysis and comparison, The common development platform Visual Studio was chosen and the development language VB was used in the risk assessment system, and the database system Access could provide data services.

### B. Weight Calculation Module

In this paper the Analytic Hierarchy Process is applied in the weight calculation module, which is used for calculating the risk indexes' weights. The flow chart of the AHP can be show in Fig.4

The Fig.5 shows the implementation interface of this module. When the multilevel risk tree of dam failure is constructed and the comparison matrix is imported in the system, the indexes' weights could be calculated in this module swiftly.

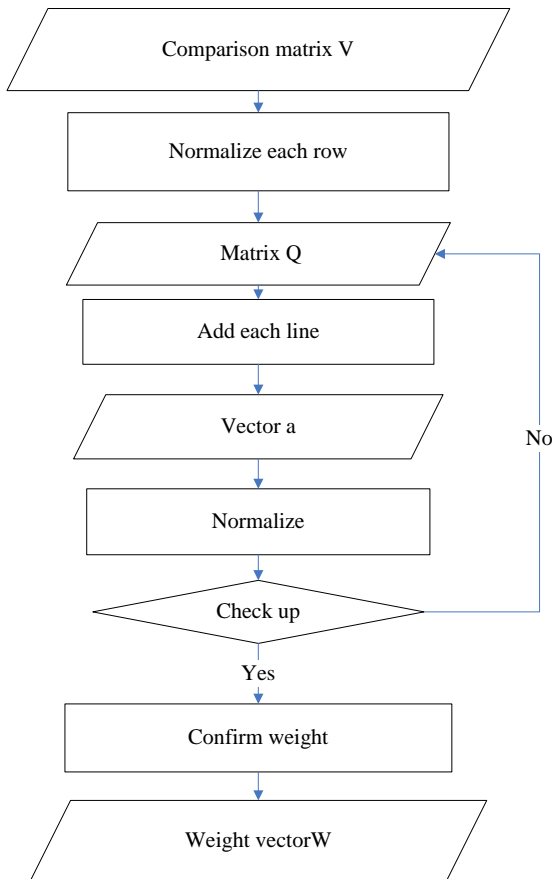


Figure 4. Flow chart of weight calculation

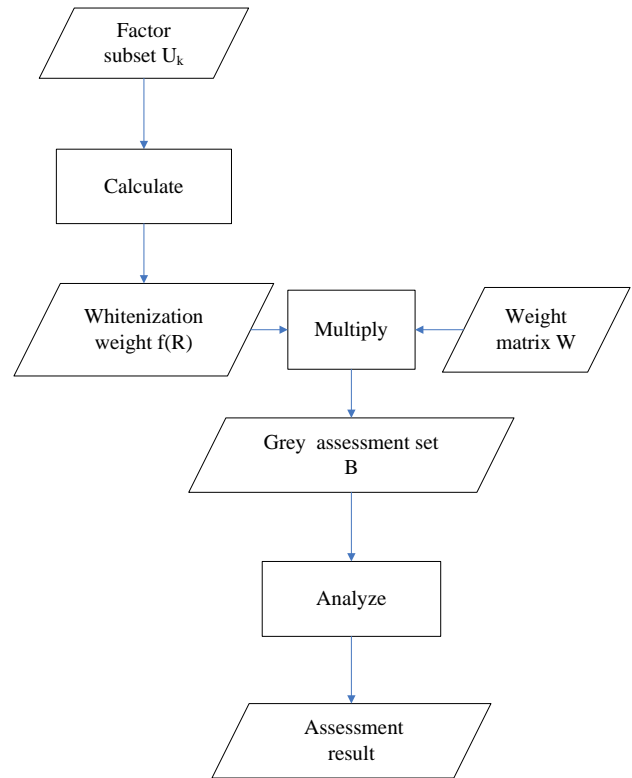


Figure 6. Flow chart of grey assessment

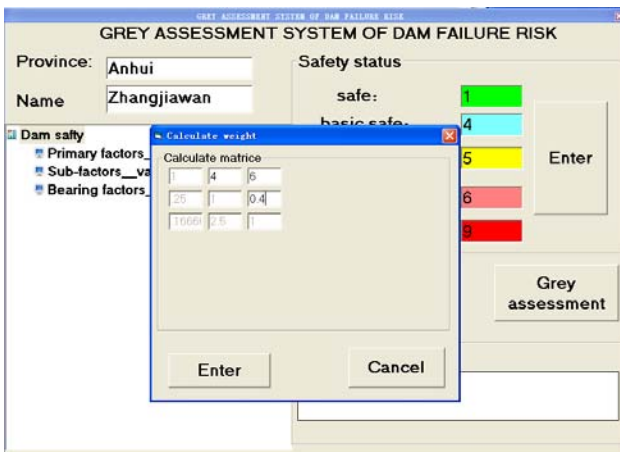


Figure 5. The interface of weight calculation

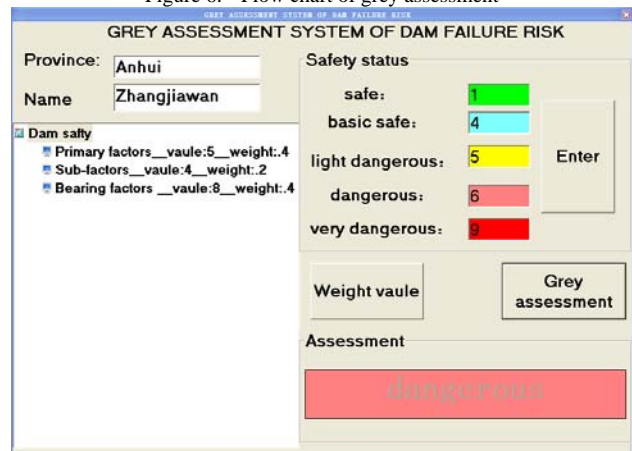


Figure 7. The interface of grey assessment

### C. Grey Assessment Module

The grey theory is used for evaluating the dam risk. The flow chart of the grey assessment method is described as Fig.6.

The interface of this module is shown in Fig.7. After all risk data are put into the system and indexes' weights are calculated, the risk grade of the dam can be assessed swiftly and shown with the corresponding color.

## V. APPLICATION

The Zhangjiawan dam in Anhui Province and the system is used for evaluating the dam failure risk. According to field knowledge and experiential knowledge, the condition attributes and decision attributes are established. The decision table of diagnosing dam risk is built by collecting historical data about the status of this dam. The continuous value of attributes is discriminated. Redundant attributes for decision table of diagnosing dam cracks are eliminated and potential genes of cracks are found with attribute reduction algorithm. Main genes of cracks are found with attribute least reduction algorithm. The fault tree of logical relation between dam risk and genesis is drawn in Fig.8, and the value of risk genes is determined as Tab.I.

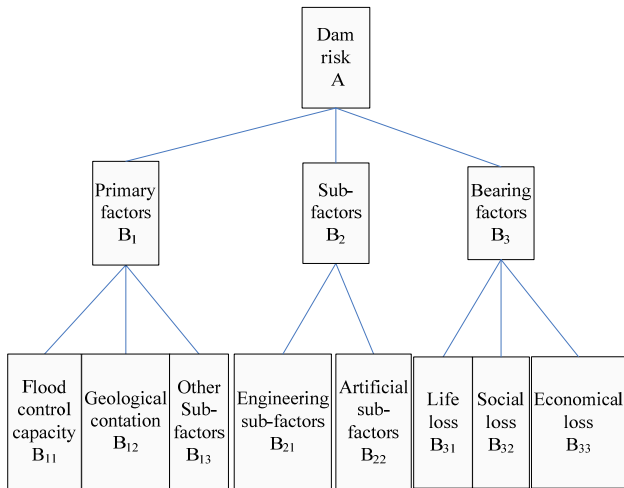


Figure 8. Sketch map of fault-tree for risk genes

TABLE I.  
VALUE OF RISK GENES

genes	$B_{11}$	$B_{12}$	$B_{13}$	$B_{21}$	$B_{22}$	$B_{31}$	$B_{32}$	$B_{33}$
value	3	2	1.5	2	1	1	1	1

TABLE II.  
PAIR-WISE COMPARISON MATRIX OF FIRST LEVEL

A	$B1$	$B2$	$B3$
$B1$	1	1/5	1/7
$B2$	5	1	1/2
$B3$	7	2	1

TABLE III.  
PAIR-WISE COMPARISON MATRIX OF PRIMARY FACTORS

$B1$	$B11$	$B12$	$B13$
$B11$	1	2	5
$B12$	1/2	1	3
$B13$	1/5	1/3	1

TABLE IV.  
PAIR-WISE COMPARISON MATRIX OF SUB-FACTORS

$B2$	$B21$	$B22$
$B21$	1	3
$B22$	1/3	1

TABLE V.  
PAIR-WISE COMPARISON MATRIX OF BEARING FACTORS

$B3$	$B31$	$B32$	$B33$
$B31$	1	2	3
$B32$	1/2	1	2
$B33$	1/3	1/2	1

TABLE VI.  
DAM RISK GRADE

Risk grade	threshold value
<i>safe</i>	$4 \leq H \leq 5$
<i>basic safe</i>	$3 \leq H \leq 4$
<i>slight dangerous</i>	$2 \leq H \leq 3$
<i>dangerous</i>	$1 \leq H \leq 2$
<i>very dangerous</i>	$0 \leq H \leq 1$

The pair-wise comparison matrix (Tab. II, III, IV, V) is constructed with the fundamental scale of the analytic hierarchy process.

The dam risk status is decomposed into 5 grades, corresponding to 5 remarks as follow: safe, basic safe, slight dangerous, dangerous, very dangerous, which can be described as Tab.VI.

The risk grade, risk genes' names and values were put into the assessment system in order. The assessment of the dam failure risk was calculated and analyzed by using the grey assessment method.

It can be shown from Fig.9 that dam safety characteristic is basic safe.

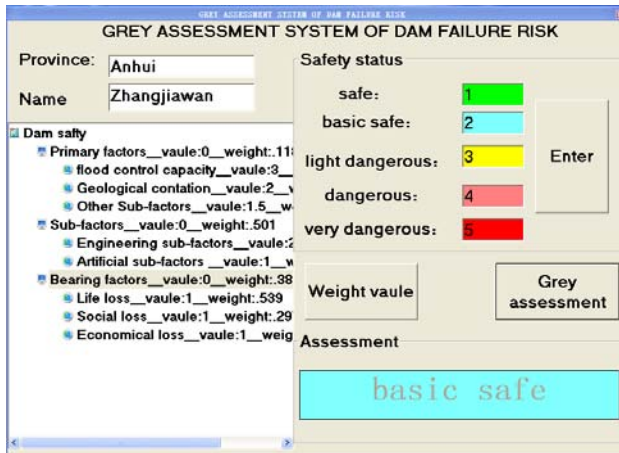


Figure 9. Results of the dam risk assessment

## VI. CONCLUSIONS

The grey evaluating model established in this research has shown to be quite valuable in practice. The grey assessment system of dam failure risk is an applied measure for optimizing the design, construction and operation of dam, ensuring the dam safety.

The application showed the measure and design are viable. The inference efficiency of system is improved by the intelligent inference models. The system can be used for advancing modern level of dam safety management, which can lessen the burden of managers' job.

## ACKNOWLEDGEMENTS

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