Neural Network Adaptive Control for a Class of Matched SISO Nonlinear Uncertain Systems with Zero Dynamics

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***Abstract*—The paper presents a direct adaptive tracking control scheme for a class of matched SISO affine nonlinear uncertain systems with zero dynamic using neural network. Through neural network approximation, neural network is used as the emulator of the unknown ideal controller. A quadratic cost function of the error between the unknown ideal controller and the used neural network controller is minimized using a gradient descent method to adjust parameters in neural network. The convergence of parameters and the uniformly ultimately boundedness of tracking error and all states of the closed-loop system are guaranteed based on Lyapunov stability theorem. The effectiveness of the proposed controller is illustrated through the simulation results.**

***Index Terms*—tracking control, matched uncertain nonlinear, gradient descent method, neural network, zero dynamic**

I. INTRODUCTION

There are some inevitable uncertainties in actual system which will cause instability and difficulties in dealing with system. Therefore, the study of uncertain nonlinear system is of vital importance. Control of uncertain nonlinear dynamic systems is still a challenging problem though it attracted many researchers in control community during the past few decades led to development of fruitful methods based on adaptive control concepts. Alternatively, in recent years, adaptive neural network [3, 7, 8, 9, 10, 12, 19, 20, 22, 25] and fuzzy logic control [1, 2, 5, 6, 13, 14, 15, 16, 17, 18, 21, 23] become an active research area. These methodologies become especially more helpful if control of highly uncertain, nonlinear and complex systems is the design issue. The main philosophy that is exploited heavily in system theory applications is the universal function approximation property of neural networks or fuzzy logic. Benefits of using neural networks or fuzzy logic for control applications include its ability to effectively control nonlinear plants while adapting to unmodeled dynamics. In general, a two-step procedure is taken. First, based on implicit function theorem an ideal controller developed to stabilize the underlying system and make the tracking approach a neighborhood of zero. Then, a neural network or fuzzy logic to approximate this ideal controller is designed. Based on the Lyapunov stability analysis, an adaptation law is devised to update the adjustable parameters. However a bounding controller may also be added for more performance robustness. In the above most methods the parameter adaptation laws are designed based on a Lyapunov approach, where an error signal between the desired output and the actual output is used to update the adjustable parameters and the control laws are composed of three control terms: a linear control term, an adaptive neural network control term and a robustifying control term used to compensate for disturbances and approximation errors. On the other hand, almost all of the above works don't considered the zero dynamics, though it plays an important role in nonlinear system control. Considering that zero dynamics exist in many practical systems, including isothermal continuous stirred tank reactors, aircraft trajectory tracking control systems and others, it is necessary to investigate their influence on nonlinear system. In the paper, according to [5], we introduce a direct adaptive neural network control approach for a class of matched SISO affine nonlinear uncertain systems with zero dynamics. The basic idea is to use neural network to adaptively construct an unknown ideal controller and the parameter adaptive laws is designed ,based on the gradient descent method, to directly minimizing the error between the unknown ideal controller and the neural network controller And no robustifying control term is used in controller. This paper proves the availability of the method in both theory and simulation experiment.

The paper is organized as follows. First, the problem is formulated in Section II. Designing a control law with on-line tuning of neural network weighting factors is given in Section III. In Section IV, convergence and stability analysis of control system is given. In Section V, simulation results are presented to confirm the effectiveness and applicability of the proposed method. Finally, conclusions are included.

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II. Problem Formulation

Consider the following matched SISO affine nonlinear uncertain system:

$$\begin{cases} \dot{x} = f(x) + \Delta f(x) + \left(g(x) + \Delta g(x)\right)u \\ y = h(x) \end{cases} \quad (1)$$

Where $x \in R^n$ and $u, y \in R$ are system state, system input and output respectively. $\Omega_x \subset R^n$, $\Omega_u \subset R$ are two compact sets. $f(x)$ and $g(x)$ are smooth vector fields. $\Delta f(x)$ and $\Delta g(x)$ are bounded uncertain terms. $h(x) \in R$ is smooth scalar function.

Assumption 1: Nominal system(1)possesses a strong relative degree $r < n$.

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \quad (2)$$

Assumption 2: Uncertainties meet the match conditions.

$$\begin{aligned} \Delta f(x) &= g(x)\delta_1(x) \\ \Delta g(x) &= g(x)\delta_2(x) \end{aligned} \quad (3)$$

According to differential geometry theory of nonlinear system, we know that there is a nonlinear transformation $\mathrm{T}(x) = \left(\xi^T, \eta^T\right)^T$, which turns system(1)to

$$\begin{cases} \dfrac{d\xi_i}{dt} = \xi_{i+1} \quad i = 1, \cdots, r-1 \\ \dfrac{d\xi_r}{dt} = \alpha(\xi,\eta) + \beta(\xi,\eta)\left\{\delta_1(x) + \left[1 + \delta_2(x)\right]u\right\} \\ \dot{\eta} = q(\xi,\eta) \\ y = \xi_1 \end{cases} \quad (4)$$

where $\xi_i = L_f^{i-1}h(x)$, $\alpha(\xi,\eta) = L_f^r h(x)$ and $\beta(\xi,\eta) = L_g L_f^{r-1} h(x) \neq 0$. For all $(x,u) \in \Omega_x \times R$ the function $\beta(\xi,\eta)$ is nonzero and bounded. This implies that $\beta(\xi,\eta)$ is strictly either positive or negative. Without loss of generality, it is assumed that it exists a positive constant c such that $\beta(\xi,\eta) \geq c > 0$ for all $(x,u) \in \Omega_x \times R$.

Assumption 3: For all $x \in R^n$, we have

$$1 + \delta_2(x) \geq \vartheta(x) > 0 \quad (5)$$

Assumption 4: Zero dynamics $\dfrac{d\eta}{dt} = q(0,\eta)$ is exponentially stable, and the function $q(\xi,\eta)$ is Lipschitz in ξ . By Lyapunov converse theorem[11], there is a Lyapunov function $V_0(\eta)$ which satisfies

$$\sigma_1 \|\eta\|^2 \leq V_0(\eta) \leq \sigma_2 \|\eta\|^2 \quad (6)$$

$$\frac{\partial V_0(\eta)}{\partial \eta} q(0,\eta) \leq -\sigma_3 \|\eta\|^2 \quad (7)$$

$$\left\| \frac{\partial V_0(\eta)}{\partial \eta} \right\| \leq \sigma_4 \|\eta\| \quad (8)$$

Where $\sigma_i, i = 1,2,3,4$ are positive constant.

$$\|q(\xi,\eta) - q(0,\eta)\| \leq L_\xi \|\xi\| \quad (9)$$

Where L_ξ is Lipschitz constant.

Define the reference vector

$$\underline{y}_d = (y_d \quad \dot{y}_d \quad \cdots \quad y_d^{(r-1)})^T \in R^r$$

The reference signal y_d and its time derivative are assumed to be smooth and bounded. We also define the tracking error as

$$e = y_d - y$$

and corresponding error vector as

$$\underline{e} = (e, \dot{e}, \quad \cdots \quad e^{(r-1)})^T \in R^r$$

Then the error equation in the new coordinate is as follows:

$$\underline{\dot{e}} = A_0 \underline{e} + b\left[y_d^{(r)} - \alpha(\xi,\eta) - \beta(\xi,\eta)\left\{\delta_1(x) + \left[1 + \delta_2(x)\right]u\right\}\right] \quad (10)$$

Where $A_0 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{r\times r}$, $b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{r\times 1}$.

Obviously, if (A_0, b) can be controllable, then there will exist a constant matrix $K = \left[k_0, k_1, \cdots k_{r-1}\right]^T$ which makes eigenvalues of matrix $A_c = A_0 - bK^T$ all have negative real part. Thus, for any given positive definite symmetric matrix Q , there exists a unique positive definite symmetric solution P to the following Lyapunov algebraic equation:

$$A_c^T P + P A_c = -Q \quad (11)$$

The control objective is to design an adaptive neural network controller for a class of matched SISO affine nonlinear systems (1) such that the system output follows a desired trajectory while all signals in the closed-loop system remain bounded.

Assumption 5: Desired output y_d and its r-order time derivative are assumed to be smooth and bounded. Then there exists a positive constant b_d which makes

$$\left\|(y_d \quad y_d^{(1)} \quad \cdots \quad y_d^{(r)})^T\right\| \leq b_d \quad (12)$$

III. DESIGN OF CONTROLLER

Define a signal

$$v = y_d^{(r)} + K^T \underline{e} + \lambda \tanh\left(\frac{b^T P \underline{e}}{\Xi}\right)$$

where $\tanh(\bullet)$ is the hyperbolic tangent function, Ξ, λ are the positive design parameters $\tanh(\bullet) \in (-1,1)$, when error $e \to +\infty$, the value of $\tanh\left(\frac{b^T P\underline{e}}{\Xi}\right) \to +\infty$. And when error $e \to -\infty$, the value of $\tanh\left(\frac{b^T P\underline{e}}{\Xi}\right) \to -\infty$. When $e \to 0$, $\tanh\left(\frac{b^T P\underline{e}}{\Xi}\right) \to 0$ .The term $\lambda \tanh\left(\frac{b^T P\underline{e}}{\Xi}\right)$ is a smooth approximation of the discontinuous term $\lambda sign\left(b^T P\underline{e}\right)$ usually used in robust control. So, λ is selected larger than the magnitude of the uncertainty and it will affect the convergence rate of the tracking error, and Ξ is chosen very small to best approximate the sign function and it will affect the size of the residual set to which the tracking error will converge. The sign function is not used here to avoid problems associated with it as chattering and solutions existence.

By adding and subtracting v in (10), we obtain

$$\begin{aligned}\dot{\underline{e}} &= \left(A_0 - bK^T\right)\underline{e} - b\lambda \tanh\left(\frac{b^T P\underline{e}}{\Xi}\right) - \cdots \\ &+ b\left[\alpha(\xi,\eta) + \beta\left\{\delta_1(x) + \left(1+\delta_2(x)\right)u\right\} - v\right]\end{aligned} \quad (13)$$

if $\delta_1(x), \delta_2(x)$ are known, there exists some ideal controller $u^*(z)$ satisfying the following equality :

$$u^*(z) = \left(\beta(\xi,\eta)\left[1+\delta_2(x)\right]\right)^{-1}\left(v - \alpha(\xi,\eta) - \beta(\xi,\eta)\delta_1(x)\right) \quad (14)$$

The closed-loop error dynamic is reduced to (15)

$$\dot{\underline{e}} = \left(A_0 - bK^T\right)\underline{e} - b\lambda \tanh\left(\frac{b^T P\underline{e}}{\Xi}\right) \quad (15)$$

Considering the following Lyapunov function:

$$V = \underline{e}^T P\underline{e} \quad (16)$$

According to (11) and (15), we have

$$\dot{V} = -\underline{e}^T Q\underline{e} - 2\lambda b^T P\underline{e} \tanh\left(\frac{b^T P\underline{e}}{\Xi}\right) \quad (17)$$

Since the term $b^T P\underline{e} \tanh\left(\frac{b^T P\underline{e}}{\Xi}\right)$ is always positive,we conclude that $\dot{V} \le 0$,and only when $e = 0$, $\dot{V} = 0$ which means $\lim_{t\to\infty} |e| = 0$.

However, in ideal controller (14) $\delta_1(x), \delta_2(x)$ are unknown ,so $u^*(z)$ is not available. In what follows, a neural network will be used to construct the unknown ideal controller.

In control engineering, radial basis function (RBF) NNs are usually used as a tool for modeling nonlinear functions because of their good capabilities in function approximation. In this paper, the following RBF NN $u(z) = \phi^T(z)\theta$ is used to approximate the ideal controller $u^*(z)$, where $z = \left[\xi^T, \eta^T, v\right]^T$, $\theta = (\theta_1, \cdots, \theta_M)^T$ is the adjustable parameter, and $\phi(z) = (\phi_1(z), \cdots, \phi_M(z))^T$ is the basic function vector. It has been proven that network can approximate any smooth function over a compact set $\Omega_Z \subset R^q$ to arbitrarily any accuracy as

$$u^*(z) = \phi^T(z)\theta^* + \delta(z) \quad (18)$$

with bounded function approximation error $\delta(z)$ satisfying $|\delta(z)| \le \bar{\delta}$ .Where θ^* is an ideal parameter vector which minimizes the function $|\delta(z)|$ .In this paper ,we assume that the used neural network does not violate the universal approximation property on the compact set Ω_Z ,which is assumed large enough so that the variable z remains inside it under closed-loop control.

RBFNN represents a class of linearly parameterized approximators and can be replaced by any other linearly parameterized approximators such as spline functions[24] or fuzzy systems[23]. Moreover, nonlinearly parameterized approximators, such as multilayer neural network(MNN), can be linearized as linearly parameterized approximators, with the higher order terms of Taylor series expansions being taken as part of the modeling error, as shown in [19],[20].

Let us define the error between the controllers $u(z)$ and $u^*(z)$ as

$$e_u = u^*(z) - u(z) = \phi^T(z)\tilde{\theta} + \delta(z) \quad (19)$$

Where $\tilde{\theta} = \theta^* - \theta$ is the parameter estimation error vector.

By substituting $u^*(z)$ into the equation (13) and considering (19), we get

$$\begin{aligned}\dot{\underline{e}} &= A_c\underline{e} - b\lambda \tanh\left(\frac{b^T P\underline{e}}{\Xi}\right) - b\left[\alpha(\xi,\eta) + \beta(\xi,\eta)\delta_1(x) - v\right] - \cdots \\ &-b\left[\beta(\xi,\eta)\left[1+\delta_2(x)\right]u + \beta(\xi,\eta)\left[1+\delta_2(x)\right]u^*(z) - \cdots \right.\\ &\left. -\beta(\xi,\eta)\left[1+\delta_2(x)\right]u^*(z)\right] \\ &= A_c\underline{e} - b\lambda \tanh\left(\frac{b^T P\underline{e}}{\Xi}\right) - b\beta(\xi,\eta)\left[1+\delta_2(x)\right]\left(u(z) - u^*(z)\right)\end{aligned} \quad (20)$$

Considering $A_c = A_0 - bK^T$, then (20) can be rewritten as

$$e^{(r)} + K^T\underline{e} + \lambda \tanh\left(\frac{b^T P\underline{e}}{\Xi}\right) = \beta(\xi,\eta)\left[1+\delta_2(x)\right]e_u \quad (21)$$

We notice here that $u^*(z)$ is an unknown quantity , so the signal e_u defined in (19) is not available. Eq.(21) will be used to overcome the difficulty. Indeed ,from(21), we see that even if the signal e_u is not available for measurement, the quantity $\beta(\xi,\eta)\left[1+\delta_2(x)\right]e_u$ is measureable. This fact will be exploited in the design of the parameters adaptive law.

Now, consider a quadratic cost function defined as

$$J_\theta = \frac{1}{2}[1+\delta_2(x)]e_u^2 = \frac{1}{2}[1+\delta_2(x)]\left(u^*(z)-\phi^T(z)\theta\right)^2 \tag{22}$$

By applying the gradient descent method, we obtain as an adaptive law for the parameters θ

$$\dot{\theta} = -\gamma\nabla_\theta J(\theta) = \gamma\left[1+\delta_2(x)\right]\phi(z)e_u \tag{23}$$

Since e_u and $\delta_2(x)$ are not available, the adaptive law (23) can not be implemented. In order to render (23) computable ,from Eq.(21), we select the design parameter $\gamma=\gamma_\theta\beta(\xi,\eta)$, where γ_θ is a positive constant. At the same time ,to improve the robustness of adaptive law in the presence of the approximation error ,we modify it by introducing a σ-modification term as follows:

$$\begin{aligned}\dot{\theta} &= \gamma_\theta\left(\phi(z)\beta(\xi,\eta)[1+\delta_2(x)]e_u - \sigma\theta\right)\\ &= \gamma_\theta\phi(z)\left\{e^{(r)} + K^T\underline{e} + \lambda\tanh\left(\frac{b^TP\underline{e}}{\Xi}\right)\right\} - \gamma_\theta\sigma\theta\end{aligned} \tag{24}$$

where σ is a small positive constant

Because the aim of the σ-modification adaptive law is to avoid parameter drift, it does not need to be active when the estimated parameters are within some acceptable bound. The proposed adaptive controller is only composed of a neural network part without additional control terms and the system stability relies entirely on the neural network. The term $\lambda\tanh\left(\frac{b^TP\underline{e}}{\Xi}\right)$ in the parameter adaptive law (24) plays, in some way, the role of a robustifying control term. Thus the robustness of the controller can be improved by selecting a large positive value for the design parameter λ and a small positive value for the parameter Ξ.

IV. CONVERGENCE AND STABILITY ANALYSIS OF CONTROL SYSTEM

Firstly, let us consider the convergence of neural network parameters. Considering the following positive function:

$$V_\theta = \frac{1}{2\gamma_\theta}\tilde{\theta}^T\tilde{\theta} \tag{25}$$

Using (19) and (24), the time derivative of (25) can be written as

$$\begin{aligned}\dot{V}_\theta &= -\tilde{\theta}^T\left(\phi(z)\beta(\xi,\eta)\left(1+\delta_2(x)\right)e_u - \sigma\theta\right)\\ &= -\phi^T(z)\tilde{\theta}\beta(\xi,\eta)\left(1+\delta_2(x)\right)e_u + \sigma\tilde{\theta}^T\theta\\ &= -\beta(\xi,\eta)\left(1+\delta_2(x)\right)e_u^2 + \beta(\xi,\eta)\left(1+\delta_2(x)\right)\delta(z)e_u + \sigma\tilde{\theta}^T\theta\end{aligned} \tag{26}$$

Using the inequalities

$$\begin{aligned}\sigma\tilde{\theta}^T\theta &= -\frac{\sigma}{2}\|\tilde{\theta}\|^2 - \frac{\sigma}{2}\|\theta\|^2 + \frac{\sigma}{2}\|\tilde{\theta}+\theta\|^2\\ &\le -\frac{\sigma}{2}\|\tilde{\theta}\|^2 + \frac{\sigma}{2}\|\theta^*\|^2\end{aligned} \tag{27}$$

$$\begin{aligned}-e_u^2 + \delta(z)e_u &= -\frac{1}{2}e_u^2 + \frac{1}{2}\delta^2(z) - \frac{1}{2}\left(e_u - \delta(z)\right)^2\\ &\le -\frac{1}{2}e_u^2 + \frac{1}{2}\delta^2(z)\end{aligned} \tag{28}$$

Considering (27) and (28), Eq.(26) can be bounded as

$$\begin{aligned}\dot{V}_\theta \le & -\frac{1}{2}\beta(\xi,\eta)\left(1+\delta_2(x)\right)e_u^2 + \frac{1}{2}\beta(\xi,\eta)\left(1+\delta_2(x)\right)\delta^2(z) - \cdots\\ & -\frac{\sigma}{2}\|\tilde{\theta}\|^2 + \frac{\sigma}{2}\|\theta^*\|^2\end{aligned} \tag{29}$$

Since the parameters θ^* are constants, and the functions $\delta(z)$ and $\beta(\xi,\eta),\delta_2(x)$ are assumed bounded in this paper, so we can define a positive constant bound ψ as

$$\psi = \sup_t\left(\frac{1}{2}\beta(\xi,\eta)\left(1+\delta_2(x)\right)\delta^2(z)\right) + \frac{\sigma}{2}\|\theta^*\|^2 \tag{30}$$

Then

$$\begin{aligned}\dot{V}_\theta &\le -\frac{1}{2}\rho V_\theta + \psi - \frac{1}{2}\beta(\xi,\eta)\left(1+\delta_2(x)\right)e_u^2\\ &\le -\rho V_\theta + \psi\end{aligned} \tag{31}$$

where $\rho=\sigma\gamma_\theta$ .Eq.(31) implies that for $V_\theta > \psi/\rho$, $\dot{V}_\theta < 0$ and , therefore, $\tilde{\theta}$ is bounded. By integrating (31), we can establish that:

$$\|\tilde{\theta}\|^2 \le \|\tilde{\theta}(0)\|^2 e^{-\rho t} + 2\gamma_\theta\frac{\psi}{\rho} \tag{32}$$

From (32) we have

$$\|\tilde{\theta}\| \le \|\tilde{\theta}(0)\|e^{-0.5\rho t} + \sqrt{2\gamma_\theta\psi/\rho} \tag{33}$$

Using (33) and the fact that $\delta(z)$ and $\beta(\xi,\eta),\delta_2(x)$ are bounded, we can write

$$\begin{aligned}&\left|\beta(\xi,\eta)\left(1+\delta_2(x)\right)\left(\phi^T(z)\tilde{\theta}+\delta(z)\right)\right|\\ &\le \left|\beta(\xi,\eta)\left(1+\delta_2(x)\right)\phi^T(z)\tilde{\theta}\right| + \cdots\\ &\quad + \left|\beta(\xi,\eta)\left(1+\delta_2(x)\right)\delta(z)\right|\\ &\le \left|\beta(\xi,\eta)\left(1+\delta_2(x)\right)\right|\left\|\phi^T(z)\right\|\left\|\tilde{\theta}\right\| + \cdots\\ &\quad + \beta(\xi,\eta)\left(1+\delta_2(x)\right)\delta(z)\Big|\\ &\le \left|\beta(\xi,\eta)\left(1+\delta_2(x)\right)\right|\left\|\phi^T(z)\right\|\left\|\tilde{\theta}(0)\right\|e^{-0.5\rho t} + \cdots\\ &\quad + \left|\beta(\xi,\eta)\left(1+\delta_2(x)\right)\right|\left\|\phi^T(z)\right\|\sqrt{2\gamma_\theta\psi/\rho} + \cdots\\ &\quad + \left|\beta(\xi,\eta)\left(1+\delta_2(x)\right)\delta(z)\right|\\ &\le \psi_0 e^{-0.5\rho t} + \psi_1\end{aligned} \tag{34}$$

where ψ_0, ψ_1 are some finite positive constants.

Lemma 2[11]: The following inequality holds for all $\Xi > 0$ and $\varsigma \in R$ with $K_c = 0.2785$.

$$0 \le |\varsigma| - \varsigma \cdot \tanh\left(\frac{\varsigma}{\Xi}\right) \le K_c \Xi \tag{35}$$

Theorem 1: Consider the system (1).Suppose that Assumption1-5 are satisfied, then the neural network controller and adaptation law given by (24) guarantees the convergence of the neural network parameters and to be uniformly ultimately bounded of all the signal in the closed-loop system.

Proof: Consider the Lyapunov function candidate:

$$V(\underline{e},\eta) = \underline{e}^T P \underline{e} + \mu V_0(\eta) \tag{36}$$

Differentiating (36) with respect to time and using (11),(20),(34),(35),and lemma 1,we have

$$0 \le |\varsigma| - \varsigma \cdot \tanh\left(\frac{\varsigma}{\Xi}\right) \le K_c \Xi$$

with $K_c = 0.2785$,we obtain

$$\begin{aligned}
\dot{V}(\underline{e}) &= \underline{e}^T\left(A_c^T P + P A_c\right)\underline{e} - 2b^T P \underline{e} \lambda \tanh\left(\frac{b^T P \underline{e}}{\Xi}\right) + \cdots \\
&\quad + 2b^T P \underline{e} \beta(\xi,\eta)(1+\delta_2(x))(u^* - u) + \mu \dot{V}_0(\eta) \\
&= -\underline{e}^T Q \underline{e} - 2b^T P \underline{e} \lambda \tanh\left(\frac{b^T P \underline{e}}{\Xi}\right) + \cdots \\
&\quad + 2b^T P \underline{e} \beta(\xi,\eta)(1+\delta_2(x))\left(\phi^T(z)\tilde{\theta} + \delta(z)\right) + \mu \dot{V}_0(\eta) \\
&\le -\underline{e}^T Q \underline{e} - 2b^T P \underline{e} \lambda \tanh\left(\frac{b^T P \underline{e}}{\Xi}\right) + 2\left|b^T P \underline{e}\right|\left(\psi_0 e^{-0.5\rho t} + \psi_1\right) \mu \dot{V}_0(\eta) \\
&\le -\underline{e}^T Q \underline{e} + 2\left|b^T P \underline{e}\right| \psi_0 e^{-0.5\rho t} + 2\psi_1 K_c \Xi + \mu \dot{V}_0(\eta)
\end{aligned} \tag{37}$$

Considering assumption 4, we have

$$\begin{aligned}
\dot{V}(\underline{e},\eta) &\le -\underline{e}^T Q \underline{e} + 2\left|b^T P \underline{e}\right| \psi_0 e^{-0.5\rho t} + 2\psi_1 K_c \Xi + \cdots \\
&\quad + \mu \frac{\partial V_0(\eta)}{\partial \eta}\left[q(0,\eta) + q(\xi,\eta) - q(0,\eta)\right] \\
&\le -\underline{e}^T Q \underline{e} - \mu\sigma_3 \|\eta\|^2 + \mu\sigma_4 L_\xi \|\xi\|\|\eta\| + 2\psi_1 K_c \Xi
\end{aligned} \tag{38}$$

Considering assumption 5 and

$$\|\xi\| \le \|\underline{e}\| + \left\|(y_d \quad y_d^{(1)} \quad \cdots \quad y_d^{(r-1)})^T\right\| \le \|\underline{e}\| + b_d$$

Then

$$\begin{aligned}
\dot{V}(\underline{e},\eta) &\le -\lambda_{\min}(Q)\|\underline{e}\|^2 - \mu\sigma_3\|\eta\|^2 + \mu\sigma_4 L_\xi \|\underline{e}\|\|\eta\| + \cdots \\
&\quad + \mu\sigma_4 L_\xi b_d \|\eta\| + 2\psi_1 K_c \Xi
\end{aligned} \tag{39}$$

Using the inequality

$$\mu\sigma_4 L_\xi \|\underline{e}\|\|\eta\| \le \frac{1}{2}\mu\sigma_4 L_\xi \varepsilon_1 \|\eta\|^2 + \frac{1}{2\varepsilon_1}\mu\sigma_4 L_\xi \|\underline{e}\|^2 \tag{40}$$

$$\mu\sigma_4 L_\xi b_d \|\eta\| \le \left(\mu\sigma_4 L_\xi \varepsilon_2 b_d\right)^2 \|\eta\|^2 + \frac{1}{4\varepsilon_2^2} \tag{41}$$

Then (39) satisfies

$$\begin{aligned}
\dot{V}(\underline{e},\eta) &\le -\lambda_{\min}(Q)\|\underline{e}\|^2 - \mu\sigma_3\|\eta\|^2 + \frac{1}{2}\mu\sigma_4 L_\xi \varepsilon_1 \|\eta\|^2 + \ldots \\
&\quad \frac{1}{2\varepsilon_1}\mu\sigma_4 L_\xi \|\underline{e}\|^2 + \left(\mu\sigma_4 L_\xi \varepsilon_2 b_d\right)^2\|\eta\|^2 + \frac{1}{4\varepsilon_2^2} + 2\psi_1 K_c \Xi \\
&\le -\left(\lambda_{\min}(Q) - \frac{1}{2\varepsilon_1}\mu\sigma_4 L_\xi\right)\|\underline{e}\|^2 + \frac{1}{4\varepsilon_2^2} + 2\psi_1 K_c \Xi - \cdots \\
&\quad -\mu\left[\sigma_3 - \frac{1}{2}\sigma_4 L_\xi \varepsilon_1 - \mu\left(\sigma_4 L_\xi \varepsilon_2 b_d\right)^2\right]\|\eta\|^2
\end{aligned} \tag{42}$$

Where $\varepsilon_1, \varepsilon_2$ are suitable positive constants. Adjusting $\varepsilon_1, \varepsilon_2$ to make $\sigma_3 - \frac{1}{2}\sigma_4 L_\xi \varepsilon_1 - \mu\left(\sigma_4 L_\xi \varepsilon_2 b_d\right)^2 > 0$.

Selecting $\mu = \dfrac{\frac{1}{2}\left(\sigma_3 - \frac{1}{2}\sigma_4 L_\xi \varepsilon_1\right)}{\left(\sigma_4 L_\xi \varepsilon_2 b_d\right)^2}$, then

$$\dot{V}(\underline{e},\eta) \le -\left(\lambda_{\min}(Q) - 0.5 - \frac{1}{2\varepsilon_1}\mu\sigma_4 L_\xi\right)\|\underline{e}\|^2 - \frac{1}{4}\frac{\left(\sigma_3 - \frac{1}{2}\sigma_4 L_\xi \varepsilon_1\right)^2}{\left(\sigma_4 L_\xi \varepsilon_2 b_d\right)^2}\|\eta\|^2 + \varepsilon \tag{43}$$

Where $\varepsilon = \frac{1}{4\varepsilon_2^2} + 2\left\|b^T P\right\|^2 \psi_0^2 e^{-\rho t} + 2\psi_1 K_c \Xi$. Selecting Q to make $\lambda_{\min}(Q) - 0.5 - \frac{1}{2\varepsilon_1}\mu\sigma_4 L_\xi > 0$. From the above equation, we can know that tracking error and internal states η are all uniformly ultimately bounded. Besides, since $\|\xi\| \le \|\underline{e}\| + \left\|(y_d \quad y_d^{(1)} \quad \cdots \quad y_d^{(r-1)})^T\right\| \le \|\underline{e}\| + b_d$, then the state ξ is uniformly ultimately bounded too. This completes the proof.

VI. SIMULATION STUDY

In this section, to illustrate the validity of the proposed adaptive neural network controller, the following SISO affine nonlinear uncertain system with zero dynamics is simulated. The affine nonlinear system is described by the following differential equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4x_1 + 2x_1 x_2^2 \\ 0 \end{bmatrix} + \Delta f(x) + \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \Delta g(x)\right)u \tag{44}$$
$$y = x_2$$

where $\Delta f(x) = \begin{bmatrix} 0 \\ 0.1\sin x_1 \end{bmatrix}$, $\Delta g(x) = \begin{bmatrix} 0 \\ 0.1\sin x_2 \end{bmatrix}$ .The control objective is to force the system output y to track the desired trajectory $y_d = \sin t + 2\cos(0.5t)$ .The relative degree $r = 1$ .Selecting nonlinear transformation $T(x) = \left[\xi^T, \eta^T\right]^T = \left[x_2, x_1\right]^T$, then the zero dynamic $\dot{x}_1 = -4x_1$ which is exponential stable. We know $\delta_1(x) = 0.1\sin x_1$ and $\delta_2(x) = 0.1\sin x_2$ satisfy the matched condition. The system initial conditions are $x(0) = [2 \quad 1]^T$ .The design parameters used in this simulation are selected as follows $Q = diag[10,10]$, $P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$, $K = [1,2]^T$, $\Xi = 0.03$, $\gamma_\theta = 6$, $\sigma = 0.08$ .The simulation result is shown in Fig1,2,3,4.

The simulation result for the output is shown in Fig.1 ,and the control input signal is shown in Fig.2. the node changes are shown in Fig.3. Fig.4 shows the evolution of the Euclidian norm of the parameter estimates It can be seen that the actual trajectories converge rapidly to the desired ones. The computer simulation results show that the adaptive neural network controller can perform successful control and achieve desired performance.

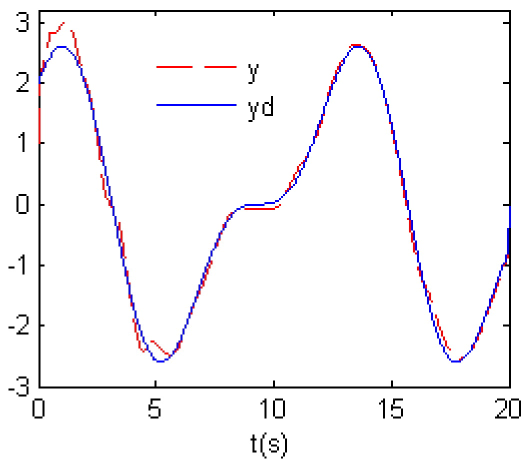

Figure 1. Plots of output tracking of system

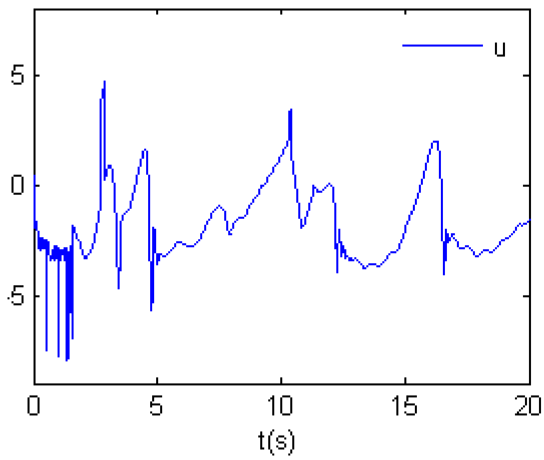

Fig.2 Plots of Control input

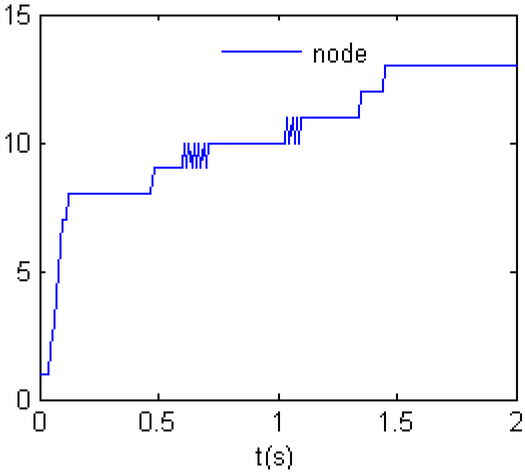

Figure 3 Node Number of Hidden Layer

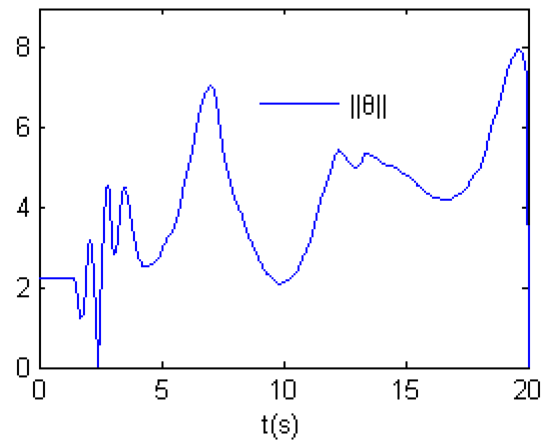

Fig 4.Norm of the weight vector θ

V. CONCLUSIONS

In this paper, a new adaptive neural network control scheme is presented for a class of matched SISO affine nonlinear uncertain systems with zero dynamics. The error between ideal controller and neural network controller is used to update the adjustable parameters by the gradient descent algorithm. The overall adaptive scheme guarantees that all signals involved are uniformly ultimately bounded and the output of the closed-loop system tracks the desired output trajectory. Simulation results show the controller can achieve a satisfactory performance.

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