

# Strip/Foil Rolling Mill Stochastic Excitation Model and Its Stability

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**Abstract**—Based on the stochastic rolling force data from aluminum hot strip tandem mill, the ARMA time series model and the stochastic excitation power spectral density (PSD) model are established, and the stochastic rolling forces excitation model is established by utilizing Levenberg-Marquardt combined with generalized global planning algorithm. A two dimensional stochastic nonlinear dynamical model of rolls is presented considering the stochastic factor of the rolling force. The Hamilton function is also described as one dimension diffusion process by using stochastic average method, the singular boundary theory was taken for analyzing the global stochastic stability of the system, and the system's stochastic stability was researched by solving the Fokker-Planck-Kolmogorov (FPK) equation. The results show that the stochastic excitation model obtained has significance for analyzing and researching stochastic dynamics characteristics to the system, and also generalized energy H in the range of 0.02 to 0.4, the system's response has the minimum transition probability density, and the system state is not easy to change, therefore the system generalized energy H should be to limit in this range in the design and operation of the rolling mill.

**Index Terms**—rolling force, ARMA model, hot strip tandem mill, stochastic excitation model, stochastic stability, Fokker-Planck-Kolmogorov equation

## I. INTRODUCTION

The modern large-scale mill has been developed into very complicated system combined with rolling technology, computer technology, sensing technology, hydraulics and automatic control technology. It is needed the good dynamic performance to rolling mill for precision, high velocity, heavy load and automation, and a series operation, maintenance and adjusting measure also must be perfected for ensuring the stable operation of the equipment. So far it is not perfect for studying on the

dynamic performance of the rolling mill, and there has no determining evaluation criteria to the dynamic performance of the rolling mill. In fact, with the increasing complexity of the system, the higher rolling speed and the various new technology applied in the rolling mill, the causes of rolling mill vibration is more complex. The vibration occurred in rolling can greatly deteriorate the quality of the product, and cause the deviation of the rolling even strip breakage. It can serious threat to the safety of mill equipment. Therefore, the theory and research to vibration of rolling mill has become an integral component of the mill designing and operating.

There have so many factors can induced vibration, such as rolling part (geometric and physical properties), rolling technology (such as pickling, rolling reduction, rolling speed, tension, roll shape, roller surface roughness and rolling solution etc.), rolling equipments (such as designing mill, manufacture precision, installation accuracy, abrasion etc.), control system (such as electric drive control, hydraulic screw down control, tension control, transducer accuracy, signal delay etc.). In fact, all of the factors can induced the vibration of rolling mill above mentioned, which make the vary of the rolling forces, and excite the rolling equipment system, and further makes the rolling equipment system vibrations. However, so far there also different knowledge about the vibration cause to the rolling equipment system.

A lot of useful work has been done in rolling mill and rolling technology using time series method. Zhu Xiang-yang, Yang Shu-zhi etc. investigated the nonlinear vibration of machining and identification method of exponential autoregressive model, and analyzed the sampling signal, and established its exponential autoregressive model, obtained the parameters varying range of the system stability, it is concluded that when the parameters satisfy certain conditions, the EAR model can be used to investigate the limit circle of system's nonlinear vibration [1]. Lu Yong and Wang Zhi-gang established the time series model to the rolling mill vibration signal, and investigated the modeling method of

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nonlinear autoregressive moving average, and deduced the vibration severity model of rolling mill [2]. Xiong Si-bo etc. presented a dynamic analyzing and modifying method based on multiple parameters test simultaneously, the cause of self-excited vibration and the way to eliminate it can be analyzed by the ARX parameter model established, the ARX parameter model takes the hydraulic AGC system servo valve current as the input of model, and the output the roll displacement sensor feedback signal [3]. Yang Jing-ming etc. investigated the close-loop control system of rolling mill for the identification and analysis, established the time series model, their model takes servo-valve current as the input and the output the feedback signal of roll gap transducer, analyzed the identifiability of the rolling mill and Regulator parameters on the results of identification[4]. Wu huai-yu etc. established the ARMA time series model of strip tension, and analyzed the time domain characteristics, revealed the dynamic characteristics of the strip time series model, and established the foundation for the strip tension identification and parameter estimation of the ARMA time series model [5].

So far, the rolling mill's random excitation model was not established, and its dynamic characteristics under random excitation were not investigated. One of this paper's intent is focusing on the rolling mill's dynamic characteristic under random excitation through establishing the time series model, and the random excitation model.

Aluminum strip/foil mill rollers stability of rolling mill is the key factor for high-speed rolling, improving quality of aluminum foil product and extending the life of the components. This is the second intent of this paper.

Usually to improve the quality of aluminum strip/foil rolling process should be strictly formulated rules and parameters set within the rolling process control, determined the fluctuations scope of the process parameters and predict the reliability of the warranty specified limits. To solve these problems, the stochastic analysis of rolling process is a promising treatment approach.

Currently, the researching to vertical vibration of rolling mill rolls mainly focused on the third-octave-mode chatter and five-octave-mode chatter, which has been published a lot of literatures at home and abroad. These studies are almost belonging to the linear and nonlinear vibration based on determining parameters, and there was no vertical nonlinear random-related vibration about rollers. Therefore, investigating the stability and reliability of the mill by taking into account the nonlinear stochastic characteristics of rolling force under the rolling mill vibration process, and deducing the four-high stand stability of nonlinear random vertical vibration discrimination method and numerical calculation, It has important theoretical significance and technical value for improving product accuracy, providing theory and practice foundation and the modernization technological innovating to rolling aluminum foil and strip mill [6~8].

## II. ROLLING MILL FORCE MEASUREMENT AND DATA ACQUISITION-PROCESSING

The "1+4" aluminum strip/foil rolling tandem mill was imported from abroad in nineties of last century by Chinese Southwest Aluminum Company, used for rolling aluminum strip/foil that less than 8mm thickness and mainly consist of one roughing mill and four finishing mills, shown in Figure 1.

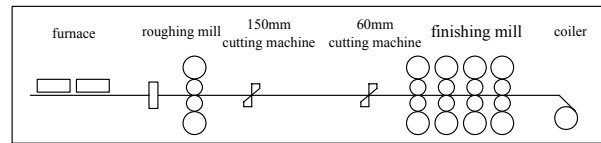


Figure1. "1+4" Aluminum rolling tandem mill main equipment.

In general, abnormal vibration of rolling mill line usually occurs in the third-stand (F3) [9, 10] and then forward and reverse pass to the other. Therefore, the vast amounts of data were gathered and tested to the third-stand mill, and the data were processed primarily by smoothing and eliminating trend etc [11, 12].

### A. Eliminating Trend

The data obtained by vibration testing usually changes against the baseline, even the value of deviation from the baseline would change with time. The deviations from baseline over time is called signal trend. The trend directly affects the accuracy of the signal, and should be eliminated.

The sampling data  $\{x_k\}(k=1, 2, 3, \dots, n)$  is equal time intervals, the polynomial function:

$$\hat{x}_k = a_0 + a_1k + a_2k^2 + \dots + a_mk^m \quad (k = 1, 2, 3, \dots, n) \quad (1)$$

Determining the undetermined coefficient  $a_j (j=0, 1 \dots m)$ , making minimum square error of the function  $\hat{x}_k$  and the discrete data  $x_k$ , and creating linear equations:

$$\sum_{k=1}^n \sum_{j=0}^m a_j k^{j+i} - \sum_{k=1}^n x_k k^i = 0 \quad (i = 0, 1, 2, \dots, m) \quad (2)$$

Solving the equations above, it can determine the  $m+1$  undetermined coefficients  $a_j (j=0, 1 \dots m)$ . Where,  $m$  is the order of the polynomial, and its value range  $0 \leq j \leq m$ , especially, when  $m=0$  or  $1$ , they are constant trend and linear trend respectively, when  $m \geq 3$  the curve trend.

The formula to eliminate trend is

$$y = x_k - \hat{x}_k \quad (k = 1, 2, 3, \dots, n) \quad (3)$$

In practical signal processing, it usually takes  $m=1 \sim 3$  eliminating the polynomial trend to the sampling data.

### B. Smoothing

One purpose of the sampling data smoothing is to eliminate irregular random noise signal, improve the smoothness of the curve vibration; the second is to eliminate the irregular trend. In both cases above, it all can be processed by moving average method.

Three five-point sampling data smoothing method can be used to smooth sampling signals in time and frequency domain. To time-domain data processing can reduce the

mixing of the high-frequency random noise, and frequency-domain data can smooth the spectral curve, so the better fitting results can be got in parameter identification. The original data processed shows in Figure 2.

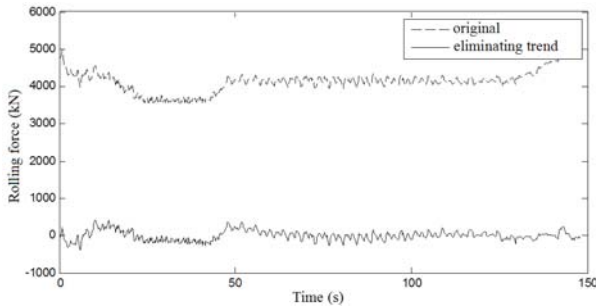


Figure2. Original rolling force signal, elimination trend and smoothing signal.

### III. TIME SERIES MODEL of ROLLING FORCE and ROLLING STOCHASTIC EXCITATION MODEL

A large number of experimental results show that the rolling force changes with a significant random characteristics. This random characteristic shows that rolling force time history curve measured is not the same under the same conditions. Large numbers of literature focused on the ocean wave force model, the earthquake ground acceleration model, the driving dynamic model of the uneven pavement acting on the moving vehicle chassis, and so on. These random dynamic load model established promote the application of random vibration theory and the development, and further reveal deeply the mechanism of structure vibration. So to say, stochastic excitation model is the essential for random vibration analysis [13]. However, in published literatures, there have not seen the rolling force stochastic excitation model, it has seriously hampered the exploration of the mechanism of rolling mill vibration and vibration suppression of rolling mill.

The stochastic excitation model is divided into two major categories of stationary and non stationary. As the rolling mill random noise factors by many uncertainties, it can be assumed to have the normal characteristics. A large number of experimental results show that the rolling force is stationary random process. For the stationary stochastic excitation model, as long as the mean and correlation function, or mean and spectral density determined, the statistical properties can be fully determined.

The ARMA model built up based on the dynamic signal time series  $\{x_k\}$  ( $k=1, 2, 3 \dots n$ ) is most basic, widely used. It not only reveals the structure of dynamic signals, but also can investigate the basic characteristics of dynamic systems. Therefore, this article first established the rolling force ARMA time series models based on the measured signal, and obtained the stochastic excitation model of rolling force.

#### A. Rolling Force Time Series Model

According to the time series modeling theory [14], for any time series  $\{x_k\}$  ( $k=1, 2, 3, \dots, n$ ) of stationary random process, it can be fitted to a mathematical model that the white noise as the input, its general form be the ARMA ( $p, q$ ) model, it can be expressed as

$$\begin{cases} x_k - \sum_{i=1}^p \phi_i x_{k-i} = a_k - \sum_{j=1}^q \theta_j a_{k-j} \\ a_k \sim NID(0, \sigma_a^2) \end{cases} \quad (4)$$

Where,  $\phi_i$  is autoregressive coefficient ( $i=1, 2 \dots p$ );  $\theta_j$  the moving average coefficient ( $j=1, 2 \dots q$ );  $p$  and  $q$  the order of autoregressive and moving average parts respectively; and  $x_k$  the output time series ( $k=1, 2 \dots k$ ).

Introducing the backward shift operator  $B$  to Eq. 4, the Eq.4 can be expressed simply as follows

$$\begin{cases} \phi(B)x_k = \theta(B)a_k \\ a_k \sim NID(0, \sigma_a^2) \\ \phi(B) = 1 - \phi_1 B^{-1} - \phi_2 B^{-2} - \dots - \phi_p B^{-p} \\ \theta(B) = 1 - \theta_1 B^{-1} - \theta_2 B^{-2} - \dots - \theta_p B^{-p} \end{cases} \quad (5)$$

From the dynamic coefficient modeling strategy, first estimating the inverse function of model shown in Eq. 5, and determining the initial value (initial estimate) of  $\{\phi_i\}$  and  $\{\theta_j\}$  by the inverse function, and the model parameters can be estimated by the nonlinear damping least square method to meet the smallest residual sum of square, and finally determining the model order combining with the AIC, BIC, F criteria etc.

According to the measured vibration test data from the site, finally obtaining the ARMA (12, 10) model as follows

$$\begin{cases} \phi(B) = 1 - 0.8774B^{-1} - 0.1341B^{-2} + 0.2866B^{-3} - 0.4207B^{-4} + \\ 0.2516B^{-5} + 0.09433B^{-6} - 0.2111B^{-7} + 0.1211B^{-8} - 0.13478B^{-9} \\ - 0.7298B^{-10} + 0.7028B^{-11} - 0.1354B^{-12} \\ \theta(B) = 1 + 0.1325B^{-1} + 0.02929B^{-2} + 0.3611B^{-3} + 0.02278B^{-4} + \\ 0.1504B^{-5} + 0.1407B^{-6} + 0.1784B^{-7} + 0.1178B^{-8} - 0.09209B^{-9} \\ - 0.4875B^{-10} \end{cases} \quad (6)$$

#### B. Rolling Stochastic Excitation Model

The rolling stochastic excitation model mainly refers to its rolling force power spectrum model for investigating the stochastic dynamic characteristics of rolling mill and rolling process. The power spectrum model includes frequency components of the system, and each frequency corresponds to the magnitude of the total power spectrum. Comparing with the time-domain analysis, it can further reveal the structure and laws of the rolling force signal itself, and can extract information about the characteristics. Owing to the Time series spectral estimation based on model identification and parameter estimation on the output time series data of the system. So, the power spectrum characteristics can be computed by using the timing series model obtained. It overcomes the inherent shortcomings in traditional FFT spectrum analysis, such as spectral leakage, side lobe, low resolution; weak signals submerged and improve the frequency resolution.

Transforming ARMA (12, 10) of the rolling force, finally obtaining the bilateral power spectrum as follows

$$S_{xx}^{ARMA}(f) = \sigma_a^2 \left| \frac{1 - \sum_{k=1}^q \theta_k e^{-j2\pi fk\Delta}}{1 - \sum_{h=1}^p \theta_h e^{-j2\pi fh\Delta}} \right|^2 \quad (7)$$

Where,  $(-1/2\Delta) \leq f \leq (1/2\Delta)$ .

For practical engineering, the general unilateral power spectrum is used. According to the dual function characteristics of bilateral power spectrum and the sided normalized power spectrum of rolling force can be obtained as follows

$$S_{xx}^{ARMA}(f) = \frac{2\sigma_a^2}{\sigma_x^2} \left| \frac{1 - \sum_{k=1}^q \theta_k e^{-j2\pi fk\Delta}}{1 - \sum_{h=1}^p \theta_h e^{-j2\pi fh\Delta}} \right|^2 \quad (8)$$

Where,  $0 \leq f \leq 1/2\Delta$ ,  $\sigma_a^2$  is the variance of input white noise, and  $\sigma_x^2$  is the variance of rolling force.

The power spectrum estimated by timing series model and periodogram to the rolling force show in Figure 3, Figure 4, the dominant frequency are all 50 Hz.

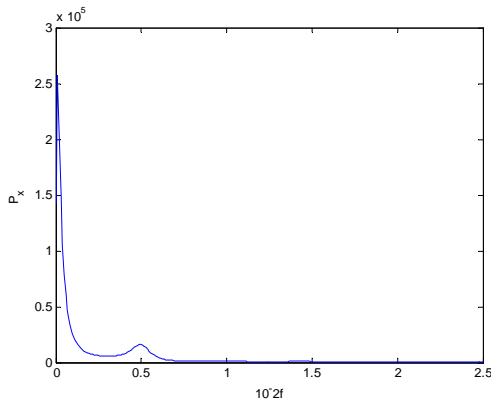


Figure3. PSD estimated by time series method.

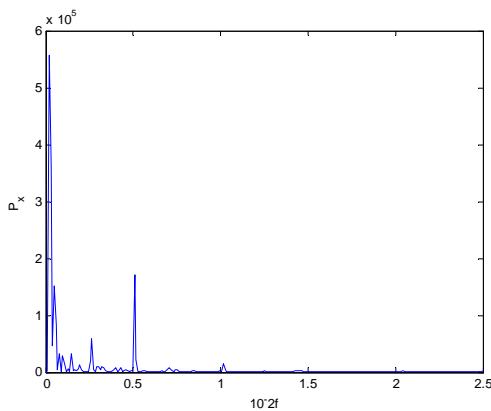


Figure4. PSD estimated by periodogram.

Figure 3 and Figure 4 show that the power spectrum estimated by time series model is smooth, clear than the periodogram method.

Although the precise power spectrum expression can be derived based on the time series model, but the expression is complex, and the parameters physical meaning are not clear. Therefore, it is necessary fitting a rolling force power spectrum of expression that the physical meaning of parameters certainly, relatively simple, convenient application, that is, rolling force stochastic excitation model. According to Figure 3 determining the form of the fitted power spectrum as follows

$$S_{xx}(f) = \frac{a + bf^2}{c + df^2 + f^4} \quad (9)$$

Where,  $f$  is frequency,  $a, b, c, d$  are the constants.

By taking advantage of the Michael Marquardt optimization algorithm and general global optimization method, convergence criteria is  $1.0e-10$ , maximum number of iterations are 10000, and obtained:

$$S_{xx}(f) = \frac{6.6 \times 10^{10} + 3.0 \times 10^8 f^2}{-2.1 \times 10^3 + 3.9 \times 10^3 f^2 + f^4} \quad (10)$$

#### IV. ROLL'S TWO DEGREES OF FREEDOM (MECHANICS) MODEL

Four-high mill vertical vibration inherent characteristic is the important to design and operating the modern rolling mill. Therefore, comprehensive knowledge and understanding of the natural frequency and vibration characteristics of four-high mill vibration is basic condition for analyzing the vibration of rolling mill.

Analyzing the four-high mill system vertical vibration, as the study's focus is different; the different simplified model should be adopted. According to the interesting and accuracy needed, it is usually simplified the system to a single degree of freedom, two degrees of freedom, four degrees of freedom, or six degrees of freedom vibration systems etc. To facilitate the analysis, considering the symmetry of the upper and lower rolls of four-high mill, the two degrees of freedom vertical vibration model was adopted, shown in Figure 5.

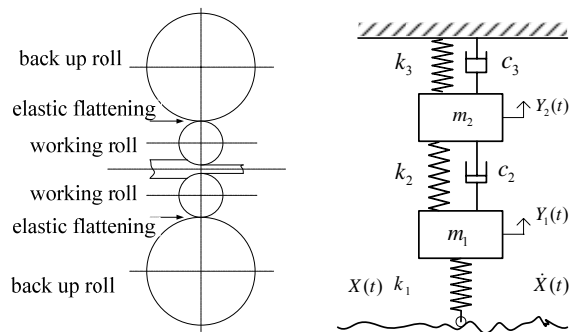


Figure 5. Four-high mill's two degrees of freedom vertical vibration model.

In Figure5,  $Y_1(t)$  is the absolute displacement of work roll, and  $Y_2(t)$  is the absolute displacement of back roll, and  $X(t)$  is the excitation of strip roughness, which can be considered as the Gaussian white noise with zero mean

and 2D intensity.  $k_1$  is the nonlinear contact stiffness between work rolls and strip/foil, the  $k_1$  consists of two parts, one is the rigidity of the mean roll gap  $\bar{k}_1$ , and the second is the fluctuating amount of stiffness within the roll gap  $\Delta k_1 \cos \omega t$ , paralleling the two on behalf of the actual value of the roll gap stiffness. Supposing the  $k_p$  is mean stiffness of rolling part,  $\Delta k_p \cos \omega t$  on behalf of the fluctuates in stiffness, and in parallel the  $k_p$  and  $\Delta k_p \cos \omega t$  on behalf of the actual value of the rolling stiffness, and then series with elastic flattening of work roll stiffness  $k_1^*$ , that is

$$k_1 = \bar{k}_1 + \Delta \cos \omega t = \frac{k_1^* (k_p + \Delta k_p \cos \omega t)}{k_1^* + k_p + \Delta k_p \cos \omega t} \approx \frac{k_1^* (k_p + \Delta k_p \cos \omega t)}{k_1^* + k_p} = \frac{k_1^* k_p}{k_1^* + k_p} + \frac{k_1^* \Delta k_p \cos \omega t}{k_1^* + k_p} \quad (11)$$

Respectively,  $k_2$  is the contact stiffness, and  $c_2$  the contact damping between the work roll and backup roll,  $k_3$  and  $c_2$  the stiffness and damping determined by frame, screw-down structure, and the bearing block of back roll, in fact, it is the contact stiffness and contact damping between rolls and frame. The  $m_1$  and  $m_2$  are the mass of work roll and backup roll respectively.

Lists the vibration equation

$$\begin{cases} m_1 \ddot{Y}_1 + c_2 \dot{Y}_1 + (k_1 + k_2)Y_1 - c_2 \dot{Y}_2 - k_2 Y_2 = k_1 X \\ m_2 \ddot{Y}_2 + (c_2 + c_3) \dot{Y}_2 - c_2 \dot{Y}_1 + (k_2 + k_3)Y_2 - k_2 Y_1 = 0 \end{cases} \quad (12)$$

Suppose  $q_1=Y_1$ ,  $p_1=\dot{Y}_1$ ,  $q_2=Y_2$ ,  $p_2=\dot{Y}_2$ , and the Eq. 12 can be turned into

$$\begin{cases} dq_1 = p_1 dt \\ dp_1 = [-(c_2 p_1 + k_1 q_1 + k_2 q_1 - c_2 p_2 - k_2 q_2) dt + k_1 X] / m_1 \\ dq_2 = p_2 dt \\ dp_2 = [-(c_2 p_2 + c_3 p_2 - c_2 p_1 + k_2 q_2 + k_3 q_2 - k_2 q_1) dt] / m_2 \end{cases} \quad (13)$$

The Hamilton function of the dynamical system can be expressed as

$$H = \frac{1}{2} (p_1^2 + p_2^2 + \omega_1^2 q_1^2 + \omega_2^2 q_2^2) \quad (14)$$

Where,  $\omega_1^2 = (k_1 + k_2) / m_1$  and  $\omega_2^2 = (k_2 + k_3) / m_2$  are the generalized angular frequency.

Thus stochastic model of this dynamical system can be established in the sense of Stratonovich

$$\begin{cases} dq_i = \frac{\partial H}{\partial p_i} dt \\ dp_i = \left[ \frac{\partial H}{\partial q_i} + m_{ij}(q, p) \frac{\partial H}{\partial p_j} \right] dt + \sigma_{jk}(q, p) \circ dB_k(t) \end{cases} \quad (15)$$

Where,  $i, j, k=1, 2; j, k$  are the sum tag from the Eq. 15

$$\begin{cases} \frac{\partial H}{\partial p_i} = p_i \\ \frac{\partial H}{\partial q_i} = \omega_i^2 q_i \end{cases} \quad (16)$$

$$\begin{cases} \sigma_{11} = \sigma_{22} = \sqrt{2D} \frac{k_1}{m_1} \\ \sigma_{12} = \sigma_{21} = 0 \end{cases} \quad (17)$$

$$\begin{cases} m_{11} = (c_2 p_1 + k_1 q_1 + k_2 q_1) / (m_1 p_1) \\ m_{12} = (-c_2 p_2 - k_2 q_2) / (m_2 p_2) \\ m_{21} = (c_2 p_2 + c_3 p_2 - c_2 p_1 - k_2 q_1) / (m_2 p_1) \\ m_{22} = 0 \end{cases} \quad (18)$$

“ $\circ$ ” sense that the product of Stratonovich,  $B_i(t)$  ( $i=1, 2 \dots$ ) is the independent Wiener diffusion process.

According to the definition and characteristic of quasi-non-integrable-hamiltonian systems, the dynamical system converges in probability 1 to one-dimensional Ito diffusion process

$$dH = m(H)dt + \sigma(H)dB(t) \quad (19)$$

Where,  $B(t)$  is Standard Wiener process,  $m(H)$  and  $\sigma(H)$  are the Ito stochastic process drift coefficient and diffusion coefficient respectively.

The stochastic average method of quasi-non-integrable hamiltonian systems is adopted [15].

$$\begin{cases} m(H) = 2D\omega_1^4 - \frac{1}{2} \frac{c_2}{m_2} H - \frac{8\sqrt{2}}{15\pi m_2} (k_2 + 2c_2) H^{3/2} \\ \sigma^2(H) = D\omega_1^4 H \end{cases} \quad (20)$$

### V. STOCHASTIC STABILITY OF THE ROLL

The stability of dynamic behavior of rolling mill is critical for ensuring final product quality [16]; therefore, the system stability under random excitation probability of research is very significant.

For one-dimensional Ito stochastic diffusion process Eq. 19, it can be used to investigate the global stochastic stability by singular boundary theory. The stochastic stability of the system can be judged by diffusion index  $\alpha_s$ , drift index  $\beta_s$  and characteristics of indicators  $c_s$ .  $H$  is generalized energy of the system, and  $H_s$  the global energy boundary. The subscript  $s=R, L$ , and they denote the right boundary and left boundary respectively.

When  $H \rightarrow H_s$ , it meets the following conditions

$$\begin{cases} \sigma^2(H) = 0 | H - H_s |^{\alpha_s} & \alpha_s \geq 0 \\ m(H) = 0 | H - H_s |^{\beta_s} & \beta_s \geq 0 \end{cases} \quad (21)$$

Where, 0 is the small number of marker.

$$\begin{cases} c_L = \lim_{H \rightarrow H_L} \frac{2m(H)(H - H_L)^{\alpha_L - \beta_L}}{\sigma^2(H)} \\ c_R = \lim_{H \rightarrow H_R} \frac{2m(H)(H_R - H)^{\alpha_R - \beta_R}}{\sigma^2(H)} \end{cases} \quad (22)$$

The left boundary  $H=0$  is the point to make  $\sigma_2(H) = 0$ , so it is the first class of singular boundary. From the Eq. 21 and 22, it can be derived that the diffusion index  $\alpha_L=1$  the drift index  $\beta_L=0$  and the characteristics of indicators  $c_L=4$  respectively, so  $H=0$  is the boundary to the inside. The right boundary  $H=\infty$  makes the  $m(H)$  infinity, and its diffusion  $\alpha_R=1$ ,  $\beta_R=0$ , respectively, it is the second class singular boundary, and also the boundary to the inside. The left and right boundaries are the entire boundary to the inside, so the all of tracks are from the border into the interior, the system must have a stationary solution [17]. The Figure 6 shows the distribution of the Hamilton function of the system in the whole range border.

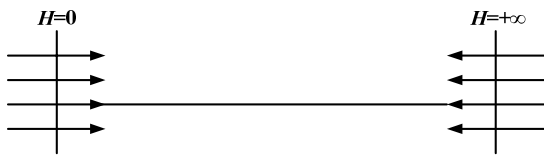


Figure 6. The system's Hamilton function on the boundary.

Because of the system's stationary solution within the energy range, so it can be obtained by building and solving the Fokker-Planck-Kolmogorov (FPK) equation of the system. The system's FPK equation reflects the transition probability density evolution process of the system response. The FPK equation of the system is

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial H} [m(H)f] + \frac{1}{2} \frac{\partial^2 [\sigma^2(H)f]}{\partial H^2} \quad (23)$$

Where  $f$  is the smooth transition probability density function, and its stationary solution be

$$f(H) = A \exp \left\{ -\int_0^H \left[ \frac{d\sigma^2(h)}{dh} - 2m(h) \right] / \sigma^2(h) dh \right\} \quad (24)$$

Where,  $A$  is the normalization constant, solution the Eq. 24 and obtained

$$f(H) = A \exp \left\{ -20.7233 - \ln H + [64\sqrt{2}H^{7/2} (2c_2 + k_2) - 7H^2(8D\omega_1^4 - c_2Hm_2)] / 28D\omega_1^4 \right\} \quad (25)$$

From the data of F3 stack based on field test,  $D=0.1$ ,  $m_1=6.8 \times 10^3$ kg,  $k_{1min}=2.3 \times 10^7$ N/m,  $m_2=7.6 \times 10^3$ kg,  $k_2=6.74 \times 10^7$ N/m,  $c_2=1.7 \times 10^6$ Ns/m,  $k_3=4.71 \times 10^7$ N/m,  $c_3=2.1 \times 10^6$  Ns/m. The system's steady state transition probability density shows in Figure 7. As can be seen from Figure 7, the extreme point is  $H=0.2$ ,  $f(H)_{min}=2.11 \times 10^{-8}$ . When the system of generalized energy value less

than 0.04 or greater than 0.4, the system's transfer probability density dramatically increases.

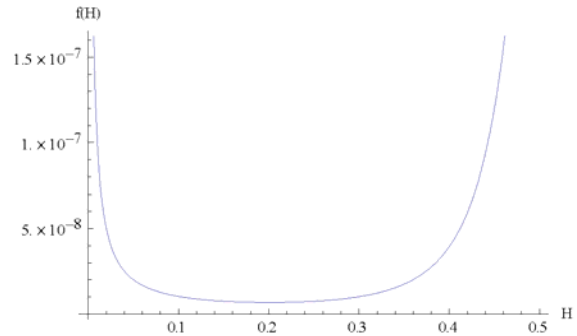


Figure 7. Stationary transition probability density of the rolling mill.

## VI. SUMMARY

In this paper, the rolling force stochastic excitation model of F3 (third-stand) to Chinese Southwest Aluminum Company of "1 + 4" hot rolling aluminum strip/foil production lines in China is established by using time series analysis method, and for the aluminum strip/foil rolling mill dynamics model by the random excitation, with the stochastic averaging method of the quasi non-integrable Hamilton system, the system's Hamilton function was simplified as one-dimensional diffusion process and the stochastic rolls stability were investigated by utilizing singular boundary theory. The results show as following:

a. The time series analysis method is with less sample data, and can obtained accurate spectrum formula, time spectral resolution is high efficiency, smooth and clear.

b. Under the normal rolling conditions, the rolling force signal measured have the same orders of the time series model and the same spectral structure, this indicate that the rolling force signal can be seen essentially as a stationary random signal, and using ARMA models to fit the rolling normal force signal is appropriate.

c. The time domain analysis to rolling force time series show that the ARMA (12, 10) model obtained can be used to predict the rolling force of the rolling mill.

d. From Figure 3 the rolling force signal frequency greater than 65Hz can be seen as white noise. Therefore, the third-octave and five-octave stochastic vibration characteristics of rolling mill can be investigated using the white noise as excitation in high frequency, and formula (10) as stochastic torsional vibration excitation in the low frequency (less than 65Hz).

e. Through analyzing the boundary of the system, it is clear that the left and right boundary of the dynamical system are both entered boundary, the system's probability density function can be obtained by solving the Fokker-Planck-Kolmogorov (FPK) equation derived.

f. In working-out technical process operation instructions, the working point of the rolling mill should be made in the 0.02 to 0.4 range of Hamilton function value  $H$ . Thus the system's stochastic stability can be ensured to the rolling mill excited by strip or sheet.



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## REFERENCES

- [1] Zhu Xiangyang, Yang Shuzhi, Huang Ren, "Time Series Model for Nonlinear Random Vibrations", *Journal of Vibration Engineering*, Vol.7, No.3, 1994, pp. 195-200
- [2] Lu Yong, Wang Zhigang, "Nonlinear Modeling Method and Its Application to Mill Vibration Signal", *Research on Iron and Steel*, No.4, 2005, pp. 35-37
- [3] Xiong Shibo, etc., "Self-excited Vibration Diagnosis of the Rolling Mills and Structure Dynamics Modification", *Chinese Journal of Mechanical Engineering*, Vol.7, No.41, 2005, pp. 147-151
- [4] Yang Jingming etc., "Identification of the Mathematical Model of the Rolling Mill", *Metallurgical Industry Automation* 21, 2004, pp. 610-612
- [5] Wu Huaiyu, Qi Zhenhuan, "Time Domain Characteristics Analysis of Strip Tension ARMA Time Series Model", *Journal of Wuhan Yejin University of Science and Technology*, Vol.20, No.1, 1997, pp. 95-98
- [6] Roberts W L, "Four H Mill Stand Chatter of Fifth Octave Mode". *Iron and Steel Engineer*, Vol. 55, No. 10, 1978, pp. 41-47
- [7] Gerald L Nessler, James F Cory, "Cause and Solution of Fifth Octave Backup Roll Chatter on 4-H Cold Mills and Temper Mills", *Iron and Steel Engineer*, Vol. 65, No.12, 1989, pp. 33-37
- [8] HOU Dong-xiao, CHEN Hao, "Analysis on parametrically excited nonlinear vertical vibration of roller system in rolling mills", *Journal of Vibration and Shock*, Vol. 28, No.11, 2009, pp.1-6
- [9] Zou Jiexiang, Xu Lejiang, "Vibration Control of Cold Tandem Mill System", *Beijing: Metallurgical Industry Press*, 1998
- [10] Zou Jiexiang etc., "Modern Design Theory of Rolling Mill". *Beijing: Metallurgical Industry Press*, 1991
- [11] Wang Ji, Hu Xiao, "MATLAB application in vibration signal processing", *Beijing: WaterPower Press*, 2006
- [12] Li Shexin, "Application of the ARMA Model to Vibration Signal Analysis", *Mechanical Science and Technology*, Vol. 29, No. 11, 2010, pp.1586-1588
- [13] Ou Jinping, Wang Guangyuan, "Structure random vibration", *Beijing: Higher Education Press*, 1998
- [14] Lennart.L, "System Identification-Theory for the User", *Beijing: Tsinghua University Press*, 2002
- [15] Roberts J B, Spanos P D, "Stochastic Averaging: an Approximate Method of Solving Random Vibration Problem", *Int J Nonlinear Mechanics*, No. 21, 1986, pp. 111-134
- [16] GAO Jia-yi, "Vibration Stability Analysis of Rolling Process of Seam Less Steel Tube", *Journal of Tianjin University of Technology*, Vol.25 No.6, 2009, pp. 46-48
- [17] Zhu WeiQiu, "Nonlinear Stochastic Dynamics and Control", *Beijing: Science Press*, 2003



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