Quantum Competition Network Model Based On Quantum Entanglement

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***Abstract*—This paper proposes a quantum competition neural network model compared to its classical counterpart from the relative parts of the complex system localizing operation without changing the perspective of entanglement measure. It shows that the pseudo-state is an inevitable part of the quantum competitive model. After the initialization of the quantum neural network; quantum competitive network is capable of associative memory through local area of operations because of the existence of these pseudo-states. Furthermore, Competitive algorithms of quantum theory are given, and finally an example of pattern recognition for simulation. Simulation results show that a quantum competitive learning algorithm in the learning rate and convergence rate is far better than the basic competitive artificial neural network.**

***Index Terms*—Quantum competition network model; quantum associative memory; Pattern recognition; Quantum entanglement**

I. INTRODUCTION

In the past ten years, the academic papers and reports about quantum computing have been widely published; the theory of quantum computing has made great progress, however, to the successful development of a practical value of the quantum computer, its physical implementation is very difficult, the main reason is to make the quantum state with the outside world. On one hand, the Quantum particles and the nteraction between the external environments will destroy the superposition of qubits, resulting in an error. On the other hand, with the development of artificial neural networks, neural computing limitations and shortcomings have gradually become prominent, especially Radovan in 1997 proved that this method of neural connectionism and its ability to express the traditional doctrine of symbolic logic methods are equivalent, so there is a neural network approach with traditional methods of symbolic logic, the same limitations. Since then, the neural network research returns to low. However, neural networks and quantum theoretical description of the system has a striking similarity, so the advantages of quantum computing can be used to compensate for the shortcomings of neural networks. In this sense, the current method of calculating and the emerging quantum neural network have become an important direction for further development.

Quantum computing demonstrated the amazing potential and unusual features are all derived from the traditional calculation of the quantum transformation, and neural computation is the biological behavior of the analog information processing method, and its kinetic characteristics of quantum systems have many similarities place. In the literature [1-3], we also discussed the quantum competitive learning algorithm, but the algorithm is relatively simple. The following of the paper introduces some key concepts from quantum mechanics, briefly discussing some of the well-known quantum algorithms, and then details a quantum version of competitive learning. Preliminary empirical results (obtained through simulation on a classical computer) are presented, and these results demonstrate that a quantum competitive learning system is indeed capable of performance that is impossible.

II. QUANTUM CONCEPTS

Quantum computation is based upon physical principles from the theory of quantum mechanics, which is in many ways counterintuitive. Yet it has provided us with perhaps the most accurate physical theory ever devised. The theory is well-established and is covered in its basic form by many textbooks. Several ideas are briefly reviewed here.

Linear superposition is closely related to the familiar mathematical principle of linear combination of vectors. Quantum systems are described by a wave function ψ that exists in a Hilbert space. The Hilbert space has a set of states $|\phi_i>$ that form a basis, and the system is described by a quantum state,

$$|\psi>=\sum_i|\phi_i> \tag{1}$$

$|\psi>$ is said to be in a linear superposition of the basis states $|\phi_i>$, and in the general case, the coefficients ci maybe complex. Use is made here of the Dirac bracket notation, where the ket $|\cdot>$ is analogous to a column

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vector, and the bra $<\cdot|$ is analogous to the complex conjugate transpose of the ket.

Coherence and decoherence are closely related to the idea of linear superposition. A quantum system is said to be coherent if it is in a linear superposition of its basis states.According to quantum mechanics, if a coherent system interacts in any way with its environment, the superposition is destroyed. This loss of coherence is called decoherence and is governed by the wave function ψ. The coefficients c_i are called probability amplitudes, and gives the probability of $|\psi>$ collapsing into state $|\phi_i>$ if it decoheres. Note that the wave function ψ describes a real physical system that must collapse to exactly one basis state. Therefore, the probabilities governed by the amplitudes c_i must sum to unity. This necessary constraint is expressed as the unitarity condition

$$\sum_i |c_i|^2 = 1 \tag{2}$$

Consider, for example, a discrete physical variable called spin. The simplest spin system is a two-state system, called a spin-1/2 system, whose basis states are usually represented as $|\uparrow>$ (spin up) and $|\downarrow>$ (spin down). In this system the wave function ψ is a distribution over two values and a coherent state $|\psi>$ is a linear superposition of $|\uparrow>$ and $|\downarrow>$. One such state might be

$$|\psi>=\frac{2}{\sqrt{5}}|\uparrow>+\frac{1}{\sqrt{5}}|\downarrow> \tag{3}$$

As long as the system maintains its quantum coherence it cannot be said to be either spin up or spin down. It is in some sense both at once. When this system decoheres the result is, for example, the $|\uparrow>$ state with probability $(2/\sqrt{5})^2 = 0.8$.

A simple two-state quantum system, such as the spin-1/2 system just introduced, is used as the basic unit of quantum computation. Such a system is referred to as a quantum bit or qubit, and renaming the two states $|0>$ and $|1>$, it is easy to see why this is so.

Operators on a Hilbert space describe how one wave function is changed into another. Here they will be denoted by a capital letter with a hat, such as $\hat{A}$, and they may be represented as matrices acting on vectors. Using operators, an eigenvalue equation can be written

$$\hat{A}|\phi_i>=a_i|\phi_i> \tag{4}$$

Where a_i is the eigenvalue. The solutions $|\phi_i>$ to such an equation are called eigenstates and can be used to construct the basis of a Hilbert space as discussed above. In the quantum formalism, all properties are represented as operators whose eigenstates are the basis for the Hilbert space associated with that property and whose eigen values are the quantum allowed values for that property. Operators in quantum mechanics must be linear and further, operators that describe the time evolution of a state must be unitary so that $\hat{A}^\dagger\hat{A}=\hat{A}\hat{A}^\dagger=\hat{I}$,where $\hat{I}$ is the identity operator, and $\hat{A}^\dagger$ is the complex conjugate transpose of $\hat{A}$.

Interference is a familiar wave phenomenon. Wave peaks that are in phase interfere constructively while those that are out of phase interfere destructively. This phenomenon is common to all kinds of wave mechanics from water waves to optics, and the well-known double slit experiment proves empirically that interference also applies to the probability waves of quantum mechanics.

III. THE GENERAL COMPETITIVE NEURAL NETWORK

Artificial Neural Network [6][8] is a system loosely modeled based on the human brain. The field goes by many names, such as connectionism, parallel distributed processing, neuro-computing, natural intelligent systems, machine learning algorithms, and artificial neural networks. It is inherently multiprocessor-friendly architecture and without much modification, and it goes beyond one or even two processors of the von Neumann architecture. It has ability to account for any functional dependency. The network discovers (learns, models) the nature of the dependency without needing to be prompted. No need to postulate a model, or to amend it, etc.

Competitive learning is an important learning style of artificial neural network. It can achieve pattern classification and associative memory. The Hamming neural network is a typical competitive learning model, which first introduced the following network structure and competitive learning process.

The Hamming neural network topology was shown in figure 1, it can be divided into two basic sections: input layer – a layer built with neurons, all of which neurons are connected to all of the network inputs; output layer – which is called MaxNet layer; the output of each neuron of this layer is connected to input of each neuron of this layer, besides, every neuron of this layer is connected to exactly one neuron of the input layer (as in the picture right).

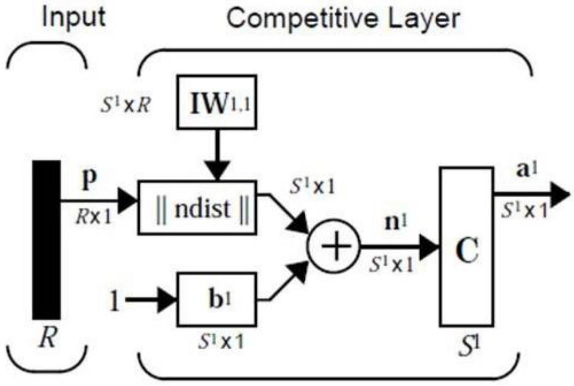

Figure 1. Hamming neural network model

For example, the dimension of the input mode is m, the number of samples stored patterns (i.e., network capacity) is n, and the learning process can be described as follows:

Left: pattern match

Assuming the input patterns is $X = \{x_1, x_2, \cdots x_m\}$, patterns are stored in the network which is $(P^j = p_i^j, 0 \le i \le m, 0 \le j \le n)$,calculate the Hamming distance between each of X and P , that is

$$HD(X, P^j) = \frac{m - X \cdot P^j}{2} \quad (0 \le j \le n), \qquad (5)$$

Here, $X \cdot P^j$ is the inner product of two patterns. Simply, if expressed in binary patterns, then the Hamming distance can be defined as two patterns for different values of vector (I.e., the opposite value) the number of components, which is used to measure the difference between two patterns, the smaller the value, the closer the two patterns, and vice versa, Otherwise, two of the same patterns of Hamming distance are zero, m is the maximum distance.

According to the minimum Hamming distance criteria, $y_j = X \cdot P^j$ is maximum, and the following operations can be classified results:

$$y_j(t+1) = f\left[y_j(t) - \varepsilon \sum_{k \ne j} y_k(t) \right], \text{ here } \varepsilon < \frac{1}{n} \qquad (6)$$

IV. QUANTUM-COMPETITION NEURAL NETWORK

Quantum computing is to use the interference properties of quantum states, so that the results you want enhanced, making unnecessary results weakened, so the results you want at the time of measurement will occur in the higher probability, In the whole calculation process, the quantum algorithm is actually a series of unitary operators of continuous operation, it describes that the quantum state changes from the initial state to the final state of evolution. Similarly, we believe that quantum learning algorithms also have similar characteristics. Controlled Hamming neural networks, quantum competition network model and its learning algorithm are given below.

A. Quantum Neural Network Model

Consider a discrete physical variable called spin. The simplest spin system is a two-state system, called a spin-1/2 system, whose basis states are usually represented as $|0>$ (spin up) and $|1>$ (spin down). In this system the wave function φ is a distribution over two values and a coherent state φ is a linear superposition of $|0>$ and $|1>$. One such state might be $|\varphi>= a|0> + b|1>$, here a, b are complex numbers such that $|a|^2 + |b|^2 = 1$. A composed system with N qubits is described using $N = 2^n$ independent states obtained through the tensor product of the Hilbert Space associated with each qubit.

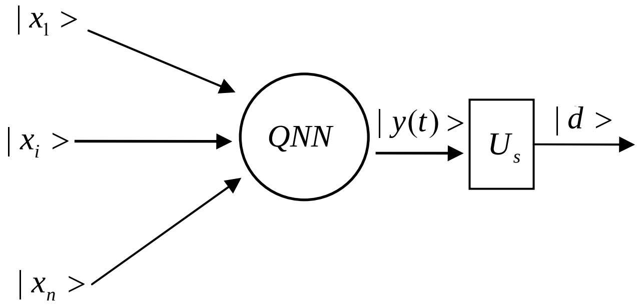

Figure 2. The quantum neural model

To better describe the quantum Competition Network, the concept of the amount of entanglement will be introduced; Entanglement is described in the quantum mechanics properties between several parts of the same system state, suppose a system is composed of A, B, C etc, general States of composite systems with the density matrix $\hat{\rho}$ to describe. There are several properties of the amount of entanglement.

(1) If the density matrix $\hat{\rho}$ described state is not entanglement and Separable, it can be expressed as belonging to different parts of the state of the tensor product

$$\hat{\rho} = \sum_i P_i \hat{\rho}_i^A \otimes \rho_i^B \otimes \cdots, \qquad (7)$$

Here, $\hat{\rho}_i^A, \hat{\rho}_i^A \cdots$ is the description of the various parts of the density operator, $P_i \ge 0, \sum_i P_i = 1$, Of non-entanglement, entanglement is zero, $E(\hat{\rho}) = 0$.

(2) Related to the various parts in the local area under the unitary transformation (Relative to various parts of the local operation).These operations can be part of the base of the unitary transformation, and the implementation is of the general measure, part of the Hilbert space to expand or give up some space, then the total entanglement remains unchanged in this system.

$$E(\hat{\rho}) = E(U_A \otimes U_B \hat{\rho} U_A^+ \otimes U_B^+), \qquad (8)$$

B. Quantum Storage Mode

Traditional artificial neural networks (such as Hopfield networks) allow association mode response, but its main drawback is the limited storage capacity. For example, to deposit a pattern of length n requires n-neuron network, the model can be stored number $m \le kn$, in general $0.15 \le k \le 0.5$; the use of quantum associative memory can greatly expand the memory capacity. In the given incomplete distorted input samples can be relatively large probability to restore a complete prototype.

In storage mode, with a quantum associative memory proposed by Ventura and Martinez difference is that a corresponds to a neuron, the longest field mode number of bits n as the number of neurons, uses Hamiltonian transformation $H^{(n)} = H \otimes H \otimes \cdots \otimes H$ (n is the number)to the initial state $|0\cdots 0>$ to get equal weight superposition state of qubits. Each basic state represents an initial mode. That n qubits can store 2^n patterns. When the input pattern number is less than n, it is inevitable that there are some pseudo-states. For example, to store the three pattern $|000>$, $|010>$, $|111>$, and need three qubits, there are five pseudo-states, and probability amplitude for each patter $1/2\sqrt{2}$. Prepare all the qubits in the state $|0>$, using Hamilton equations to qubits is written as follows:

$$|s> = H^{(n)}|00\cdots 0> = \frac{1}{2^{n/2}}\sum_{x=0}^{2^n-1}|x> \quad . \tag{9}$$

In general, the n qubits open 2^n dimensional Hilbert space, there are 2^n orthogonal states, and the 2^n basic state is expressed by $|x>$,

$$|\varphi> = \sum_{i=1}^{2^n} C_i\,|i> \quad , \tag{10}$$

Note that $|i>$ is one of the 2^n basic states, C_i is Stacking factor. x is n-bit string made of 0 and 1, $H^{(n)}|x>$ is an equal weight superposition state of binary number from 0 to 2^n-1 . State $|s>$ can be expressed as the following formula:

$$|s> = H^{(n)}|0^i>|0>|0^{n-i-1}>$$
$$= \left(\frac{1}{2^{i/2}}\sum_{x=0}^{2^i-1}|x^i>\right)\frac{1}{\sqrt{2}}(|0>+|1>)\left(\frac{1}{2^{(n--i-1)/2}}\sum_{x=0}^{2^{n-i-1}-1}|x^{n-i-1}>\right)$$
$$= \left(\frac{1}{2^{(n-1)/2}}\sum_{x=0}^{2^{n-1}-1}|x^{n-1}>\right)\frac{1}{\sqrt{2}}(|0>+|1>) \quad , \tag{11}$$

Here j is the binary digit string. From the above known equation, the location of the binary character in the quantum system is equivalent. The location of the binary character in the quantum system is equivalent.

C. *Response mode*

Now the system is divided into any two parts, part A (input layer) and Part B (competitive layer) as shown in Figure 2.A part searches known part string , and the character of B part may be incomplete state. These two parts are connected by certain entanglement.

Suppose we want to recall the incomplete mode $|y_1 y_2 \cdots y_{i-1} ? y_{i+1} \cdots y_n>$, the Symbol ? means that it did not know the i bits but now need to association it. By the formula (7) we know that system does not change the whole entanglement when using Grover's algorithm to find a matching string $|y_1 y_2 \cdots y_{i-1} y_{i+1} \cdots y_n>$ in 2^{n-1} dimensional Hilbert space, no matter what one bit data is not clear. So after about $T \cong \frac{\pi}{4}\sqrt{N}$ iterations, system state changes as fallow:

$$|s> = |y_1 y_2 \cdots y_i y_{i+2} \cdots y_n> \frac{1}{\sqrt{2}}(|0>+|1>) \quad , \tag{12}$$

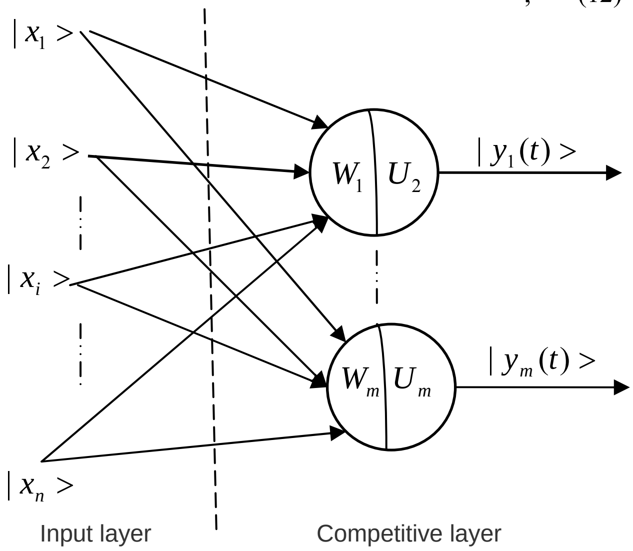

Figure 3. quantum competition network models

Then, with some measurements, the system will collapse to a 50% probability to state $|y_1 y_2 \cdots y_i \otimes |0> \otimes y_{i+2} \cdots y_n>$ and $|y_1 y_2 \cdots y_i \otimes |1> \otimes y_{i+2} \cdots y_n>$

V. Pattern Recognition used Quantum competition network

To test Quantum competition network (QCNN) algorithm performance, we selected a typical example of pattern recognition to simulate, with the General competitive neural network for performance comparison. For example, to store the four models $|000>$, $|010>$, $|111>$, $|110>$, requires four q-ubits, there are four pseudo-states, the probability amplitude of each mode is $1/2\sqrt{2}$. Now we give an incomplete pattern $|11?>$, it's the first two quantum bits constitute Input part, implementation of Grover iteration algorithm [4-5] on the input part will find the matching state $|11>$ in $T \cong \frac{\pi}{4}\sqrt{N} = \frac{\pi}{4}\sqrt{2^{3-1}} = 1$ times. Using formula (8) can be obtained

$$|11> \frac{1}{\sqrt{2}}(|0>+|1>) = \frac{1}{\sqrt{2}}|110> + \frac{1}{\sqrt{2}}|111>$$
, (9)

The system will collapse to a 50% probability to state $|111>$ and $|110>$.

We choose simulation of typical examples of pattern recognition and General competitive neural network (GCNN) for performance comparison.

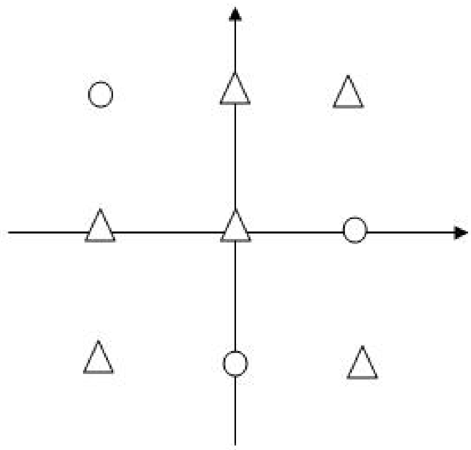

Figure 4. The nine-point pattern distribution map

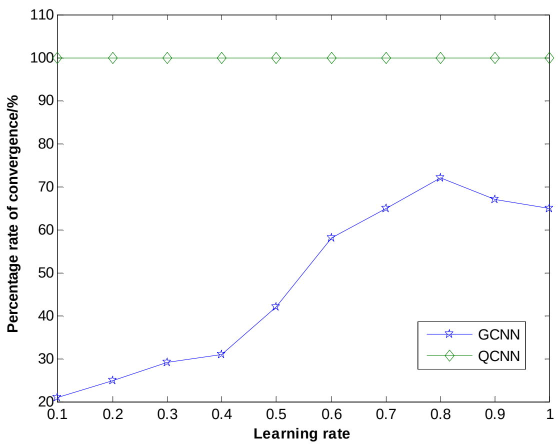

Figure 5. The comparison chart of GCNN and QCNN convergence rate

Nine samples of pattern recognition are shown in Figure 3. This mode is a typical two-class classification problem, which can be seen as "exclusive or" generalization of the problem, often as the inspection algorithms of classification ability scale. Since GCNN and QCNN are respectively as a classifier, network structure are taken by a 2-10-1 model, limited the number of iteration step is 15,000, limited error precision of 0.01, learning rate from 0.1, 0.2,0 ...1 in the value. For each learning rate, respectively QCNN and GCNN to 100 times the simulation, convergence times were recorded as the evaluation index, such as the maximum number of Iterative steps, the minimum number of Iterative steps, and the average number of iterative steps. When the learning rate is changed, the comparisons of the convergence of the two models are shown in Figure 5; the comparisons of the number of Iterative step are shown in Figure 6.

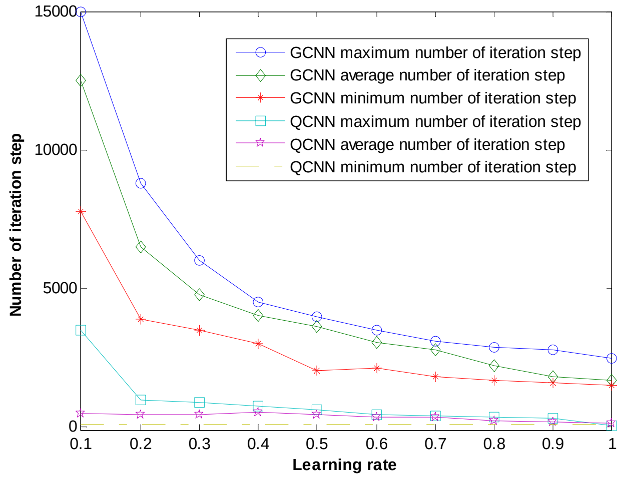

Figure 6. The comparison chart of QCNN and GCNN number of iterative step

Figure 5 shows that when learning rate changes, QCNN convergence rate is of 100%, while the CBP convergence rate of the minimum is 22%, Max is only 69%.Figure 6 shows that when learning rate changes, QCNN the average number of Iterative steps up to 687.70, down to 275.01, fluctuation range is only 412.69.Average number of GCNN iteration steps up to 12335.45, about 6 times of Q QCNN, and fluctuation range is up to 10638.21, about 26 times of QCNN. Simulation results show that QCNN are not only small number of iteration step, also high rate of convergence, when the parameters change with strong robustness.

VI. CONCLUSIONS

Ideas from classical neural network theory are recast in a quantum computational framework, using the language of wave functions and operators. The unique characteristics of quantum systems are utilized to produce a quantum competitive learning network capable of storing exponentially more prototype patterns than possible classically. This demonstrates that quantum computational ideas can be combined with concepts from the field of neural networks to produce useful and interesting results. Simulations using real-world data show that the quantum competitive learner performs very favorably during pattern recall.

Ongoing work focuses on discovering new operators to improve performance by weighted feature discovery. Future work includes searching for further applications of quantum computation to neural networks and generally further developing the field of quantum computational learning.

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