A New Hybrid Grid Multiple Model Estimation Based on STF

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***Abstract*—The paper presents a hybrid model grid variable structure multiple model algorithm basing on strong tracking filter (STF-VSMM) which is used to state estimation for complicated system. The total model set for STF-VSMM is the combination of a coarse model grid and an adaptive fine grid moving freely in the system mode space. During the produce of the fine model grid, the paper uses a strong tracking filter to get the center position. Then, STF-VSMM can form a two-double bestrow of the system mode space. At last, the paper realizes the accurate tracking of maneuvering target using STF-VSMM. Simulation results demonstrate that STF-VSMM estimator outperforms the corresponding fixed structure multiple model (FSMM) at a negligible extra computational cost.**

***Index Terms*—maneuver target tracking; multiple model estimation; strong tracking filter; VSMM**

I. INTRODUCTION

A hybrid system involves two types of components: the base state which varies continuously and the model state which may jump only. For the estimation problem of hybrid systems, multiple model (MM) approach is a valid solution. It is cost-effective and robust and has a parallel structure. In the MM approach, a set of models is designed to cover the possible system behavior patterns. This model set has fixed structure during the algorithm running. In the result, this approach is always been named fixed structure multiple method (FSMM). However, when applying the FSMM to hybrid estimation, we sometimes encounter two problems: First, the chosen model set may not cover the full range of the mode, the truth may lie between the adjacent models; second, even if the chosen model set is large enough to cover the full range, use of all those models does not necessarily guarantee performance improvement, not to mention the prohibitively large computational cost. It was demonstrated that use of too many models may be as bad as use of too few models.

To overcome the limit of FSMM, X. R. Li presented variable structure multiple model method (VSMM) in 1992. The basic idea of VSMM is that uses a model set whose structure is changing to replace the fixed structure model set in FSMM. VSMM is a probabilistically weighted sum of all estimators based on admissible mode sequences that are mutually exclusive and exhaustive, while FSMM is of all estimators based on possible mode sequences. Generally, VSMM estimation consists of two functional components: model set adaptation (MSA) and model-set sequence conditioned estimation (MSSCE). MSSCE aims to provide the best possible estimation given a model-set sequence. MSA, which is unique for VSMM, aims to determine the model set at each time for the MM estimation, using the information contained in measurements as well as a priori knowledge. Different VSMM algorithms differ from one another primarily with respect to how the model set adapts.

Under the frame of VSMM, the paper develops a practical algorithm for MM estimation, called the variable structure multiple model basing on strong target filter (STF-VSMM). STF-VSMM uses a hybrid model grid consisted of a fixed coarse model grid (CMG) and an adaptive fine model grid (FMG). The area center of FMG is updated by STF in real time. It is an online processing scheme, and is particularly advantageous when the mode space is continuous and large, and the mode involves jumps of small or medium magnitudes. Via simulation in the context of maneuvering target tracking in different scenarios, STF-VSMM estimation is shown to have a good adaptive ability and better performance than the corresponding FSMM.

II. DESCRIPTION OF STF-VSMM

As is known, the performance of a STF-VSMM depends highly on how close the model set used in the

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approach is to the true mode. It is best that the true mode can be added to the model set within the MM estimation framework. But it is impossible because that the true mode of system is unknown. An EMA method is provided in [2], which improves the overall estimation through adding an optimal estimation of system to the fixed structure coarse model grid. The optimal estimation of system can close to the real system mode in statistical probability. STF-VSMM improves the EMA on two aspects: first, STF-VSMM improves the result of FSMM by adding a batch of models, so, the amendatory power of the optimal estimation of system is enhanced; second, STF-VSMM introduces the STF method to decide the center position of fine model grid. So, it is possible to get the fine model set which is closer to the real system mode, no matter the target makes strong maneuver or not.

As the result, the hybrid model grid in STF-VSMM can be depicted as followed. The model set in effect at the current time is composed of two model subsets: fixed coarse and adaptive fine. Firstly, the coarse subset is quantized from the mode space crudely, that is, the spacing between the quantization levels is large, and it is fixed at all times. Secondly, the fine subset is quantized from the region surrounding the optimal estimate of the true mode, and the quantization is finer than the coarse subset. The true mode may jump, so the fine subset is adaptive and time-varying.

Using the hybrid grid has the following advantages:

1) The coarse grid provides a robust scheme to handle abrupt jumps of the system mode and directs the placement of the fine grid to be based on. The fine grid can be adapted in a relative small and better unit, which intuitively makes the mode and state estimate more accurate.

2) The HG can be also viewed as a generalization of the EMA, for the mode estimation-error, as well as the mode estimate, is incorporated into the model inference. The HG scheme is suitable for the estimation of the system whose mode involves jumps of different magnitudes.

III. DESIGN AND REALIZATION OF STF-VSMM

One important task of STF-VSMM is the decision of FMG, and the important task has two steps: first, the decision of the center position; second, the decision of the radius.

A. Get the Center Position of FMG

We must get the optimal system estimation if we want to decide the center of FMG, at the same time, the value of determines the position of FMG in the total system mode space. So the distance between and real system mode affect the precision of the STF-VSMM, greatly. In EMA, is calculated by (1).

$$\hat{s}_k = \sum_{a_j \in M_{k-1}} u_j(k-1)a_j \qquad (1)$$

M_{k-1} is the fixed CMG at the time k-1, a_j is the acceleration of model j, uj is the probability of model j at the time k-1. When the target is not in the maneuver style, the by (1) is close to real system mode; but when the target is in the style of maneuver, will be aberrant and degrade the accuracy of STF-VSMM. The paper [4] provides a strong target tracking method. This filter selects the proper time varying filter gain *K(k+1)* online to make (a) the mean of residual is least; (b) the residual approximates Gaussian white noise. When the model matches the actual system mode, the import residual of kalman filter is not auto-correlational Gaussian white noise sequence. As the result, when the target makes great maneuver, the STF still can get the preferable tracking result. The STF can adaptively adjust the filter gain basing on residual, through importing an attenuation gene. After getting the filter result of STF, STF-VSMM obtains the area center of FMG by (2).

$$\begin{aligned}\hat{s}_{STF}(k) &= (1-\delta)*\hat{a}_{CMG}(k-1)+\delta*\hat{a}_{STF}(k-1)\\ &= (1-\delta)*\sum_{a_j\in M_{k-1}} u_j(k-1)a_j(k-1)+\delta*\hat{a}_{STF}(k-1) \qquad (2)\end{aligned}$$

δ is adjust gene, usually, δ=0.5. is the estimation of acceleration basing on coarse model set. is the estimation of acceleration basing on fine model set.

B. Strong Tracking Filter

In STF, the linear discrete-time model for a maneuvering target is represented by (3~4).

$$X(k+1) = F(k+1,k)X(k) + U(k)\bar{a}(k) + w(k) \qquad (3)$$

$$Y(k+1) = H(k+1)X(k+1) + v(k) \qquad (4)$$

Where X is state vector, $X = [x, v_x, a_x, y, v_y, a_y]'$; Y is measurement vector; H is observe matrix and H=[1,0,0,0,0,0; 0,0,0,1,0,0]; $\bar{a}(k)$ is the mean of current acceleration, F is the state transition matrix refer to (5). U is control input matrix as (6). T is the sampling period, α is the maneuver frequency . *v(k)* and *w(k)* are process noise and measurement noise respectively. Suppose they are independent white Gaussian noise with zero mean and known variance matrix *Q(k)* and *R(k)*. The value of *Q* is illustrated in [4].

$$F(k+1,k) = \begin{bmatrix} 1 & T & (-1+\alpha T+e^{-\alpha T})/\alpha \\ 0 & 1 & (1-e^{-\alpha T})/\alpha \\ 0 & 0 & e^{-\alpha T} \end{bmatrix} \qquad (5)$$

$$U(k) = \begin{bmatrix} (-T+\alpha T^2/2+(1-e^{-\alpha T})/\alpha)/\alpha \\ T-(1-e^{-\alpha T})/\alpha \\ 1-e^{-\alpha T} \end{bmatrix} \qquad (6)$$

The process of strong tracking filter is as following (7~12):

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)r(k+1) \qquad (7)$$

$$\hat{x}(k+1|k) = F(k+1,k)\hat{x}(k|k) + U(k)\bar{a}(k) \qquad (8)$$

$$r(k+1) = y(k+1) - H(k+1)\hat{x}(k+1|k) \qquad (9)$$

$$K(k+1) = P(k+1|k)H^T(k+1) \bullet [H(k+1)P(k+1|k)H^T(k+1)+R(k)]^{-1} \quad (10)$$

$$P(k+1|k) = \lambda(k+1)F(k)P(k|k)F^T(k) + \Gamma(k)Q(k)\Gamma^T(k) \quad (11)$$

$$P(k+1|k+1) = [I - K(k+1)H(k+1)] \cdot P(k+1|k) \quad (12)$$

The key part of STF is the decision of attenuation gene.

$$\lambda(k+1) = \begin{cases} \eta(k+1), & \eta(k+1) > 1 \\ 1, & \eta(k+1) < 1 \end{cases} \quad (13)$$

$$\eta(k+1) = \frac{tr[N(k+1)]}{tr[M(k+1)]} \quad (14)$$

$$N(k+1) = V_0(k+1) - \beta R(k+1) - H(k+1)\Gamma(k)Q(k)\Gamma^T(k)H^T(k+1) \quad (15)$$

$$M(k+1) = H(k+1)F(k)P(k|k)F^T(k)H^T(k+1) \quad (16)$$

$$V_0(k+1) = E[r(k+1)r^T(k+1)] = \begin{cases} (\rho V_0(k) + r(k+1)r^T(k+1))/(1+\rho), & k >= 1 \\ r(1)^T r(1), & k = 0 \end{cases} \quad (17)$$

Where ρ is the forget gene, usually ρ = 0.95, r(1) is initial residual. After we get the filter result of STF, we can calculate the area center of FMG as (2).

C. Get the FMG

Here, we present a simple approach to produce the FMG. The fine model grid is formed by quantizing the region whose center is , and it has a fixed area radius. The grid distance is predefined with the prior knowledge.

D. The realization of the STF-VSMM

Traditional FSMM and STF whose filter results are used to update the area center of FMG run parallel in the STF-VSMM. Then a fine model grid which has strong amendatory capability is produced and the overall system estimation basing on optimal fusion theory will be obtained. The details of STF-VSMM is as Fig.1

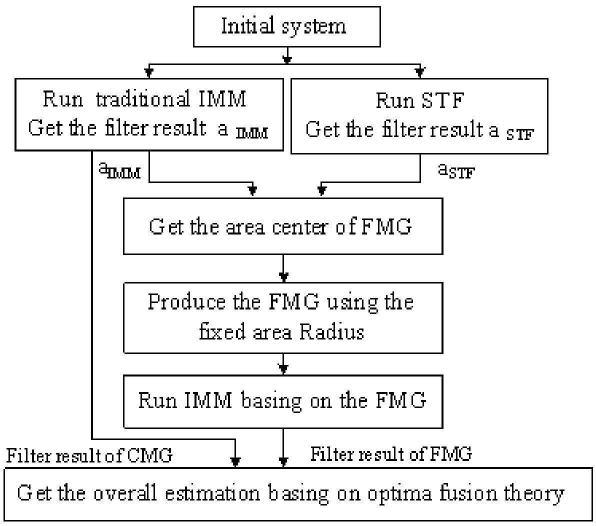


Figure 1 Flow Chart of STF-VSMM

IV STF-VSMM IN MANEUVER TARGET TRACKING

The state and observation equation of the traditional FSMM which bases on coarse model grid (CMG) can refer to (18~19).

$$X(k+1) = FX(k) + G(a_k + w_k) \quad (18)$$

$$Y(k+1) = H(k+1)X(k+1) + v(k) \quad (19)$$

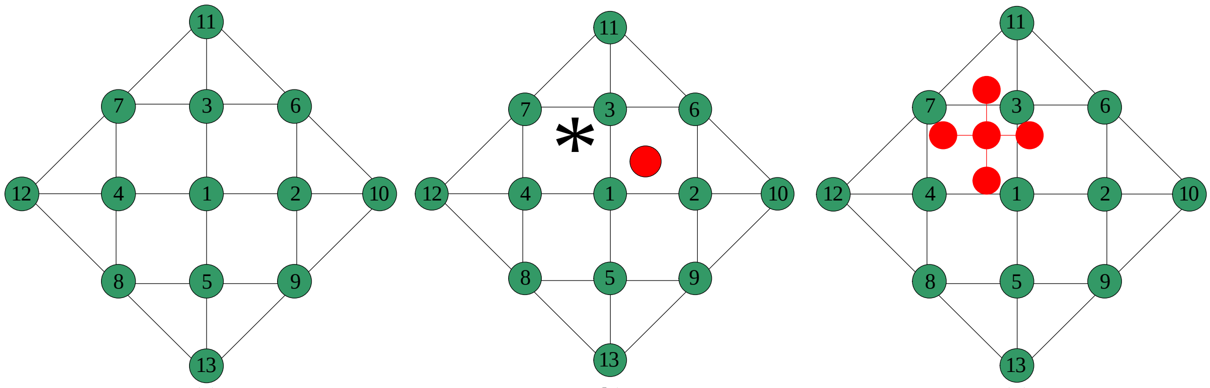
where $X = (x, v_x, y, v_y)'$ is the state vector, z is the measurement vector, $a = (a_x, a_y)'$ is the acceleration. $w \sim N(0,Q)$ and $v \sim N(0,R)$ are mode-dependent Gaussian process and measurement noises respectively and mutually independent, $F = diag\ [F_2,F_2]$ and $G = diag\ [G_2,G_2]$.

$$F_2 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

In addition, supposes the acceleration of maneuver target will be obtained by quantizing of the acceleration space:

$$A^c = \{(a_x, a_y): |a_x| + |a_y| < a_{max}\} \quad (20)$$

And the jump among the acceleration governed by a Markov process with a transition probability matrix. The model set in FSMM and STF-VSMM is depicted in Fig 2.



(a) (b) (c)

Figure 2 The mode set in FSMM/EMA/STF-VSMM

Figure 2(a) represents the model set in FSMM, whose structure is fixed. Figure 2(b) depicts the model set belonging to EMA. The model (1~13) is the coarse model grid whose structure is fixed, and the symbol "*"represents the real system mode, the symbol "●"represents the optimal estimation basing on coarse model grid. When the target makes strong maneuver, the estimation will depart from real system mode. Figure 2(c) represents the model set which is used by STF-VSMM. As above mentioned, model (1~13) is the coarse model grid, and the red fine model grid is the amendatory model set whose center area is the probabilistically weighted sum of filter results belonging to FSMM and STF respectively. Even the target makes strong maneuver, the optimal estimation can be close to the real system mode.

V. SIMULATION

A. Design of Simulation Scene

The parameters of coarse model grid in simulation are decided as following: T = 1s, the models in coarse model grid are initialized as {m_1,m_2,m_3,m_4,m_5}, the initial probility is P_{CMG}={u_1=u_2=u_3=u_4=u_5=1/5}, the state vector is X_0={0,10;0,10}. The process noise covariance Q_v=0.6 and the measure noise covariance R_x=100.

To prove the validity of STF-VSMM, the paper designs three different simulation scenes, DS_1 and DS_2 belong to decided scenes. The concrete parameters are described by Figure 3, and the sequence pairs in the table denote the accelerations with x/y axis in different time.

Scene/Time	DS1	DS2
0~10	[0,0]	[0,0]
10~20	[38,0]	[45,36]
20~30	[37,40]	[0,42]
30~40	[0,42]	[-42,0]
40~50	[-37,40]	[-37,0]
50~60	[-40,0]	[0,0]
60~70	[-43,-38]	[38,-38]
70~80	[0,-41]	[80,-41]
80~90	[42,-36]	[42,45]
90~100	[0,0]	[0,40]

Figure 3 The Scene of Simulation

The first DS_1 assumes that the real acceleration jumps only among the nodes that are borders upon each other in Figure 2(a). The second DS_2 assumes that the real acceleration jumps in arbitrary nodes in Fig 2(a). The models in coarse model grid are illustrated by (21). The compared algorithm is FSMM.

It should be emphasized that the evaluation and thus comparison of MM algorithms depend to a large degree on the scenarios used. Both deterministic and random scenarios were designed for this example.

For the random scenario, it is assumed that the acceleration vector $a(t)=a(t)\cdot\Delta\theta(t)$ is a semi-markov process. It is a 2-dimensional process that would be Markov were the sojourn time τ for each of its states not random. In simple terms, it implies that the acceleration process undergoes sudden jumps from a state with a magnitude a and phase θ to another one after staying in it for a random period of time.

1) The sojourn time τ_k of the state ak conditioned on ak has a truncated ($\tau_k > 0$) Gaussian density with mean τ_k and variance σ^2_τ..

2) The acceleration magnitude a_{k+1} has probability masses of P_0 and P_M to be zero and maximum, respectively, and is uniform over the values in between, where P_0 and PM are in general functions of a_k.

3) The angle θ_{k+1} of acceleration is uniform over 2π if a_k=0 and is Gaussian with mean θ_k and variance σ^2_θ if $a_k \neq 0$.

The following parameters were used in our design:

$$\bar{\tau} = \bar{\tau}_M + (a_{\max} - a)(\bar{\tau}_0 - \bar{\tau}_M) / a_{\max}$$

$$\sigma_\tau = \bar{\tau}_a / 12, \quad \bar{\tau}_M = 10, \quad \bar{\tau}_0 = 30$$

$$P_M = 0.1, \quad a_{\max} = 80, \quad \sigma_\tau = \pi / 12$$

$$P_0 = \begin{cases} 0.6, & a_k \neq a_{\max} \\ 0.8, & a_k = a_{\max} \end{cases}$$

The random sojourn time τ is rounded to its nearest integer and the initial acceleration a_1 was set to zero. The Monte Carlo simulation runs 50、100、150 in the three scenarios respectively.

B. The Results and Analysis of Simulation

Here, the paper presents the filter results of position and velocity, at the same time, the standard deviation is a so important standard of the algorithm performance that we present the results in the figures.

Figure 4 depicts the position result of simulation in DS_1. Figure 5 presents the standard deviation of estimation error belong to position and velocity in DS_1; Figure 6 depicts the position result of simulation in DS_2. Figure 7 presents the standard deviation of estimation error belong to position and velocity in DS_2; Figure 8 depicts the position result of simulation in DS_3. Figure 9 presents the standard deviation of estimation error belong to position and velocity in DS_3.

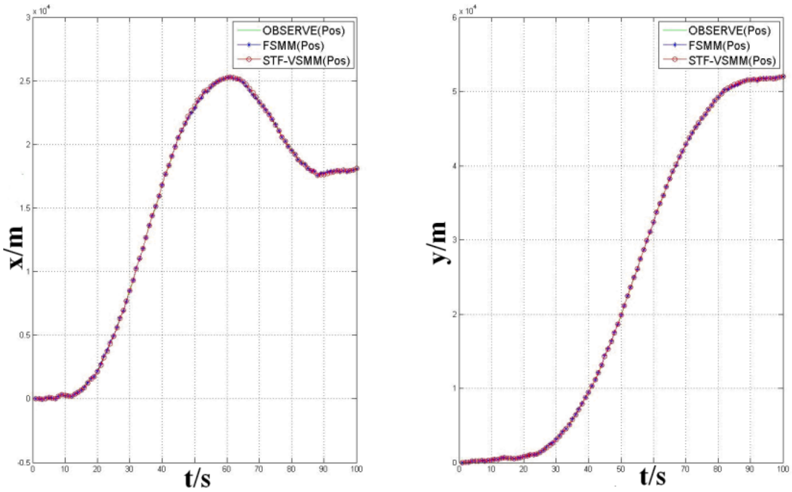
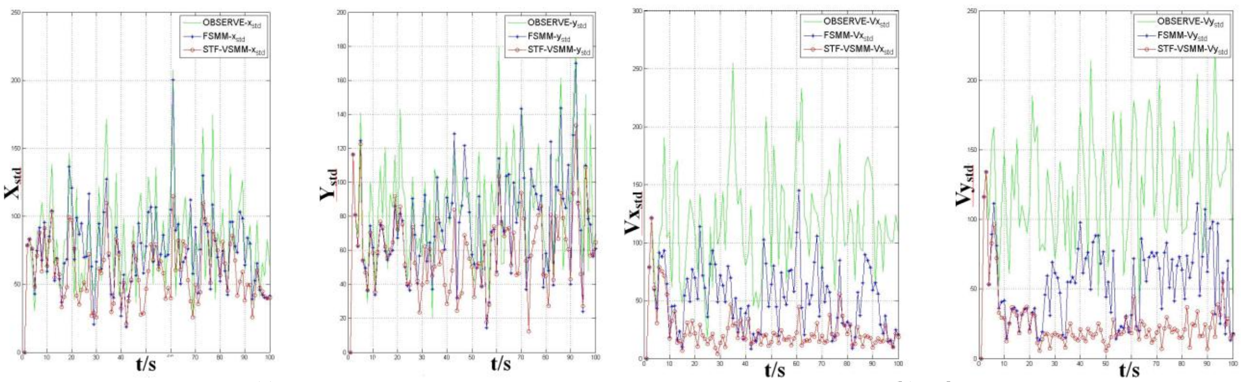


Figure 4 The Filter Results of Position in DS1



(a) Position (b) Velocity

Figure5 The Standard Deviation of Estimation Error Belong to Position and Velocity in DS1

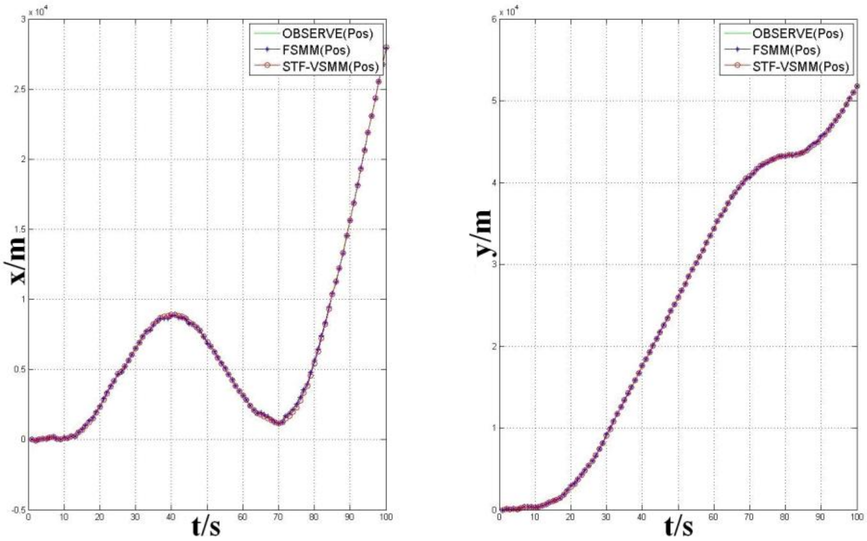
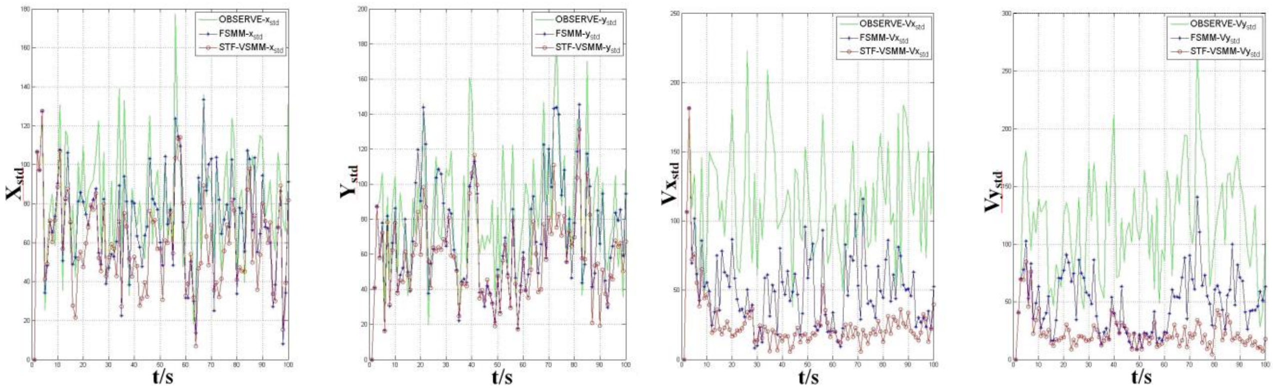


Figure 6 The Filter Results of Position in DS2



(a) Position (b) Velocity

Figure 7 The Standard Deviation of Estimation Error Belong to Position and Velocity in DS2

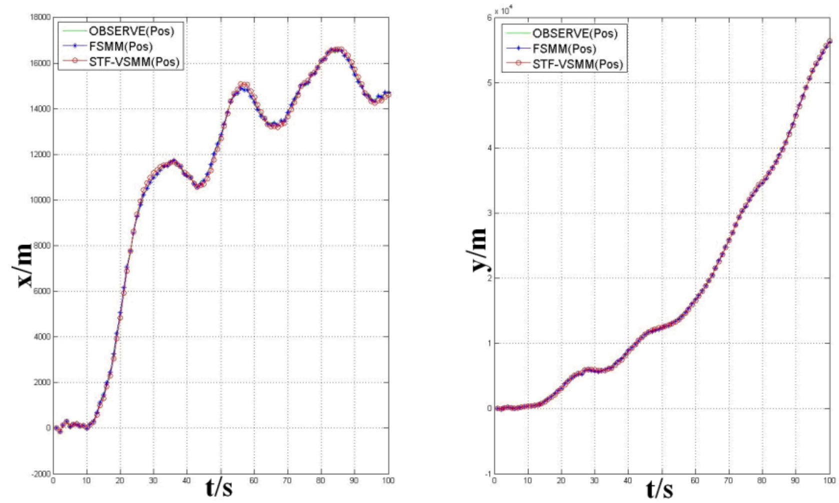
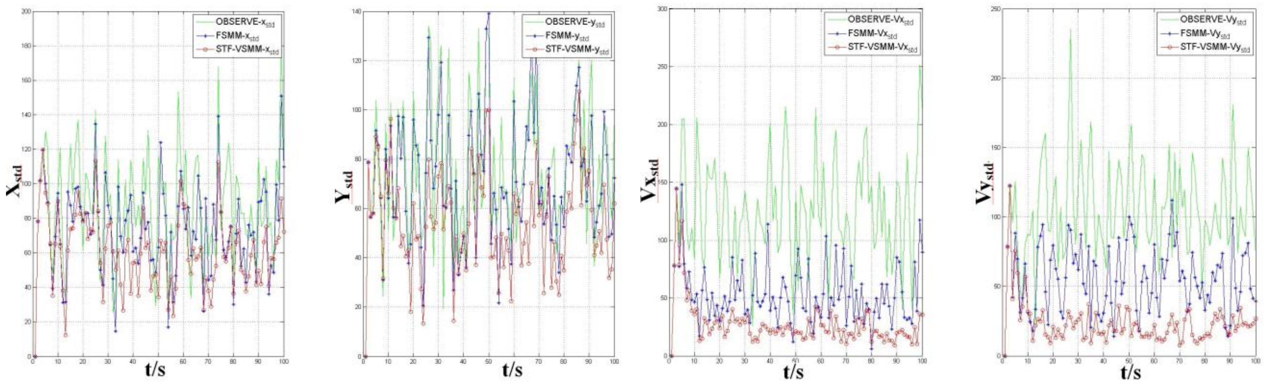


Figure 8 The Filter Results of Position in DS3



(a) Position (b) Velocity

Figure 9 The Standard Deviation of Estimation Error Belong to Position and Velocity in DS3

The TABLE I statistics the mean of estimation error belongs to position and velocity of FSMM and STF-VSMM. The TABLE II statistics the standard deviation of estimation error belongs to position and velocity of FSMM and STF-VSMM.

TABLE I.
THE COMPARATION OF THE MEAN FOR ESTIMATION ERROR BELONGING TO FSMM AND STF-VSMM

SCENE	X_{mean} /m		$V_{x,mean}$ /m s^{-1}		Y_{mean} /m		$V_{Y,mean}$ /m s^{-1}	
	FSMM	STF-VSMM	FSMM	STF-VSMM	FSMM	STF-VSMM	FSMM	STF-VSMM
DS1	-0.01	-0.10	1.16	0.96	2.13	1.29	1.08	1.05
DS2	-8.20	-6.88	-1.17	0.73	-6.17	-6.04	-0.59	-0.71
DS3	6.03	-3.17	1.28	4.22	8.20	8.13	1.35	-1.20

TABLE II.
THE COMPARATION OF THE STANDARD DEVIATION FOR ESTIMATION ERROR BELONGING TO FSMM AND STF-VSMM

SCENE	X_{std}		$V_{x,std}$		Y_{std}		$V_{Y,std}$	
	FSMM	STF-VSMM	FSMM	STF-VSMM	FSMM	STF-VSMM	FSMM	STF-VSMM
DS1	47.74	44.70	30.21	23.22	43.86	40.85	27.76	20.27
DS2	53.74	41.03	46.38	22.30	56.79	44.76	46.29	23.11
DS3	50.68	39.80	42.55	19.59	53.18	40.87	46.38	20.36

Under the DS3, FSMM and STF-VSMM run 50, 100, 150 times, at the same time, the TABLE III compares the mean value belong to position and velocity of FSMM and STF-VSMM, the TABLE IV compares the standard deviation belong to position and velocity of FSMM and STF-VSMM. The results indicate that the time of simulation has a little influence on the precision of algorithms, and the performance of STF-VSMM is steady.

TABLE III.
THE COMPARATION OF THE MEAN FOR ESTIMATION ERROR WHEN FSMM AND STF-VSMM RUN 50, 100, 150

ALGORITHM	X_{mean}			$V_{x,\,mean}$			Y_{mean}			$V_{Y,\,mean}$		
	50	100	150	50	100	150	50	100	150	50	100	150
FSMM	-5.45	-6.03	-4.95	1.78	1.47	1.28	-6.34	-5.04	-6.84	-0.56	-0.71	-0.57
STF-VSMM	-3.98	-3.17	-4.09	4.90	4.22	4.86	7.85	8.13	8.37	-1.85	-1.20	-1.96

TABLE IV.
THE COMPARATION OF THE STANDARD DEVIATION FOR ESTIMATION ERROR WHEN FSMM AND STF-VSMM RUN 50, 100, 150

ALGORITHM	X_{std}			$V_{x,std}$			Y_{std}			$V_{Y,std}$		
	50	100	150	50	100	150	50	100	150	50	100	150
FSMM	53.36	50.68	52.04	46.36	42.55	43.17	53.85	53.18	55.20	46.07	46.38	43.45
STF-VSMM	36.45	39.80	40.56	20.45	19.59	22.24	40.58	40.87	41.34	19.57	20.36	18.77

VI. INCLUSION

The paper presents a variable structure multiple model method basing on strong tracking filter -- STF-VSMM. The new approach imports the STF, and adjusts the center position of FMG in real time. It is possible that the optimal estimation of system is closer to the real system mode, no matter the target has small or great maneuver. Secondly, the STF-VSMM gets the fine model grid and runs a period of IMM. At last, STF-VSMM realizes the accurate tracking of maneuver target basing on optimal fusion theory. Simulation results demonstrate that STF-VSMM estimator outperforms the corresponding fixed structure multiple model (FSMM) at a negligible extra computational cost.

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The important papers that she has published include:

1)A New Variable Structure Multi-model Method of Maneuvering Target Tracking Importing Multi-Rate model. Journal of Sichuan University.

2) A Variable Structure Multiple-model Estimation with Fuzzy Inference and Strong Tracking Filter. Journal of Xi’an Jiao Tong University. (EI)

3) Survey of Typical Variable Structure Multi-model Estimation Algorithm. 2010 International Conference on Computer, Control and Electronic Engineering. (EI)

4) Variable Structure Multi-model Algorithm for Maneuvering Target Tracking Basing on Two-layer Grid. Journal of University of posts and telecommunications. (EI)

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