Erdös Conjecture on Connected Residual Graphs

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***Abstract*—A graph G is said to be F-residual if for every point u in G, the graph obtained by removing the closed neighborhood of u from G is isomorphic to F. Similarly, if the remove of m consecutive closed neighborhoods yields Kn, then G is called m-Kn-residual graph. Erdös determine the minimum order of the m-Kn-residual graph for all m and n, the minimum order of the connected Kn-residual graph is found and all the extremal graphs are specified. Jiangdong Liao and Shihui Yang determine the minimum order of the connected 2-Kn-residual graph is found and all the extremal graphs are specified expected for n=3, and in this paper, we prove that the minimum order of the connected 3-Kn-residual graph is found and all the extremal graphs are specified expected for n=5, 7, 9,10, and we revised Erdös conjecture.**

***Index Terms*—Residual-graph, Closed neighborhood, Adjacent, Cartesian product**

I. INTRODUCTION

A graph G is said to be F-residual if for every point u in G, the graph obtained by removing the closed neighborhood of u from G is isomorphic to F. If G is a graph such that the deletion from G of the points in each closed neighborhood results in the complete graph Kn-residual graph. We inductively define multiply-Kn-residual graph by saying that G is m-F-residual if the removal of the closed neighborhood of any point of G result in an (m−1)-F-residual graph, where of course a 1-F-residual graph is simply an F-residual graph.

It is natural to ask what is the minimum number of points that an m-Kn-residual graph must contain. We easily prove that this number is (m+1)n and that the only m-Kn-residual graph with this number of point is (m+1)Kn. In [2] they show that a connected Kn-residual graph must have at least 2n+2 points if $n \neq 2$. Furthermore, the cartesian product $G \cong Kn+1 \times K2$ is the only such graph with 2n+2 points for $n \neq 2; 3; 4$. They complete the result by determining all connected Kn-residual graph of minimal order for n = 2, 3, 4.

The concept of residual graphs was firrst in-duced[1], by Paul. Erdös, Frank. Harary and Maria.Klawe.They studied residually complete graphs, determined the minimum order of m-*Kn*-residual graphs are (m+1)n, and (m+1)*Kn* is the corresponding extremal graph for any posi-tive integers *m* and *n*.C5 is the unique connected *K*2-residual graph with least order 5. For$1< n \neq 2$, the least order of connected *K*n-residual graphs is 2(n+1), for $n \neq 2; 3; 4$ Kn+1×K2 is unique connected Kn-residual graphs with least order. The authors[2] proved that for any positive integers *n* and *k*, there exist *K*n-residual graphs with even order 2(n + k). For n= 2; 3; 4 there exist Kn-residual graphs with odd order 2n+3. And for n = 6, C5[K3] is the unique connected. K6-residual graph with least odd order 15. In this paper we proved that for any positive odd number t and n = 2t, C5[Kt] is the unique connected *K*n-residual graphs with least odd order 5t. The least odd order of *K*n-residual graphs is 5(n + 1)/2. It is easy to prove that for any odd number *n*, there is no *K*n-residual graphs with odd order. For t = 5,n = 2; 4; 6; 8, we construct the corresponding connected *K*n-residual graphs with odd order 2n+t = 9; 13; 17; 21 respectively. For t is odd, n = 2t−2 and n = 2t−4, we constructed the corresponding connected *K*n-residual graphs with odd order 2n + t as well.

We state the following conjecture [2].

Conjecture 1. If $n \neq 2$, then every connected m-Kn-residual graph has at least Min{2n(m + 1); (n + m)(m + 1)} points.
Conjecture 2. For n large, there is a unique smallest connected m-Kn-residual graph.
The known supportting results are summarized in the following theorem.

Theorem 1.1 (Erdös [2]). (1) If G is F-residual, then for any point u in G, the degree

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$d(u) = P(G) - P(F) - 1$.

(2) Every m-Kn-residual graph has at least (m+1)n points, and (m+1)Kn is the only m-Kn-residual graph with (m + 1)n points.

(3) Every connected Kn-residual graph has at least 2n + 2 points if $n \neq 2$.

(4) If $n \neq 2$, then $G \cong Kn+1 \times K2$ is a connected Kn-residual graph of minimum order, and except for n = 3 and n = 4, it is the only such graph.

In this paper, [Theorem 3.1, Theorem 3.2] we show that a connected 3-Kn-residual graph must have at least 4n+12 points if $n \geq 11$. Furthermore, the cartesian product $G \cong Kn+3 \times K4$ is the only such graph with 4n + 12 points for $n \geq 11$, we construct the result by determining all connected 3-Kn-residual graph of minimal order for n = 3, 4, 6, and connected 3-K8-residual graph has only one graph $G \cong K11 \times K4$ with mini- mum order. In Section 4, we revise Erdös conjecture.

In general the notation follows that of [1]. In particular P(G) is the number of points in a graph G, N(u) is the neighbor- hood a point u consisting of all points adjacent to u. $N^*(u)$ is the closed neigh- borhood of u.

Definition 1.1. Let $F \subset G$, then dG(F)=

$$\sum_{x \in F} d_G(x) - \sum_{x \in F} d_F(x)$$

Definition 1.2. If $x \in X$ and $y \in Y$ are adjacent, then X and Y are said to be adjacent, and vice verse. For any point $x \in X$ and any point $y \in Y$, if x is adjacent to y then said to X is complete adjacent to Y .

Definition 1.3. $G = \bigcup_{i=1}^{m} G_i$, where Gi is a subgraph G, $G \cong F$, $Gi \cap Gj = \varnothing$, and Gi is nonadjacent to Gj(i, j = 1; $2 \cdots$ m; $i \neq j$), denoted by $G \cong mF$.

II. 2-Kn-residual Graph

We begin third section with a simple obser-vation which will turn out to be extremely useful lemmas, these lemmas in [3].

Lemma 2.1. Let G be a Kn-residual graph for n is odd,then P(G) is even.

Lemma 2.2. Let G be a Kn-residual graph with $P(G) \neq 2n + 3$ for $n \neq 2, 4, 6$.

Lemma 2.3. Let G be a connected 2-Kn- residual graph with P(G) =3n+t for $n \geqslant 3$, where $1 \leqslant t \leqslant 2n$. Then

(1) $n \leqslant d(u) \leqslant n+t-1$, $\forall u \in G$;

(2) $d(u) \neq n + t - 2$, $\forall u \in G$;

(3)There exist some point $u \in G$, thus $d(u) \neq n + t - 1$;

(4) $d(u) \neq n + t - 4$, for n 6= 4, 6.

Lemma 2.4. Let G be a connected 2-Kn-residual graph with P(G)=3n +t for $n \geqslant 3$, where $1 \leqslant t \leqslant 2n$. Then $d(u) \neq n$, $\forall u \in G$.

Lemma 2.5. Let G be a 2-Kn-residual graph with $P(G) \geqslant 3n+4$ for $n \geqslant 3$.

Lemma 2.6. The connected 2-Kn-residual graph with P(G) =3n+t for $n \geqslant 3$. If there exist $u \in G$, d(u) = n+1, then exist there d(v)=n+t− 1, where $v \in G$.

Lemma 2.7. The connected 2-Kn-residual graph with P(G)=3n+t for n and t are odd, then d(u) is odd.

Lemma 2.8. Let $G=\langle H1 \cup H2 \cup X \rangle$ be a connected Kn-residual graph with P(G)=2n+t for $n \geqslant 3$ and t < 2n, where $H1 \cong Kn$ and H2=Kn, |X| =t, then

(1). H1 is adjacent to H2;

(2). Hj is not complete adjacent to X, where j= 1, 2.

Lemma 2.9. Let G be a connected 2-Kn-residual graph with P(G) = 3n+t for $n \geq 5$ and $4 \leq t \leq 6$, then there dose not exist three mutually non adjacent points whose degree are n + t−1.

Lemma 2.10. Let G be a connected 2-Kn-residual graph with P(G) = 3n+t for $n \geq 5$ and $n \neq 6$, where $4 \leq t \leq 6$, then there does not exist mutually nonadjacent points whose degree are n + t−1.

Lemma 2.11. Let G be a connected 2-Kn- residual graph with P(G) =3n + t for $n \geq 5$, where $4 \leq t \leq 6$, then there does not exist complete mutually adjacent points whose degree are n+ t−1.

Theorem 2.1. Every connected 2-Kn-residual graph has at least 3n+6 for $n \geq 5$.

Lemma 2.12. Let G be a connected 2-Kn-residual graph with P(G)=3n+6 for $n \geq 5$ and $n \neq 6$, then d(u) = n + 3, $\forall u \in G$.

Theorem 2.2. If $n \geq 5$, then G□Kn+2×K3 is a connected 2-Kn-residual graph of minimum order, and expect for n=6, it is only such graph.

We now prove the remainder of the Theorem 2.2 involving the small cases $n \leq 4$. For n = 1, a connected 2-K1-residual graph is the only regular graph C5. For n = 2, Erdös [2] construct a connected 2-K2-residual graph in Fig. 3. For n =4 suppose G is a connected 2-K4-residual graph with P(G) = 16<3×4 + 6=18, the graph in Fig.1. For n = 6 we construct a connected 2-K6- residual graph $G \cong K8 \times K3$ in Fig. 1

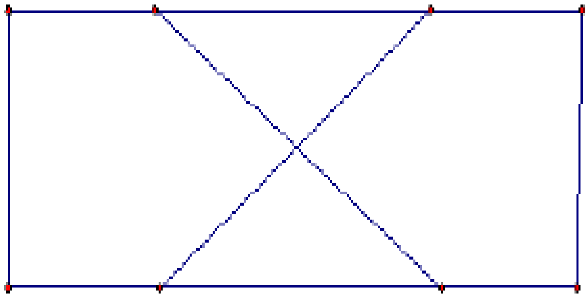

Fig. 1. 2-K2-residual graph

III. 3-KN-RESIDUAL GRAPH

Lemma 3.1. Let G be a connected 3-Kn- residual graph with P(G) = 4n+t for n≥ 7, where 1≤t≤2n. Then

(1) n + 3 ≤d(u)≤ n + t-1; $\forall u \in G$;

(2) d(u) ≠ n + t-4,

d(u) ≠n + t-5,

d(u) ≠n + t-6,

d(u) ≠n + t-8 for $\forall u \in G$.

Proof.(1) Since G−N*(u) =G1 is a 2-Kn-residual graph, let d(u) = n+ t, then P(G1)= 3n, by Theorem 2.2 we have

P(G1) ≥3n + 6, a contradiction.

By Theorem 2.2 we have

d(u)≥n + 3.

So n + 3≤d(u)≤n + t-1, $\forall u \in G$.

(2) Set G−N*(u) = G1; $\forall u \in G$, G1 is a 2-Kn-residual graph, by Theorem 2.2 we have

P(G1) ≠3n+3; 3n+4; 3n + 5 for n≥7,

P(G1) ≠3n + 7 for n ≠4; 6,

then

d(u) = P(G)−P(G1)−1 ≠n +t−4; n +t−5; n + t−6; n + t −8, for n ≥7.

By proof method of Lemma 2.9, we have

Lemma 3.2. Let G be a connected 3-Kn-residual graph with P(G) =4n+ t for n ≥ 11 and 1≤ t ≤2n, then there dose not exist four mutually nonadjacent vertices whose degree are n + t −1.

By proof method of Lemma 2.10, we have

Lemma 3.3. Let G be a connected 3-Kn- residual graph with P(G)=4n+t for n ≥11 and 1 ≤ t ≤2n, then there dose not exist three mutually nonadjacent points whose degree are n + t − 1.

Lemma 3.4. Let G be a connected 3-Kn-residual graph with P(G) =4n+t for n≥11, where 1≤t≤2n, then there does not exist mutually nonadjacent points whose degree are n + t − 1.

By proof method of Lemma 2.11, we have

Lemma 3.5. Let G be a connected 3-Kn-residual graph with P(G) = 4n + t, for n ≥ 11, where 1≤t≤2n, then there does not exist complete mutually adjacent points whose degree are n + t − 1.

By proof method of Lemma 2.12, we have

Lemma 3.6. Let G be a connected 3-Kn-residual graph with P(G)=4n+t, for n≥ 11, where 1 ≤ t ≤ 2n, then there does not exist case that G has just only one point of degree is n + t − 1.

So, let G be a connected 3-Kn-residual graph with P(G) = 4n + t, where 1 ≤ t ≤ 2n, by Lemma 3.2, 3.3, 3.4, 3.5, 3.6, we have d(u)≠ n + t − 1 for $\forall$u ∈ G, Shihui Yang and Huiming Duan [3] determined d(u)≠ n+t−3, d(u)≠ n+t−5 for $\forall$ u ∈G. By proof method of Theorem 2.2 and Theorem 2.3, we have

Theorem 3.1. Every connected 3-Kn-residual graph has at least 4n + 12 for n≥11.

Proof. Let P(G)=4n+t, by Lemma 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 we have

n+4 ≤d(u)≤ n+t-7

then t≥11.

We now prove that t ≠11.

Suppose that t=11, then d(u) = n + 4, $\forall u \in G$, if u is nonadjacent to v, where u, v∈ G, set

N*(u)∩ N*(v) = X;

G1 = G−N*(u);

G2 = G−N*(v),

then u∈ G2, v ∈ G1. Set

G-N*(u)-N*(v) = G-N*(v) −N*(u) = H ≅ Kn. By P(G1) = P(G2) = 3n + 6,

dG1(w) = dG2(w) = n + 3, $\forall$ w∈ H,

then

N*(v)-X has (n+3)-dH(w) = (n+3)−(n−1) = 4 points adjacent to w∈ H in G1, and

N*(u)−X has four points adjacent to w∈ H in G2.

By (N*(u) −X)∩N*(v)−X) = ∅, thus

d(w)≥ dH(w) + 4 + 4 = n−1+ 4 + 4 = n + 7, contrary to d(w) = n + 4.

So P(G) ≥4n+12.

Lemma 3.7. Let G be a connected 3-Kn-residual graph with P(G)=4n+12 for n≥11,then d(u) = n + 5; $\forall u \in G$.

Proof. Since d(u) ≥n+3, t = 12, d(u) ≠ n+t−8 = n+4, then d(u)≥n+5.

By Lemma 3.1, 3.2,3.3,3.4,3.5,3.6 and Theorem 3.1 we have d(u) = n+5; $\forall u \in G$

Theorem 3.2. If n ≥11, then G□Kn+3×K4 is a connected 3-Kn-residual graph of minimum order, it is only such graph.

Proof. Fact 1. Let F =<H1 ∪ H2 ∪ H3> ≅ Kn+2×K3, where H1 ≅ H2 ≅ H3 ≅ Kn+2, then

Hi and Hj have bijection

θ: V (Hi) → V (Hj),

and u1 ∈ Hi is adjacent to μ(u1) ∈ Hj ,

where i ≠j; i; j = 1; 2; 3.

If H ⊂ F and H ≅ Ks; 3≤ s≤n + 2, then

H ⊂ H1 or H ⊂ H2 or H ⊂ H3.

Fact 2. By Lemma 3.2 we have

d(u) = n+5, G1 = G−N*(u), ∀ u ⊂ G,

P(G1) = 3n + 6

so G1 ≅ Kn+2×K3, set

G1=< $\overline{H_1} \cup \overline{H_2} \cup \overline{H_3}$ >=< $x_r^j|_{r=1,2,3}^{j=1,2,\cdots,n+2}$ >,,

where $\overline{H_r}$ =< $x_r^1, x_r^2, \ldots, x_r^{n+2}$ >,

and x_l^i is adjacent to x_m^j if i = j,

x_l^i is nonadjacent to x_m^j if i ≠ j.

wherel ≠m; l,m = 1, 2, 3 (3.1)

Set G2 = G−N*(x_2^{n+2})

=< $H_0^* \cup H_1^* \cup H_3^*$ >

≅ Kn+2 × K3,

by(3.1) we have

Kn+1 ≅ $\overline{H_1} - x_1^{n+2}$

=< $x_1^1, x_1^2, \ldots, x_1^{n+1}$ > ⊂ G2,

by Fact 1 without loss of generality we may assume that

< $x_1^1, x_1^2, \ldots, x_1^{n+1}$ > ⊂ H_1^*

=< $x_1^0, x_1^1, \ldots, x_1^{n+1}$ >,

< $x_3^1, x_3^2, \ldots, x_3^{n+1}$ > ⊂ H_3^*

=< $x_3^0, x_3^1, \ldots, x_3^{n+1}$ > (3.2)

If $x_0^j \in H_0^*$ is adjacent to x_3^j, where j=0, 1,… , n+1, obvious x_0^0 = u, then

H_0^* =< $x_0^0, x_0^1, \ldots, x_0^{n+1}$ >.

We now prove x_1^0 is adjacent to x_1^{n+2}.

Suppose the contrary, set

G3 = G−N*(x_1^{n+2}); x_1^0 ∈ G3,

by (3.1) and (3.2) we have x_1^0 is adjacent to

{ $x_1^1, x_1^2, \ldots, x_1^{n+1}$ } ⊂ N*(x_1^{n+2}), thus

d(x_1^0) ≥ dG3(x_1^0) + n + 1 = n + 2 + n + 1 > n + 5, a contradiction.

So x_1^0 is adjacent to x_1^{n+2}, hence

x_1^0 is adjacent to $\overline{H_1}$.

Set H1 =< $x_1^0, x_1^1, \ldots, x_1^{n+2}$ > ≅ Kn+3.

Similar x_3^0 is adjacent to x_3^{n+2},

x_3^0 is adjacent to $\overline{H_3}$.

Set H3 =< $x_1^0, x_1^1, \ldots, x_1^{n+2}$ > ≅ Kn+3.

Similar x_2^0 ∈ N*(u) is complete adjacent to $\overline{H_2}$, obvious $x_2^0 \neq x_1^0$, $x_2^0 \neq x_3^0$, so

H2 =< $x_2^0, x_2^1, \ldots, x_2^{n+2}$ > ≅ Kn+3.

Similar, in G−N*(x_2^{n+2})

=< $H_0^* \cup H_1^* \cup H_3^*$ >,

we have x_0^{n+2} ∈ N*(x_2^{n+2})=< $H_0^* \cup H_1^*$ >complete adjacent to H_0^*.

Obvious $\overline{x_0^{n+2}}$ ∈ (H2 ∪ x_1^{n+2} ∪ x_3^{n+2}) ⊂ N*(x_2^{n+2})

So H0 =< $x_0^0, x_0^1, \ldots, x_0^{n+2}$ > ≅ Kn+3.

Fact 3. Any point in Hr is adjacent to single point in Hs ≠ r. Suppose the contrary, let xj ∈ H0 be nonadjacent to H2, then G * =

$$G - N^*(x_0^j) \cong K_{n+2} \times K_3,$$

but H ≅ Kn+3,H2 ⊂ G *, contrary to $G^* \cong K_{n+2} \times K_3$

x_0^j is adjacent to H2. If H2 has two points adjacent to x_0^j, by dH0(x_0^j) = n + 2, d x_0^j) = n + 4, so x_0^j is nonadjacent to H1 ∪ H2, a contradiction.

Fact 4. By Fact 3 we have x_1^0 is adjacent to H2. If x_1^0 adjacent to $x_2^{j\neq 0}$, by $x_2^{j\neq 0}$ is adjacent to x_1^j, thus H1 has two points adjacent to x_2^j, contrary to Fact 3.

So x_1^0 is adjacent to x_2^0.

Similar x_3^0 is adjacent to x_2^0,

x_0^{n+2} is adjacent to x_1^{n+2},

x_0^{n+2} is adjacent to x_3^{n+2}.

Fact 5. Since x_0^j is adjacent to x_2^j for j = 0, n + 2, let x_0^j be nonadjacent to x_2^j for j ≠ 0, n + 2,

by Fact 3 we have x_0^j adjacent to $x_0^{i\neq j}$.

Since x_0^j is adjacent to x_1^j and x_3^j, set

$$G - N^*(x_0^j)$$

$$=< (H_1 - x_1^j) \cup (H_2 - x_2^j) \cup (H_2 - x_3^j) >$$

$$\cong K_{n+2} \times K_3 \tag{3.3}$$

By Fact 4 we have x_2^t adjacent to x_1^t and x_3^t for t≠ i, j, by (3.1) we have x_2^i adjacent to x_1^j and x_3^j, contrary to (3.1).

So x_0^j is adjacent to x_2^j.

Hence $G = < X >< x_r^j \mid_{r=0,1,2,3}^{j=0,1,2,n+2} >$,

where x_r^i is adjacent to x_s^j if and only if r = s, i≠ j or i = j, r≠ s.

So G≅Kn+3×K4.

We now prove the remainder of the Theorem 3.2 involving the small cases n≤11. For n = 2, Erdös [2] construct a connected 3-K2-residual graph in Fig. 2. For n = 3 suppose G is a connected 3-K3-residual graph with P(G) = 20<4 × 3+12=24, the graph in Fig.3. In the Section2, we by connected 2-K2-residual graph construct connected 2-K4-residual graph and connected 2-K6-residual graph, similar by con-nected 3-K2-residual graph construct con-nected 3-K4-residual graph G1, connected 3-K6-residual graph G2 and connected 3-K8-residual graph G3,where P(G1)= 22, P(G2) =33, P(G3) = 44. For n= 8 we construct a connected 3-K8-residual graph G3≅K11×K4.

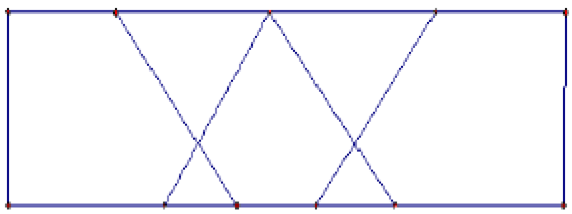

Fig. 2. 3-K2-residual graph

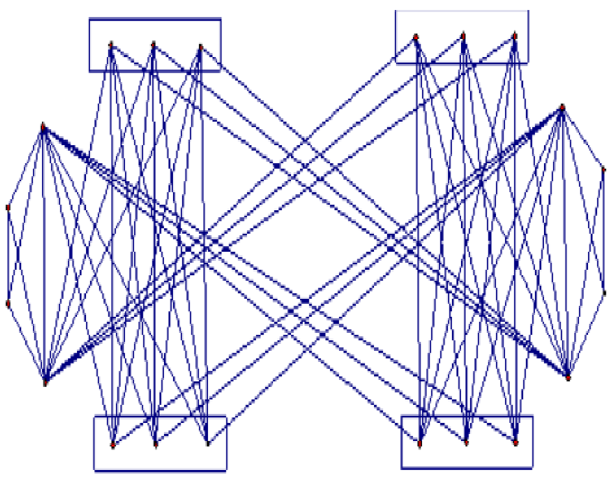

Fig. 3. 3-K3-residual graph

In the above Fig. 3, we can see that all points in the same square adjacent.

IV. MULTIPLY-KN-RESIDUAL GRAPHS

To conclude the paper, let us present a conjecture. A connected (m−1)-Kn-residual graph, denoted by (m−1)-Knm-1-residual graph, a connected m-Kn-residual graph, denoted by m-Knm-residual graph.We revised Erdös conjecture.

Conjecture **3.** For all *m* and *n*, then every connected m-Kn-residual graph has at least min{2n(m+1), (n+m)(m+1), $\frac{n}{2}(3m+2)$} points.

For example: connected 2-K2-residual graph, connected 2-K4-residual graph, connected 3-K2-residual graph, connected 3-K4-residual graph, connected 3-K6-residual graph.

Conjecture 4. If nm ≥nm−1 + 2m−1 for m≤n and m≠4y + 1, where y is a Natural Number and n1=5, then G≅Kn+m×Km+1 is a connected m-Kn-residual graph of minimum order, it is only such graph.

We have determined the minimum order of the connected m-Kn-residual graph for all *m* and *n* large, Kn+m×Km+1 is the only such graph with (n+m)(m+1) points. We suppose that Erdös conjectures is true for n≥ n0.

problem **1.** What is n0 ?

problem **2.** What are the extremal graphs on connected m-Kn-residual graph for n < n0?

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