Medical Equipment Utility Analysis based on Queuing Theory

Xiaoqing Lu
College of Science, Hebei United University, Tang Shan, China
luxq2005@126.com

Ruyu Tian, Shuming Guan
College of Science, Hebei United University, Tang Shan, China
try2012@126.com, smguan@126.com

***Abstract*—to improve the efficiency of hospital, reduce patients' waiting time, as well as meet patients' satisfaction, the optimization of patients' queuing system which is based on the knowledge of queuing theory has the practical significance in the use of medical equipments. Based on the relevant data of the usage of B- ultrasound in a hospital, the distribution model of queuing theory has been analyzed by the $\chi 2$ test of goodness of fit, then being able to describe the law of probability of queuing system in a scientific and exact way, and finally the application of the hospital instruments will be arranged in the most efficient way. The wish model can be set and decided by two indexes: one is the average time of patients' waiting time in queuing system, the other is the free rate of medical equipments, and then the director can identify the best status of the model. This method, on one hand, can optimize the hospital service, improve the efficiency as well as supply the scientific evidence and reference for the director to arrange the material and human resources reasonably; on the other hand, it can find out a balance between patients and hospitals through system optimization. In this way, not only can short the waiting time, but also can save the human resources and materials of the hospital. The paper also provides a theoretical basis for hospital administrators to manage and optimize medical equipment.**

***Index Terms*—queuing theory; medical equipment; fit test; optimization**

I. The Background And The Approaches

Since most advanced technical equipments and talents are clustered in the large scale hospital, queuing for the treatment is frequent and normal. Long time waiting not only results in the low efficiency and the waste of equipments and human resources, but also will lead to patients' dissatisfaction and left. Therefore, how to improve service and lower the operating cost together with other issues in queuing system is a primary and cardinal management issue for the large scale hospital to solve.

To solve the queuing problem, what must be done firstly is the queuing theory. Queuing theory is a discipline which solves the optimal design and control service system [3]; it is an important branch of operations research [4]. The queuing theory will be applied into the hospital service through analyzing the routine that the patients come, are treated and then leave in the hospital, and therefore based on it, a qualitative and quantitative evaluation can be received, and meanwhile based on the evaluation of the clinic procedure efficiency can discover the merits and demerits of resource distribution. When arranging the utilities of the medical equipment with queuing theory, two optimization principles ought to be noticed, as follows [12]:

- The patients' satisfaction: to well settle the queuing issue for clinics and pharmacy, the principle of patients' satisfaction should be put in the first place. That is, the hospital should truly and fully mobilize and utilize the all existed resources and aim to meet every patient requirement in the given time.
- The benefit principle: Hospital is supposed to take measures that low cost to achieve the goal of patients' satisfaction. At present, most hospitals are facing the pressure of self-financing. How to reduce the operating costs of major medical equipments is an important orientation of hospital management currently. How to utilize and optimize the all existed resources and improve efficiency and patient satisfaction is an important application of queuing theory discussed in the thesis.

This paper, through case analysis, analyzes the application of the hospitals' medical equipments, and then finds out the best arranging and utilizing way of medical equipments. Specific studies are as follows:

- The statistical inference of B-ultrasound check based on queuing system: First collect and record the customers' arrived time, service time and other data outside the B-room in a hospital during a certain period in the morning. Then process the obtained data with the mathematical statistics. At last infer the probability law of the queuing system, then establish M / M / C queuing model.
- Determine the number of optimal help desk: In order to determine the number of optimal help desk, for the established queuing model, the actual data should be calculated and analyzed with the

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formulas. As to the design or the operation management of the queuing system, the interests of both patients and service part ought to be calculated, with the aim of making the queuing system optimized while achieving any reasonable index.

II. THEORETICAL BASIS

A. Chi Square Goodness of Fit Test

The specific procedures as follows [18]:

- Divide the whole getter area of X into k intervals that do not overlap $[a_{i-1}, a_i)$, $i = 1,2,\cdots k$.
- Under the null hypothesis H_0, with maximum likelihood estimate the distribution of the unknown parameter, the contained unknown parameter can be represented by r.
- Under the null hypothesis H_0, calculate the quantum

$$p_i = P(a_{i-1} \le X < a_i) \quad (1)$$

And then get the theoretical frequency np_i.

- Suppose that x_1, x_2, …, x_n come from the sample observations of X field, then observing the quantity that the locations of x_1, x_2, …, x_n are in the arrange of $[a_{i-1}, a_i)$, that is, the actual frequency f_i, $i = 1,2,\cdots k$.

According to the formula

$$\chi^2 = \sum_{i=0}^{10} \frac{(f_i - \bar{f}_i)^2}{\bar{f}_i} \quad (2)$$

$$\bar{f}_i = nP_i \quad (3)$$

Calculate the value of χ^2.

- Based on the given significance level α, find out χ^2 distribution table on the condition of the degree of freedom of $k - r - 1$ and then get the critical value of $\chi^2(k-r-1)$.
- If $\chi^2 \ge \chi^2(k-r-1)$, the null hypothesis H_0 will be refused; otherwise, the null hypothesis H_0 will be accepted.

B. The Structure of the Queuing System

Actually the queuing system is various; however, from the main determine factors of the queuing system, there are three primary components, that is, input procedure, queuing rule and service part.

1. Input procedure

Input procedure is used to describe the source of the clients and the law of the clients arriving at the queuing system. Several various situations are concluded in this input procedure, and meanwhile they are compatible.

- The quantity of the clients (that is the source of the clients): the construction of the clients is various, maybe limited or infinitive.
- The way of arrival: the situation of the clients' arrival is also various. They may arrive in a continuous way, or in a discrete way, or one by one, or coming in large quantity.
- The intervals between the different clients can be a certain type, and also can be a random type. If all the parameters (such as Expectation, variance and so on) that are used to describe the distribution of the intervals between the different clients are all not related to the time, and then this input procedure is called the stationary input procedure, otherwise it is called the non-stationary input procedure.

2. Queuing rule

Queuing rule refers to the system whether the service system permits the clients to queue or not, and whether the clients are willing to accept the queuing. If the service system permits to wait, what will be the sequence? The queuing rule is usually divided as: losing system, waiting system as well as mixing system [18].

- Losing system: when the clients arrive, all the service desks are taken and at the same time the service agent does not allow the clients to wait, so the clients have to leave to choose other place to accept the service or have to give up the demand. If the clients leave at once, it can be called as instance system or losing system.
- Waiting system: when the clients arrive, all the service desks are working for the current clients, but the clients automatically join in the queue to wait for the service till they receive the service. In this system, according to the serving sequence, there are several rules can be adopted:

  First come first serve, that is, the service system offers the service according to the clients' arrival sequence. This is the most common serving law.

  Last come first serve, that is, the service system offers the service opposite to the clients' arrival sequence. For example, in the information system the latter part information is much more important, so has to be settled first.

  Have the priority in the service system, that is, among the waiting clients, due to the distinctiveness of some clients some service objects must be treated first.

  Random service refers to choose the client in random to offer the service not considering the arrival sequence.

In the study of the queuing system, the length of the queue is not related to the law of service, but the waiting and staying time has relationship with the law of service. Consequently, the different service law will directly affect the time cost in the queuing system.

- Mixing system: it is combined by the losing system and the mixing system, the service agent only allows limited number clients to wait, the extra clients having to leave. In addition, some clients are not willing to wait when they saw the long queue; for the short queue they stay to wait; or the waiting period of the clients can not beyond the value of T otherwise they will leave.

3. The service agent

Describe the main aspects of the service agent, as follows:

The quantity of the service desks, and if there are several desks existed, they are in series or in parallel.

The serving time of clients should obey what kind of probabilistic distribution; the service time of every client is independent or not; the services offer to the clients per person or per group and so on.

C. The Classification and the Symbolization of the Queuing Model

The ordinary form of the classification of the queuing model is put forward by D. G Kendall is $X/Y/Z/A/B/C$, among them X referring to the interval of the clients' arrival time, Y representing the service time, Z referring to the number of desk, A instead of the capacity of the system, that is, the largest quantity of the clients (containing the being served clients and the waiting clients), B replacing the source number of the clients, C referring to the serving rule. In other words, the criteria of the classification is the main characteristics of the queuing system, that is, the distribution of the intervals of the clients' arrival, the serving time and the quantity of service desks.

D. The Optimal Design of the Queuing System

To optimize the design of queuing system, the one hand, considering the patients, the patients stay as short as possible; the other side, when the device is increased, the idle of the device may increase and cause waste[14]. Therefore, adding the equipment is conditional. We can use the tools of economic analysis to minimum the costs:

$$\text{The total cost} = \text{service cost} + \text{line damages} \tag{4}$$

When the total cost is the smallest, the corresponding service level is the optimal. Optimization problem can also be considered from the service side. For example, in a loss control service system, when a customer is serviced, service organization can receive certain income. Service rate is higher, the patients' loss is littler, so the income is more .On the other hand, the service rate is higher, and the expense is greater. Then you will find the optimal service rate and make server's net income maximal.

Suppose unit time cost per each equipment is h , the loss expenses of unit time which each sufferer stops in the system is b , so the expenses function f is service cost of unit time plus linger to lose expenses [13]

$$f(C) = h \cdot C + b \cdot L(C) \tag{5}$$

L(C) is the number of sufferers that stop in the system which has C set equipments.

Adopting a marginal analysis and according to the essential condition, the expenses function has a minimum value is

$$f(C^*) \le f(C^* - \mathbf{1}) \tag{6}$$

$$f(C^*) \le f(C^* + \mathbf{1}) \tag{7}$$

Based on analysis and computation results

$$L(C^*) - L(C^* + \mathbf{1}) \le \frac{h}{b} \le L(C^* - \mathbf{1}) - L(C^*) \tag{8}$$

By computer simulation, we can calculate the difference between two items of L(1),L(2),L(3),··· ,observe the constant being located in two items, then determine the optimal solution C^* that is, the optimal quantity of equipment, which make the cost of patients losing and the cost of hospital services to be minimum[6][10].

III. THE SITUATION ANALYSIS OF THE APPLICATION OF MEDICAL EQUIPMENTS

A.Cases and Related Data

As for the data collection, the quantity of medical devices, costs of medical equipment, assignment of the operators and so on are direct and available, and don't need choose samples in random. To get the average arrival rate of patients in different time, average service rate, and the probability distributions of patients' waiting time and service time, it needs to collect the following sample data including the quantity of patients, the waiting time and the duration of examination.

Now collect, arrange and analyze the data of the arrival time and the time of the patients to receive the service. Calculate the quantity of patients that arrive at the hospital per unit time and the distribution of service time (read Table Ⅰ and Table Ⅱ). The unit time of the table is one hour. Service time is the time used to complete patients' exam, counting by minute.

TABLE I.

THE NUMBER OF B-ULTRASOUND PATIENTS WHICH REACHES IN UNIT TIME (PER HOUR)

Arrivals	0	1	2	3	4	5	6
Observed frequency	2	4	18	23	22	15	12
Arrivals	7	8	9	≥10		total	
Observed frequency	9	4	3	1		113	

TABLE II.
THE SERVICE TIME (MINUTES) DISTRIBUTION OF B-ULTRASOUND PATIENTS

Service time	0-15	15-30	30-45	≥45	total
Arrive frequency of patients	58	27	15	2	102 persons

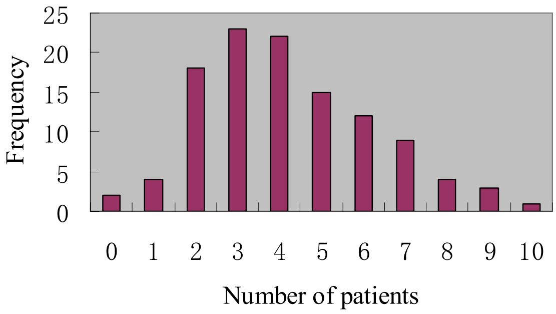

Figure 1 Frequency histogram of B-ultrasound arrival time

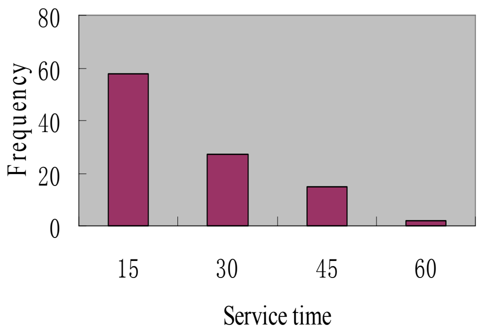

Figure 2 Frequency distribution histogram of service time

B. The Basic Characteristics of Queuing System [1, 7]

1. Input process: the patient's arrival and B-ultrasound examination time is random. So ultrasound examination system is a typical stochastic service system. The system has the following characteristics:

- Unlimited number of patients overall, all parameters (such as expectation, variance, etc.) of the distribution of patients' arriving interval time are independent of time, and their arrival is a smooth entry process;
- The patient's arrival is a one by one and independent process, and they do not affect the others.

2. Queuing rules: To a help desk, a queue lines a single team, there is no restriction on the captain, and apply the rule that first come first serve when the patients arrive. If the desk is available, then check; if the desk is obtained, the patients will be waiting in line.

3. Help desk: Suppose N Desks arranged in parallel and each desk works independently; the average service rate is the same; each desk can only serve a client per time.

4. Service hours: the service time of patients' receiving is random, and the law can be described by probability distribution, and the service time of patients' receiving is independent.

In summary, the service system is a multi-server system of $M/M/C/\infty/\infty$ queuing system.

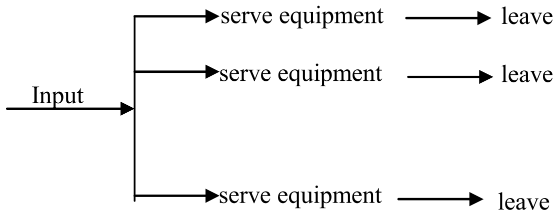

Figure 3 B-ultrasound queuing model

Figure 3 [8, 16], the queue system is a single team, and C set of serving equipments are parallel. The service devices are equipped with a medical doctor, and a desk on queuing theory also can form a queuing system.

C. Fitting Test

In order to understand and master the law of an operating queuing system, we need multiple observations and data. Secondly, with mathematical statistics process the collected data, and observe to identify the type of model in queuing system. Then we can use appropriate theory to study and solve the problem of queuing system. The statistical inference of queuing system in this case applies goodness of fit for the χ2 test to determine a given queuing system in line with distribution model [2].

- Patients arriving number in per unit time subject to fitting test of possion distribution (read Table III) [5,15].

An average arrival rate per unit time of patients

$$\lambda = \frac{\sum i\, f_i}{113} = 4.2 \text{ persons/hour} \quad (9)$$

The data in Table 3 obtained by the following Formula: Probability

$$P_i = \frac{\lambda^i}{i!} e^{-\lambda} \quad (10)$$

Theoretical frequency

$$\bar{f}_i = 113 P_i \quad (11)$$

$$\chi^2 = \sum_{i=0}^{10} \frac{(f_i - \bar{f}_i)^2}{\bar{f}_i} \quad (12)$$

Therefore, the quantity of patients' arrival to receive B-ultrasound examination in unit time is subjected to the possion distribution which parameter is 4.2.

TABLE III.

THE POSSION DISTRIBUTION χ^2 FITTING TEST FOR THE NUMBER OF PATIENTS PER UNIT TIME

Number of patients (i)	0	1	2	3
Observed frequency (f_i)	2	4	18	23
Probability (P_i)	0.0150	0.0630	0.1323	0.1852
Theoretical frequency ($\bar{f}_i$)	1.6945	7.1169	14.9455	20.9237
$\frac{(f_i-\bar{f}_i)^2}{\bar{f}_i}$	0.0551	1.3651	0.6243	0.2060
Number of patients (i)	4	5	6	7
Observed frequency (f_i)	22	15	12	9
Probability (P_i)	0.1944	0.1633	0.1143	0.0686
Theoretical frequency ($\bar{f}_i$)	21.9699	18.4547	12.9183	7.7510
$\frac{(f_i-\bar{f}_i)^2}{\bar{f}_i}$	0.0001	0.6467	0.0653	0.2013
Number of patients (i)	8	9	≥10	Total
Observed frequency (f_i)	4	3	1	113
Probability (P_i)	0.0360	0.0168	0.0071	
Theoretical frequency ($\bar{f}_i$)	4.0693	1.8990	0.7976	
$\frac{(f_i-\bar{f}_i)^2}{\bar{f}_i}$	0.0012	0.6384	0.0514	χ^2 =3.8549

Take α=0.05, γ =11-2=9.

Look-up table to be

$\chi^2{}_{0.05,9}$=16.919, $\chi^2 < \chi^2{}_{0.05,9}$, P>0.05.

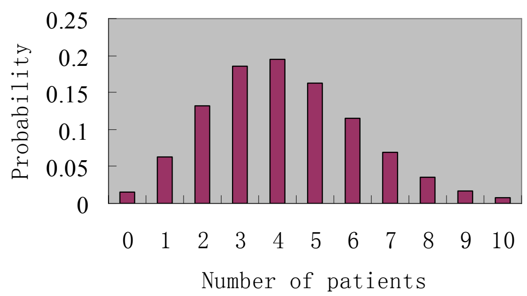

Figure 4. A possion distribution of patients arriving in B-ultrasound examination

- Service time follows a negative exponential distribution fitting test

The average service time is

$$\frac{1}{\mu}=\frac{\sum X_i f_i}{102} \qquad (13)$$

The value of the group is X_i .

Solution was μ =0.0596 person / minute.

Table IV data can be obtained by the following formula:

Probability

$$P_i = P(x_i < \xi < x_{i+1}) = e^{-\mu x_i} - e^{-\mu x_{i+1}} \qquad (14)$$

Theoretical frequency

$$\bar{f}_i = 102 P_i \qquad (15)$$

$$\chi^2 = \sum_{i=1}^{4} \frac{(f_i-\bar{f}_i)^2}{\bar{f}_i} \qquad (16)$$

TABLE IV.

χ^2 FITTING TEST FOR THE NEGATIVE EXPONENTIAL DISTRIBUTION OF SERVICE TIME

Service time (Minutes)	0--15	15--30	30--45	45--60	total
Observed Frequency (f_i)	58	27	15	2	102
Probability (P_i)	0.5913	0.2417	0.0988	0.0404	
Theoretical frequency ($\bar{f}_i$)	60.3111	24.6500	10.0748	4.1177	
$\frac{(f_i-\bar{f}_i)^2}{\bar{f}_i}$	0.3126	0.2241	2.4077	1.0891	χ^2 = 4.0335

Take α=0.05, γ =4-2=2.

Check the number of table

$\chi^2{}_{0.05,2}$ =5.99, $\chi^2 < \chi^2{}_{0.05,2}$, P>0.05.

B-ultrasound service time follows a negative exponential distribution $\mu = 0.052$.

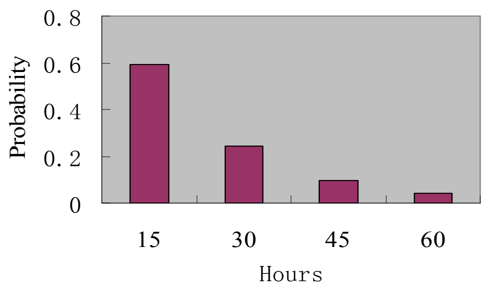

Figure 5. A negative exponential distribution of B-ultrasound service time

D. Index Calculation and Optimization of System

From the above fitting test results, the queuing system in line with $M/M/C/\infty/\infty$ queuing model [3], the application of indicators of $\lambda =$ 4.2 people / hour = 0.07 people / minute, $\mu =$ 0.0596 person / min, C=2. Calculated as follows [3, 11]:

Service intensity:

$$\rho = \frac{\lambda}{C\mu} = 0.59 \tag{17}$$

Free probability:

$$P_0 = \left[\sum_{K=0}^{C-1} \frac{1}{K!}\left(\frac{\lambda}{\mu}\right)^K + \frac{1}{C!}\cdot\frac{1}{1-\rho}\cdot\left(\frac{\lambda}{\mu}\right)^C \right]^{-1} = 0.26 \tag{18}$$

The average number of patients waiting for examination:

$$L_q = \frac{(C\rho)^C \cdot \rho}{C!\cdot(1-\rho)^2}\cdot P_0 = 0.64 \text{ persons} \tag{19}$$

The average number of patients staying in the system:

$$L_S = L_q + C\rho = 1.82 \text{ persons} \tag{20}$$

The average waiting time of patients:

$$W_q = \frac{L_q}{\lambda} = 9.14 \text{ minutes} \tag{21}$$

Average length of stay of patients:

$$W_s = W_q + \frac{1}{\mu} = 25.92 \text{ minutes} \tag{22}$$

The probability of patients waiting for examination:

$$P\left(C, \frac{\lambda}{\mu}\right) = \sum_{n=C}^{\infty} P_n = \frac{C^C \rho^n P_0}{C!\cdot(1-\rho)} = 0.44 \tag{23}$$

TABLE V.
THE OPERATION OF INSTRUMENT QUANTITY

Instrument quantity	ρ	P_0	L_q	W_q	$P\left(C,\frac{\lambda}{\mu}\right)$
2	0.59	0.26	0.64	9.14	0.44
3	0.39	0.25	0.07	1.00	0.11
4	0.29	0.19	0.01	0.14	0.02

The optimization of queuing system is actually critical to determine the optimal number of help desk. If we consider it from the cost structure, there are usually two kinds of cost in the service system. One is the loss cost of each client waiting for in the system; the other is the unit time service cost of every desk. If the average total cost of formula can be identified, it will be able to determine the optimal number of help desk and make it minimize [15].

From the above results, when there are two sets of the B-ultrasound instruments, the system utilization is 59% and idle probability is 0.26, and 44% of patients have to wait in line after arrival; the average waiting time is 9.14 minutes, and average length of stay is nearly 26 minutes. The system is busy; it is difficult to guarantee the quality of service. To optimize the system, it will now increase the number of instrument after observing the performance indicators (see TableⅤ). We can see that when the device is increased to three, the capacity utilization is 39%, and free probability is 0.25 and 11% of patients have to wait in line after arrival; average waiting time is only 1 minute. If we increase to 4 sets of equipment, although the indicators improved significantly, but capacity utilization is only 29%. While reducing the cost of waiting patients, it increases the cost of service, apparently resulting in some waste [20]. In summary, 2 equipments are basically rational, but 3 equipments is the optimal choice of the system [18].

E. Comparison of Patients with Two Kinds of Queuing Models [9, 17]

In the previous study, we assume that patients can be transferred ahead of the B- ultrasound equipment, and regarded the patient's line as a "single team", and then establish $M/M/C$ queuing model. If patient cannot be transferred in front of all B- ultrasound equipments, the each desk will become an independent queuing system. The whole queuing system will change into n $M/M/\mathbf{1}$ queuing models.

Here comparing the two queuing approaches, think about a situation that the set quantity is 3.

If we assume that there is no patient transferring in front of each other B- ultrasound equipment, the average arrival rate is

$$\frac{\lambda}{n} = \frac{0.07}{3} = 0.023 \text{ people / minute} \tag{24}$$

Average service rates are remained, so the system becomes five M / M / 1 queuing systems with λ =0.023 people / minute and μ =0.0596 person / minute.

Free probability:

$$P_0 = 1 - \frac{\lambda}{\mu} = 0.39 \tag{25}$$

The average number of patients waiting for check:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = 0.24 \text{persons} \tag{26}$$

The average number of patients stays in the system:

$$L_S = \frac{\lambda}{\mu - \lambda} = 0.63 \text{ persons} \tag{27}$$

The average waiting time of patients:

$$W_q = \frac{L_q}{\lambda} = 10.43 \text{ minutes} \quad (28)$$

Average length of stay of patients:

$$W_s = \frac{1}{\mu - \lambda} = 27.32 \text{ minutes} \quad (29)$$

Now compare mainly indexes of M / M / C and M / M / 1 queuing system as follows (see table VI):

TABLE VI.
$M/M/C$ AND $M/M/1$ QUEUING SYSTEM PERFORMANCE COMPARISON

	P_0	L_q	W_q	W_s
$M/M/C$	0.25	0.07	1.00	17.78
$M/M/1$	0.39	0.24	10.43	27.32

The figures in tableⅥindicate that $M/M/C$ (single queue) queuing system's the length of the queue; waiting time and the average stopping time of patient are significantly lower than the $M/M/\mathbf{1}$ (multi-queue) queuing system. In a multi-team queuing system, the free rate of service equipment is 39% which also caused a waste of resources. From this, efficiency of single team in multi-server system is relatively good and is consistent with the theoretical conclusion. The reason lays in that the system operating status and indicators are different in different queue fashion. The joint service (single queue) is more effective than the decentralized services (multi-queue). Therefore, the hospital should adopt single-queue and multi-server system, it is also to say all patients need the check line up into single queue. The queue can be avoided collision and hustle with each other, and will improve the hospital's service status and efficiency.

IV. CONCLUSIONS

But we estimate the cost in practice, especially the loss cost of the customer waiting is very difficult. Therefore, a desire model is developed. It is according the level of intention to determine the best model. To determine the model number of the best desk C will involve the following two indicators:

- The average waiting time of the patients in the system W_q ;
- The free rate of medical equipment is P_0 .

Two above-mentioned indexes become the critical value. Decision-makers can accord the following two formulas to determine the range of the optimal number of devices.

$$1 - \frac{\lambda}{C\mu} \leq P_0 \quad (30)$$

$$\frac{\lambda}{C\mu} < 1 \quad (31)$$

Then according to the number of equipments, we can compute the sufferer waiting time in the system, one by one; make the sufferer average waiting time W_q in the system to the upper bound value for determining the optimal number of equipment units. If there is no C value to be satisfied with two objects, you need to correct one of the goals.

Based on the arrival routine of client flow, service system manager's task is that control and regulate the service system to make sure it is in the best operating condition. It is necessary to meet the clients' requirements and minimize the total cost of the community (or the net income of service organization is maximum) or make other index to attain the optimum. Therefore, the managers of hospital should continuously study the rule of the clients' arrival with the knowledge of queuing theory. The study and the queuing theory will help to design and regulate the service level and other indicators, making the hospital achieve the best service effect [19].

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