Distributed H_∞ coordination of multi-agent systems with directed switching topology and time-delay

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***Abstract*—In this paper, we consider multi-agent H_∞ consensus control problems with external disturbance under directed switching topologies. Both switching networks with and without time-delay are taken into consideration. By the model transformation, the multi-agent H_∞ control problem is converted into the switching linear system H_∞ control problem with special structure. Based on graph theory and common Lyapunov function method, we establish LMI sufficient conditions under which all agents can reach consensus with the desired H_∞ performance in two cases of zero and nonzero communication delays respectively. Specially, we obtain an explicit estimation expression of H_∞ performance index in undirected non-time-delay switching topology case. Finally, two numerical examples are provided to illustrate the effectiveness of the obtained results.**

***Index Terms*-Multi-agent systems, H_∞ control, switching topology, consensus, time-delay**

I. INTRODUCTION

Recently, the coordination problem of multiple autonomous agents has attracted a great deal of attention for its applications in many fields such as aggregation behavior of animals, collective motion of particles, cooperative control of unmanned air vehicles, schooling for underwater vehicles, distributed sensor networks, attitude alignment for cluster of satellites, distributed optimization of multiple mobile robotic systems, scheduling of automated highway systems and congestion control of communication networks in [1]–[5].

In the literature, a critical problem in distributed cooperative control of multi-agent systems is to design appropriate protocols that enable all agents to asymptotically reach an agreement on certain quantities of interest. This is usually called the consensus problem, which is well accepted as one of the most important and fundamental issues in the fields of automata theory and coordination control of multi-agent systems. In the fields of system and control, the development of consensus theory is primarily impelled by Vicsek's particle swarm model mentioned in [1]. Vicsek et al. proposed a simple discrete-time model for phase transition of a group of self-driven particles and simulation showed complex dynamics of the model. Jadbabaie et al. provided a theoretical explanation for the observed behavior of the Vicsek model using the graph theory [3]. Till now, many interesting results for solving similar or generalized consensus problems have also been obtained (see [4], [5] and the references therein).

There is no doubt that the stability of multi-agent systems is of utmost importance. In real applications, the interacting topology between agents may change dynamically. For example, in the case of interaction via communications, the communication links between vehicles may be unreliable due to disturbances and/or subject to communication time-delay. However, a well-known fact is that switching of the communication topology and communication time delays may lower the system performance and even cause the network system to diverge or oscillate [6]. So it is necessary for us to solve the problem with the impact of external disturbances and time-delay on the network. In [7], Lin et al. considered robust H_∞ control in the multi-agent system which involved disturbances and time-delay in both fixed topology case and switching topology case respectively. A linear matrix inequality (LMI) approach was adopted to study consensus problems in [8] and it was proved that all the nodes in a network achieved average consensus asymptotically for appropriate communication delays if the network topology was connected. From a practical standpoint, LMI approaches are appealing for applications because there are effective and powerful algorithms such as interior-point method for the solution of LMI problems and there are also a number of software packages such as Matlab to be available for solving LMI problems [9]. [10] investigated the L_2-L_∞ leader-following coordination problems with undirected switching topologies and external disturbance. Theoretically, the consensus in undirected switching topology with time delay is easier than that of directed switching topology. Recently, some preliminary results have been reported to deal with the directed switching topology with time delays (see, for instance, [7], [8] and [11]]).

Furthermore, with many practical applications, especially involving mechanical systems such as unmanned aerial vehicles and mobile robots can be controlled directly by their accelerations rather than by their velocities. Hence, it is also necessary to investigate consensus problems of agents with dynamics which is taken as a double integrator. Double integrator model is a second-order model. There are many interesting agent-related works involved double integrator agent's dynamics such as in [5], [12]–[15] and the references therein. [16] established second order consensus condition for the directed

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switching topology by using common Lyapunov function method.

Motivated by the above works, we study a group of agents with the double integrator dynamics. The main purpose of this paper is to develop a decentralized control strategy to reach the global consensus of the multi-agent systems and discuss H_∞ performance index of the closed systems under the directed switching topology. By using the model transformation, the multi-agent H_∞ control problem is converted into the switching linear system H_∞ control problem with special structure. In our leader-following construction, the interconnection topological structure among the agents keeps changing, a sufficient condition given by constructing a parameter-dependent common Lyapunov function can still guarantee global consensus stability and achieve the desired H_∞ performance. Especially, we obtain an explicit estimation expression in non-time-delay case to estimate H_∞ performance index, which may make our result be applied easily.

The rest paper is organized as follows. In Section II, we give a formulation of the coordination problem with the help of graph theory and H_∞ control theory. Then in Section III, the main results on the consensus stability and H_∞ performance index are obtained for the multi-agent system under varying interconnection topologies. Following that, Section IV provides simulation examples, and finally, the concluding remarks are given in Section V.

The notation of this paper is standard. Throughout this paper, the following notations are used: R is the real number set. I is an identity matrix with compatible dimension. A^T is denoted as transpose of a matrix A; $\mathbf{1}$=$[1\dots1]^T$ with proper dimension; For symmetric matrices A and B, $A>(\geq)B$ means $A-B$ is positive (semi-) definite. $\lambda(A)$ represents an eigenvalue of A. For symmetric matrices A, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ represent the minimum and maximum eigenvalue of A respectively. $\|\bullet\|$ denotes Euclidean norm. "$*$" denotes the entries of matrices implied by symmetry. $\otimes$ is the denotes the Kronecker product, which satisfies (1) $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$; (2) If $A\geq 0$ and $B\geq 0$, then $A\otimes B\geq 0$.

II. Graph Theory, H_∞ Control Theory and Problem Formulation

A. Graph Theory

First of all, we introduce some preliminary knowledge of graph theory that will be used throughout this paper. More details are available referring to [17]. Let $\mathcal{G}=\{\mathcal{V},\varepsilon,A\}$ be a weighted directed graph of order n, where $\mathcal{V}=\{v_1,v_2,\cdots,v_n\}$ is the set of nodes, ε is the set of edges and a weighted adjacency matrix $A=[a_{ij}]$ is of nonnegative elements. The node indexes belong to a finite index set $\mathcal{I}=\{1,2,\cdots,n\}$. The element a_{ij} associated with the edge of the directed graph is positive . i.e., $a_{ij}>0 \iff (v_i,v_j)\in\varepsilon$. Moreover, for all $i\in\mathcal{I}$ we assume $a_{ij}=0$. Throughout the paper, we assume that all the graphs have no edges from a node to itself. A weighted graph is said to be undirected if $\forall(v_i,v_j)\in\varepsilon \Longrightarrow (v_j,v_i)\in\varepsilon$ and $a_{ij}=a_{ji}$. Otherwise, the graph is called a directed graph. If $(v_i,v_j)\in\varepsilon$, then v_j is said to be a neighbor of v_i and we denote the set of all neighbor nodes of node v_i by $\mathcal{N}_i=\{j|(v_i,v_j)\in\varepsilon\}$. A path is a sequence of ordered edges of the form $(v_{i_1},v_{i_2}),(v_{i_2},v_{i_3}),\cdots,(v_{i_{j-1}},v_{i_j})$ where $i_j\in\mathcal{I}$ and $v_{i_j}\in\mathcal{V}$, which starts from v_i and ends with v_j such that consecutive pair of vertices make an edge of digraph. v_j is said to be reachable from v_i if there is a path from node v_i to another node v_j . If a node v_i is reachable from every other node of the directed graph, then it is said to be globally reachable.

The degree matrix $D=\{d_1,d_2,\cdots,d_n\}\in\mathcal{R}^{n\times n}$ of graph $\mathcal{G}$ is a diagonal matrix, where diagonal elements $d_i=\sum_{j\in\mathcal{N}_i}a_{ij}$ for $i=1,2,\cdots,n$. Then the Laplacian of $\mathcal{G}$ is defined as $L=D-A\in R^{n\times n}$. Obviously, the Laplacian matrix of any undirected graph is symmetric.

In what follows, we mainly consider a graph $\hat{\mathcal{G}}$ associated with the system with n agents (labeled by $v_i, i=1,2,\cdots,n$) and one leader (labeled by v_0). A simple and directed graph $\mathcal{G}$ describes the topology relation of these n followers, and $\hat{\mathcal{G}}$ contains graph $\mathcal{G}$ and v_0 with the directed edges from some agents to the leader describes the topology relation among all agents. The graph $\hat{\mathcal{G}}$ is said to be undirected if the induced subgraph $\mathcal{G}$ associated with n followers is undirected. A diagonal matrix $B\in R^{n\times n}$ is defined as a leader adjacency matrix associated with $\bar{\mathcal{G}}$ whose diagonal elements are $b_i(i\in\mathcal{I})$. Moreover, the information topology $\bar{\mathcal{G}}$ is time-varying. Denote $\{\bar{\mathcal{S}}=\{\bar{\mathcal{G}}_1,\bar{\mathcal{G}}_2,\cdots,\bar{\mathcal{G}}_N\}$ as a set of the graphs of all possible topologies and denote $\mathcal{P}=\{1,2,\cdots,N\}$ as its index set. To describe the variable interconnection topology, we define a switching signal $\sigma:[0,\infty)\to\mathcal{P}$, which is piecewise constant. Let $t_1=0,t_2,t_3,\cdots$ be an infinite time sequence at which the interconnection graph of the considered multi-agent system switches. Therefore, $\mathcal{N}_i$ and the connection weight $a_{ij}(i,j=1,\cdots,n)$ are time-varying, and furthermore, Laplacian $L_{\sigma(t)}(\sigma(t)\in\mathcal{P})$ associated with the switching interconnection graph is also time-varying, though it is a time-invariant matrix in any interval $[t_i,t_{i+1})$. Assume there is a constant $\tau_0>0$, often called dwell time, with $t_{i+1}-t_i\geq\tau_0$, $\forall i$.

B. H_∞ control theory

In this subsection, we introduce some basic concepts on H_∞ control theory. Consider the following switching system

$$\begin{cases} \dot{x}(t)=A_\sigma x(t)+B\omega(t)\\ z(t)=Cx(t)\end{cases} \tag{1}$$

where $x(t)\in\mathcal{R}^n$ is the system state, $z(t)\in\mathcal{R}^m$ is the objective signal to be attenuated, $\omega(t)\in\mathcal{L}_2[0,\infty)$ is the external disturbance signal and $L_2[0,\infty)$ represents the space of square integrable vector functions over$[0,\infty)$. A_σ,B,C are matrices with appropriate dimensions.

H_∞ norm of closed-loop transfer function matrix $T_{z\omega}(t)$ from external disturbance $\omega(t)$ to the controlled output $z(t)$ which is defined by

$$\|T_{z\omega}(t)\|_\infty = \sup_{\omega(t)\neq 0} \frac{\|z(t)\|_2}{\|\omega(t)\|_2}$$

Thus, the H_∞ control objective is to design an output feedback protocol $u_i(t)(i \in \mathcal{I})$ such that $\|T_{z\omega}(t)\|_\infty < \mu$, or equivalently, the closed-loop system meets the dissipation inequality

$$\int_0^\infty \|T_{z\omega}(t)\|^2 dt < \mu^2 \int_0^\infty \|\omega(t)\|^2 dt, \forall \omega \in \mathcal{L}_2[0,\infty)$$

where $\mu > 0$ is a given H_∞ performance index. The following result is about the H_∞ performance index.

Lemma 1: [7] System (1) is asymptotically stable with $\|T_{z\omega}(s)\|_\infty < \mu$ for all nonzero $\omega(t) \in \mathcal{L}_2[0,\infty)$, if there exists a positive definite matrix $P \in \mathcal{R}^{n\times n}$ and a scalar μ satisfying

$$\begin{bmatrix} A_\sigma^T P + PA_\sigma & PB & (C\otimes I_n)^T \\ B^T P & -\mu I_n & 0 \\ (C\otimes I_n) & 0 & -\mu I_{2n} \end{bmatrix} < 0$$

C. Problem Formulation

In the paper, suppose that the multi-agent system under consideration consists of n agents. Each agent is regarded as a node in a directed graph $\mathcal{G}$. $\hat{\mathcal{G}}$ is the graph with a leader in $\mathcal{G}$. Assume the ith agent $(i \in \mathcal{I})$ has the second-order dynamics and disturbance is considered as follows

$$\begin{cases} \dot{x_i}(t) = y_i(t) \\ \dot{y_i}(t) = u_i(t) + \omega_i(t) \end{cases}, \quad i = 1, \cdots, n, \tag{2}$$

where $x_i(t)$ represents the position of the node v_i, $y_i(t)$ its velocity, $u_i(t)$ its control input and $\omega_i(t) \in \mathcal{L}_2[0,\infty)$ its disturbance input. The dynamic of the leader is taken as

$$\begin{cases} \dot{x_0}(t) = y_0(t) \\ \dot{y_0}(t) = 0 \end{cases} \tag{3}$$

where y_0 is the leader's velocity and keeps unchanged. The leader of this considered multi-agent system is active, and its motion only depends on the known input y_0 and does not be influenced by the following agents. But follower nodes are affected by other followers and the leader. To the end, the controller u_i of agent i , regarded as node i in a graph, requires state information from a subset of the agent's flockmates, called the neighbor set $\mathcal{N}_i$ as above. Take the local control law as follows in case of non-delay for agent $i(i = 1, 2, \cdots, n)$, which bases on neighbor feedback law and had been proposed by several references

$$\begin{aligned} u_i = k[&\sum_{v_j\in\mathcal{N}_i} a_{ij}(t)(x_j(t) - x_i(t)) \\ &+ b_i(t)(x_i(t) - x_0(t))] \\ &+ kr[\sum_{v_j\in\mathcal{N}_i} a_{ij}(t)(y_j(t) - y_i(t)) \\ &+ b_i(t)(y_i(t) - y_0(t))] \end{aligned} \tag{4}$$

Take the local law as follows in case that there is time-varying delay,

$$\begin{aligned} u_i = k[&\sum_{v_j\in\mathcal{N}_i} a_{ij}(t)(x_j(t-\tau(t)) - x_i(t-\tau(t))) \\ &+ b_i(t)(x_i(t-\tau(t)) - x_0(t-\tau(t)))] \\ &+ kr[\sum_{v_j\in\mathcal{N}_i} a_{ij}(t)(y_j(t-\tau(t)) - y_i(t-\tau(t))) \\ &+ b_i(t)(y_i(t-\tau(t)) - y_0(t-\tau(t)))] \end{aligned} \tag{5}$$

Here k is "control" parameter, which is positive constant and will be determined later. r is positive weighted parameter, $\tau(t)$ is time-varying delay and it satisfies $0 \le \tau(t) < d, \dot{\tau}(t) \le d_1 < 1$.

At time t , the $a_{ij}(t)$ and $b_i(t)$ are chosen by

$$a_{ij}(t) = \begin{cases} \alpha_{ij} & \text{if agent } i \text{ is connected to agent } j \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$b_i(t) = \begin{cases} \beta_i & \text{if agent } i \text{ is connected to the leader} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where $\alpha_{ij} > 0(i, j = 1, ..., n)$ is connection weight constant between agent i and agent j, and $\beta_i > 0(i = 1, ..., n)$ is connection weight constant between agent i and leader.

The main purpose of this paper is to design k, r in $u_{ij}(t)$ to guarantee that any follower-agent can track the active leader, i.e.,

$$\begin{cases} \lim_{t\to\infty} x_i = x_0 \\ \lim_{t\to\infty} y_i = y_0 \end{cases}, \quad i = 1, \cdots, n, \tag{8}$$

and the closed system has smaller H_∞ performance index.

For the convenience, define

$$x := \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad y := \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \quad \omega := \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix}.$$

Then we can rewrite the closed-loop system (2-4) as

$$\begin{cases} \dot{x} = y \\ \dot{y} = -k(L_\sigma + B_\sigma)x + kB_\sigma \mathbf{1} \otimes x_0 \\ \qquad -kr(L_\sigma + B_\sigma)y + krB_\sigma \mathbf{1} \otimes y_0 + \omega \end{cases} \tag{9}$$

where $\mathbf{1} = (1, 1, ..., 1)^T$. $\sigma : [0,\infty) \to \mathcal{P} = \{1, ..., N\}$ is a piecewise-constant switching signal with successive switching times, L_σ is the Laplacian for the n agents, B_σ is a $n \times n$ diagonal matrix whose ith diagonal element is $b_i(t)$ at time t ($b_i(t)$ is a positive constant if agent i is connected to the leader).

Denote $\bar{x}(t) = x(t) - x_0(t)\mathbf{1}$ and $\bar{y}(t) = y(t) - y_0(t)\mathbf{1}$. For convenience, let $H_\sigma = L_\sigma + B_\sigma$. Due to $L\mathbf{1} = 0$, we can obtain

$$\begin{cases} \dot{\bar{x}} = \bar{y} \\ \dot{\bar{y}} = -kH_\sigma \bar{x} - krH_\sigma \bar{y} + \omega \end{cases} \tag{10}$$

Define consensus error vector $\delta = (\bar{x}^T, \bar{y}^T)^T$. Thus, using the control law (4), the error dynamics of closed system can be expressed as follows:

$$\dot{\delta}(t) = F_\sigma \delta(t) + B\omega(t) \tag{11}$$

where

$$F_\sigma = \begin{bmatrix} 0 & I_n \\ -kH_\sigma & -krH_\sigma \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \tag{12}$$

As for the time delay case, the error dynamics can be written as the following form by using the control law (5):

$$\dot{\delta}(t) = A\delta(t) - E_\sigma \delta(t - \tau(t)) + B\omega(t) \tag{13}$$

where

$$E_\sigma = \begin{bmatrix} 0 & 0 \\ kH_\sigma & krH_\sigma \end{bmatrix}, \ A = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} \tag{14}$$

Obviously, the multi-agent system achieve consensus if and only if $\lim_{t\to\infty} \delta(t) = 0$. The consensus stability of the multi-agent system (2, 3) and (4) in non-time delay case is converted into the stability of error dynamics system (11), and the consensus stability of the multi-agent system (2, 3) and (5) in time delay case is converted into the stability of error dynamics system (13)

To quantize the influence of the disturbance input on the consensus, we should investigation how the disturbance affect the relative position and relative velocity errors between each follower and the leader. Therefore, we consider the following output $z(t)$ for the error dynamic systems (11) and (13):

$$z(t) = (C \otimes I_n)\delta(t) \tag{15}$$

where

$$C = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \tag{16}$$

The weighted matrix C is erroneous scope restraint to relative position and velocity, the value of the weighted coefficients a and b may be different according to the controlled different ranges of relative position and velocity. Generally speaking, the smaller the controlled range is, the larger the weighted value is. So we can limit $\|z(t)\|_\infty$ within certain range. That is to say, we can limit relative position and velocity to the desired scope of the systems.

Since $z(t) = 0$ implies that the following agents can track the leader, the attenuating ability of multi-agent system on consensus against external disturbances can be quantitatively measured by the H_∞ norm of closed-loop transfer function matrix $T_{z\omega}(t)$ from external disturbance $\omega(t)$ to the controlled output $z(t)$. From the above transformations, we know that the H_∞ consensus problem of the multi-agent systems with external disturbances are converted into the above H_∞ control problem of the switching systems (11) and (15) or the switching time-delay system (13) and (15).

III. H_∞ Consensus Problem

Before giving the main results, some useful results are introduced, which will be used later.

Lemma 2: [18] (Schur Complement) Suppose that a symmetric matrix is partitioned as

$$\begin{bmatrix} D_1 & D_2^T \\ D_2 & D_3 \end{bmatrix} \tag{17}$$

where D_1 and D_3 are square. This matrix is positive definite if and only if D_3 is positive definite and $D_1 - D_2 D_3^{-1} D_2^T$ is positive definite.

Lemma 3: [9] For any vector u, v, and symmetric positive definite matrix S, there is $2u^T v \leq u^T S^{-1} u + v^T S v$

Lemma 4: [12] If graph $\hat{\mathcal{G}}$ is connected and undirected, then the symmetric matrix H_σ is positive definite.

Therefore, define

$$\bar{\lambda} := \min_{p\in\mathcal{P}} \{\lambda(H_p) | \hat{\mathcal{G}} \ is \ \ connnected \ and \ undirected\} \tag{18}$$

Based on lemmas 4 and the fact that the set $\mathcal{P}$ is finite, we can obtain that is fixed and greater than zero, which depends directly on the constants all constants a_{ij} and β_i $(i, j = 1, 2, \cdots, n)$ given in (6) and (7).

A. Non-time-delay Case

In this subsection, we first solve the problem of H_∞ performance index of the switching system (11) and (15) under directed switching communication topology based on the LMI approach. As a special case, the consensus condition for the switching and undirected interconnection topology case can be obtained directly.

Theorem 1: For the system (11) and (15), suppose that the interconnection graph is connected for any interval $[t_i, t_{i+1})$. If there exist a positive definite matrix $\bar{P}$ and a positive constant h such that

$$H_i^T \bar{P} + \bar{P} H_i \geq hI, i \in \mathcal{P} \tag{19}$$

and take a constant

$$k > \frac{2\lambda_{max}(\bar{P})}{hr^2}, \tag{20}$$

then the local control law (4) can guarantee that multi-agent system in leader-following directed case can achieve the consensus for any given initial condition $x(0)$ and $y(0)$ in case that $\omega(t) = 0$. Furthermore, the H_∞ disturbance attenuation from $\omega(t)$ to $z(t)$ is not greater than

$$\min_{m>0} \lambda_{\max}(Q^{-1}\Omega),$$

where the matrices Q and Ω are expressed respectively as follows

$$Q = \begin{pmatrix} kmrhI & kmr^2hI - 2m\bar{P} \\ kmr^2hI - 2m\bar{P} & kmr^3hI - 2mr\bar{P} \end{pmatrix},$$

$$\Omega = \begin{pmatrix} m^2r^2\bar{P}^2 + a^2I & m^2r^3\bar{P}^2 \\ m^2r^3\bar{P}^2 & m^2r^4\bar{P}^2 + b^2I \end{pmatrix}.$$

Proof: To prove the theorem, we consider the dynamics in each interval at first. Note that, in any interval (say

$[t_j, t_{j+1})$), the interconnection topology does not change. Therefore, $F_{\sigma(t)}$ is a constant matrix for $t \in [t_j, t_{j+1})$ for any $j \geq 0$, and then the solution to equation (11) is well defined. Choose a positive definite matrix

$$P = m \begin{bmatrix} 2\bar{P} & r\bar{P} \\ r\bar{P} & r^2\bar{P} \end{bmatrix}. \quad (21)$$

We can verify that P is positive definite by result of lemma 2. Consider a common Lyapunov function $V(t) = \delta^T(t)P\delta(t)$, where P is defined in (21). Then, for any interval $[t_l, t_{l+1})$, we can assume that $\sigma(t) = i, i \in \mathcal{P}, t \in [t_j, t_{j+1})$. To analyze stability of error system (11), we always assume that $\omega(t) = 0$. For the convenience, define

$$\Omega_i = H_i^T\bar{P} + \bar{P}H_i$$

For any interval $[t_j, t_{j+1})$, we can obtain

$$\dot{V}(t) = \delta^T(t)(F_i^TP + PF_i)\delta(t) = -\delta^T(t)Q_i\delta(t)$$

where Q_i is defined as

$$\begin{aligned} Q_i &:= -(F_i^TP + PF_i) \\ &= \begin{pmatrix} kmr\Omega_i & kmr^2\Omega_i - 2m\bar{P} \\ kmr^2\Omega_i - 2m\bar{P} & kmr^3\Omega_i - 2mr\bar{P} \end{pmatrix} \end{aligned} \quad (22)$$

Noticing that

$$\begin{pmatrix} kmr & kmr^2 \\ kmr^2 & kmr^3 \end{pmatrix} \otimes \ (\Omega_i - hI) \geq 0,$$

Thus we obtain

$$Q_i \geq Q := \begin{pmatrix} kmrhI & kmr^2hI - 2m\bar{P} \\ kmr^2hI - 2m\bar{P} & kmr^3hI - 2mr\bar{P} \end{pmatrix}. \quad (23)$$

By using the Schur Complement lemma 2, we know that Q is positive definite only if the inequality (20) is satisfied. So we have $\dot{V}(t) < 0$ when $\omega(t) = 0$. In this case, we have $\lim_{t\to\infty} \delta(t) = 0$, that is, the system can reach consensus for any given initial state $x(0)$ and $y(0)$ when $\omega(t) = 0$.

To consider H_∞ performance index, we always assume that all initial values are zero and nonzero $\omega(t) \in L_2[0, \infty)$. Moreover, by applying the result of lemma 1, the H_∞ disturbance attenuation of the multi-agent system is also not greater than μ if the following inequalities are satisfied for any $i \in \mathcal{P}$

$$\begin{bmatrix} -Q_i & PB & (C \otimes I_n)^T \\ B^TP & -\mu I_n & 0 \\ (C \otimes I_n) & 0 & -\mu I_{2n} \end{bmatrix} < 0$$

which can be guaranteed by

$$\begin{bmatrix} Q & -PB & -(C \otimes I_n)^T \\ -B^TP & \mu I_n & 0 \\ -(C \otimes I_n) & 0 & \mu I_{2n} \end{bmatrix} > 0,$$

where the matrices Q_i and Q are defined by (22) and (23) respectively.

By using lemma 2 again, the above matrix is positive definite if and only if

$$\mu Q - \Omega > 0. \quad (24)$$

It is easy to verify the matrix Ω is positive definite. Thus, the inequality (24) is satisfied only if $\mu > \lambda_{\max}(Q^{-1}\Omega)$.

Therefore, due to $m > 0$, we can obtain that the H_∞ disturbance attenuation from $\omega(t)$ to $z(t)$ is not greater than

$$\min_{m>0} \lambda_{\max}(Q^{-1}\Omega).$$

Now, the proof is completed.

Remark 1: For two positive definite matrices Q and Ω, we have $\lambda(\Omega^{-1}Q) = \lambda(\Omega^{-\frac{1}{2}}Q\Omega^{-\frac{1}{2}})$ [18]. Obviously, $\Omega^{-\frac{1}{2}}Q\Omega^{-\frac{1}{2}}$ is also a positive definite matrix. Thus, all eigenvalues of matrix $\Omega^{-1}Q$ are positive real number. The natation $\lambda_{\max}(Q^{-1}\Omega)$ used in Theorem 1 is meaningful and compatible.

As a special case, the undirected switching topology is discussed in following corollary.

Corollary 1: For the system (11) and (15), suppose that the interconnection graph is connected and undirected for any interval $[t_i, t_{i+1})$. Take the positive constants k and r satisfied

$$k > \frac{1}{r^2\bar{\lambda}}. \quad (25)$$

Then the local control law (4) can guarantee that multi-agent system in leader-following directed case can achieve the consensus for any given initial condition $x(0)$ and $y(0)$ in case that $\omega(t) = 0$. Furthermore, the H_∞ disturbance attenuation from $\omega(t)$ to $z(t)$ is not greater than

$$\min_{m>0} \lambda_{\max}(\bar{Q}^{-1}\bar{\Omega}),$$

where the matrices $\bar{Q}$ and $\bar{\Omega}$ are expressed respectively as follows:

$$\bar{Q} = \begin{pmatrix} kmrh & kmr^2h - 2m \\ kmr^2h - 2m & kmr^3h - 2mr \end{pmatrix}, \quad (26)$$

$$\bar{\Omega} = \begin{pmatrix} m^2r^2 + a^2 & m^2r^3 \\ m^2r^3 & m^2r^4 + b^2 \end{pmatrix}. \quad (27)$$

Proof: Based on lemma 4 and the definition of $\bar{\lambda}$ in (18), we have

$$H_l^TI + IH_l \geq 2\bar{\lambda}I.$$

Choose a symmetric matrix

$$P = m \begin{bmatrix} 2I & rI \\ rI & r^2I \end{bmatrix} \quad (28)$$

By using similar method as proof of Theorem 1, we can known that if the positive constants k and r satisfied

$$k > \frac{1}{r^2\bar{\lambda}}, \quad (29)$$

then the local control law (4) can guarantee that multi-agent system in leader-following directed case can achieve the consensus for any given initial condition $x(0)$ and $y(0)$ in case that $\omega(t) = 0$. Moreover, the H_∞ disturbance attenuation of the multi-agent system is also not greater than μ if

$$\mu Q - \Omega > 0. \quad (30)$$

where

$$Q = \begin{pmatrix} kmrhI & kmr^2hI - 2mI \\ kmr^2hI - 2mI & kmr^3hI - 2mrI \end{pmatrix},$$

$$\Omega = \begin{pmatrix} m^2r^2I + a^2I & m^2r^3I \\ m^2r^3I & m^2r^4I + b^2I \end{pmatrix}$$

Obviously, the matrix inequality (30) is equivalent to

$$\mu\bar{Q} - \bar{\Omega} > 0, \tag{31}$$

where the matrices $\bar{Q}$ and $\bar{\Omega}$ are defined in (26) and (27) respectively. Similarly, we can obtain that the H_∞ disturbance attenuation from $\omega(t)$ to $z(t)$ is not greater than

$$\min_{m>0} \lambda_{\max}(\bar{Q}^{-1}\bar{\Omega}).$$

Remark 2: It is easy to verify the two matrices $\bar{Q}$ and $\bar{\Omega}$ are positive definite. The two eigenvalues of matrix $\bar{Q}^{-1}\bar{\Omega}$ are positive, which can be obtained by directed calculation. Although the close-system have n-dimension, the estimation of H_∞ disturbance attenuation only based on two-dimension matrix.

B. Time-delay Case

In this section, we mainly study H_∞ control problem in the directed network of multi-agents with switching and time-varying delay.

Theorem 2: Assume that the interaction graph $\mathcal{G}$ is connected, for given positive constants $k, r > 0$ and the time-varying time-delay $\tau(t)$ which satisfies $0 \le \tau(t) < d$ and $\dot{\tau}(t) \le d_1 < 1$, the system (13) and (15) is stable for $\omega(t) = 0$ with the given H_∞ disturbance attenuation at least μ, if there exist a positive constant m and positive definite matrices $\bar{P}, Q_1, Q_2, R_1, R_2$ such that

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} \\ * & \Theta_{22} & \Theta_{23} & \Theta_{24} \\ * & * & \Theta_{33} & \Theta_{34} \\ * & * & * & \Theta_{44} \end{bmatrix} < 0 \tag{32}$$

where

$$\begin{aligned}
\Theta_{11} &= -kmr\bar{P}H_i - kmrH_i^T\bar{P} + Q_1 + a^2I_n, \\
\Theta_{12} &= 2m\bar{P} - kmr^2\bar{P}H_i - kmr^2H_i^T\bar{P} \\
\Theta_{13} &= kmr\bar{P}H_i, \\
\Theta_{14} &= kmr^2\bar{P}H_i, \\
\Theta_{22} &= 2mr\bar{P} - kmr^3\bar{P}H_i - kmr^3H_i^T\bar{P} \\
&\quad + Q_2 + dR_1 + b^2I_n \\
\Theta_{23} &= kmr^2\bar{P}H_i \\
\Theta_{24} &= kmr^3\overline{P}H_i, \\
\Theta_{33} &= -\frac{d}{1-d_1}R_1 - \mu I_n \\
\Theta_{34} &= 0, \\
\Theta_{44} &= -\frac{d}{1-d_1}R_2 - \mu I_n
\end{aligned}$$

Proof: Choose a common Lyapunov function as

$$\begin{aligned}
V(t) =& \delta^T(t)P\delta(t) + \int_{t-\tau(t)}^{t} \delta^T(s)Q\delta(s)ds \\
&+ \int_{-\tau(t)}^{0}\int_{t+\theta}^{t} \dot{\delta}^T(s)R\dot{\delta}(s)dsd\theta
\end{aligned}$$

where $Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}, R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}, P = \begin{bmatrix} 2m\bar{P} & mr\bar{P} \\ mr\bar{P} & mr^2\bar{P} \end{bmatrix}$.

For any $T > 0$, also define a performance function as

$$J(t) = \int_0^T [z^Tz - \mu^2\omega^T\omega]dt.$$

By calculating the derivation of $V(t)$ about the time t, we can get

$$\begin{aligned}
\dot{V}(t) &= 2\delta^T(t)P\dot{\delta}(t) + \delta^T(t)P\delta(t) \\
&- (1-\dot{\tau}(t))\delta^T(t-\tau(t))Q\delta(t-\tau(t)) \\
&+ \tau(t)\dot{\delta}^T(t)R\dot{\delta}(t) \\
&+ (\dot{\tau}(t)-1)\int_{t-\tau(t)}^{t} (\dot{\delta}^T(s)R\dot{\delta}(s))ds
\end{aligned} \tag{33}$$

According to the system (13) and (15), using Lemma 3 and noticing the fact that $\delta(t-\tau) = \delta(t) - \int_{t-\tau}^{t} \delta(s)ds$, we can obtain

$$\begin{aligned}
\dot{V}(t) &\le 2\delta^T(t)PA\delta(t) - 2\delta^T(t)PE_i\delta(t) \\
&+ \frac{d}{1-d_1}\delta^T(t)PE_iR^{-1}E_i^TP\delta(t) \\
&+ 2\delta^T(t)PB\omega(t) + \delta^T(t)Q\delta(t) \\
&- (1-d_1)\delta^T(t-\tau(t))Q\delta(t-\tau(t)) \\
&+ d\delta^T(t)A^TRA\delta(t) + d\delta^T(t)A^TRB\omega(t) \\
&+ d\delta^T(t-\tau(t))E_i^TRE_i\delta(t-\tau(t)) \\
&+ d\omega^T(t)B^TRA\delta(t) + d\omega^T(t)B^TRB\omega(t)
\end{aligned} \tag{34}$$

we rewrite (34) in a compact form as follows

$$\dot{V}(t) \le \begin{bmatrix} \delta(s) \\ \delta(s-\tau) \\ \omega(s) \end{bmatrix}^T M_i \begin{bmatrix} \delta(s) \\ \delta(s-\tau) \\ \omega(s) \end{bmatrix} \tag{35}$$

where M_i has the following structure

$$M_i := \begin{pmatrix} \Phi_{11} & 0 & PB + dA^TRB \\ 0 & \Phi_{22} & 0 \\ * & 0 & dB^TRB \end{pmatrix} \tag{36}$$

with

$$\begin{aligned}
\Phi_{11} &= PA + A^TP - PE_i - E_iP \\
&\quad + \frac{d}{1-d_1}PE_i^TR^{-1}E_i^TP + Q + dA^TRA \\
\Phi_{22} &= -(1-d_1)Q + dE_i^TRE_i
\end{aligned}$$

Due to zero initial conditions and using the above inequality, we know

$$J(T) = \int_0^T [z^T z - \mu^2 \omega^T \omega] dt$$
$$= \int_0^T [z^T z - \mu^2 \omega^T \omega + \dot{V}(t)] dt - V(T) + V(0)$$
$$\leq \int_0^T (\begin{bmatrix} \delta(t) \\ \delta(t-\tau(t)) \\ \omega(t) \end{bmatrix}^T (\Upsilon + M_i) \begin{bmatrix} \delta(t) \\ \delta(t-\tau(t)) \\ \omega(t) \end{bmatrix}) dt$$
$$- V(T) + V(0) \quad (37)$$

where Υ has the following form:

$$\Upsilon := \begin{pmatrix} C^T C \otimes I_n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\mu^2 I_n \end{pmatrix} \quad (38)$$

Due to $V(T) > 0$ and $V(0) = 0$, if $(\Upsilon + M_i) < 0$, then we have $J(T) < 0$. That is to say, $z^T z < \mu^2 \omega^T \omega$, which implies $\|T_{z\omega}(s)\|_\infty = sup \frac{\|z\|_2}{\|\omega\|_2} < \mu$. Following that, we provide the conditions of $(\Upsilon + M_i) < 0$. $(\Upsilon + M_i)$ can divide into $\Xi_1 + \Xi_2$, where

$$\Xi_1 := \begin{pmatrix} \Phi_{11} + C^T C \otimes I_n & 0 & PB \\ 0 & -(1-d_1)Q & 0 \\ B^T P & 0 & -\mu^2 I_n \end{pmatrix} \quad (39)$$

and

$$\Xi_2 := \begin{pmatrix} 0 & 0 & dA^T RB \\ 0 & -dE_i^T RE_i & 0 \\ dB^T RA & 0 & dB^T RB \end{pmatrix} \quad (40)$$

Noticing that $\Xi_2 \geq 0$, so $(\Upsilon + M_i) < 0$ is equivalent to $\Xi_1 < 0$.

By using the lemma 2, Ξ_1 is converted into $\Xi_3 + \Xi_4$. Ξ_3 and Ξ_4 are expressed respectively as follows:

$$\Xi_3 := \begin{pmatrix} \Phi_{11} + C^T C \otimes I_n & 0 \\ 0 & -(1-d_1)Q \end{pmatrix} \quad (41)$$

$$\Xi_4 := \frac{1}{\mu^2} \begin{pmatrix} PB \\ 0 \end{pmatrix} * \begin{pmatrix} B^T P & 0 \end{pmatrix} \quad (42)$$

Since $\Xi_4 \geq 0$, we can get $\Xi_3 < 0$. Thus, $(\Phi_{11} + C^T C \otimes I_n) < 0$, by using the lemma 2 again, the inequality can rewrite as Δ that has the following structure:

$$\Delta := \begin{pmatrix} \Delta_{11} & PE_i \\ * & -\frac{d}{1-d_1} R - \mu I_{2n} \end{pmatrix} < 0 \quad (43)$$

where $\Delta_{11} = PA + A^T P - PE_i - E_i^T P + Q + dA^T RA + C^T C \otimes I_n$

From above analysis, we can obtain the LMI condition given in the theorem 2 by substituting the block expression of A, P, Q, R, E_i into Δ. The proof is completed now.

IV. Simulation Examples

In this section, two numerical simulations will be given to illustrate the theoretical results obtained in the previous section. Without loss of generality, in case that there is no time-delay, we take $a = 1, b = 1, k = 10, r = 2, h = 1$. Consider a multi-agent system with one leader and six followers. The interconnection directed topology is arbitrarily switched with switching period 1 among three graphs $\hat{\mathcal{G}}_i (i = 1, 2, 3)$. The Laplacian matrices $L_i (i = 1, 2, 3)$ for the three subgraphs $\hat{\mathcal{G}}_i (i = 1, 2, 3)$ are

$$L_1 = \begin{bmatrix} 3.5 & -1.5 & 0 & 0 & 0 & -2 \\ -1 & 5.5 & -2.5 & 0 & -2 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -2 & 5 & -1 & -2 \\ 0 & -2 & 0 & -1 & 5 & -2 \\ -1 & 0 & 0 & -1 & -2 & 4 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 5 & -1 & -2 & 0 & -2 & 0 \\ -1.5 & 4.5 & -1 & 0 & 0 & -2 \\ -2 & -1 & 4 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 & 0 & -2 \\ -1 & 0 & -1 & 0 & 2 & 0 \\ 0 & -2 & 0 & -2 & 0 & 4 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 3 & 0 & 0 & -2 & 0 & -1 \\ 0 & 2 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 \\ -2 & -1 & 0 & 5 & -2 & 0 \\ 0 & -1 & 0 & -2 & 3 & 0 \\ -2 & 0 & -2 & 0 & 0 & 4 \end{bmatrix}$$

and the diagonal matrices for the interconnection relationship between the leader and the followers are

$$B_1 = diag\{1, 0, 0, 1, 0, 0\}$$
$$B_2 = diag\{0, 1, 0, 1, 0, 0\}$$
$$B_3 = diag\{1, 0, 0, 0, 1, 1\}.$$

From matrices L_i and B_i $(i = 1, 2, 3)$, we know that the interconnection graph $\hat{\mathcal{G}}_i$ is connected.

The positive definite

$$\bar{P} = \begin{bmatrix} 3.411 & 0.236 & 0.094 & 0.348 & 0.727 & 0.209 \\ 0.236 & 3.092 & 0.668 & 0.273 & 0.524 & 0.489 \\ 0.094 & 0.668 & 5.303 & 0.363 & 0.475 & 1.20 \\ 0.348 & 0.273 & 0.363 & 3.046 & 0.420 & 0.487 \\ 0.727 & 0.524 & 0.475 & 0.420 & 4.618 & 0.389 \\ 0.209 & 0.489 & 1.203 & 0.487 & 0.389 & 3.794 \end{bmatrix}$$

satisfies condition (19). Its Maximum and minimum eigenvalues are $\lambda_{max}(\bar{P}) = 6.72$ and $\lambda_{min}(\bar{P}) = 2.71$ respectively.

To verify H_∞ performance index, we choose disturbance model $\omega(t) = sin(5 * t)$. The initial positions and velocity of the all agents are randomly produced. The position errors in Fig. 1 are defined as $||x_i(t) - x_0(t)||$ and the velocity errors in Fig. 2 are defined as $||y_i(t) - y_0(t)||$. This two figures show that the follower-agents can track the leader.

In addition, by simple calculations, we know that the disturbance attenuation μ is not greater than $\mu_0 = 2.3077$ if $m = 0.05$.

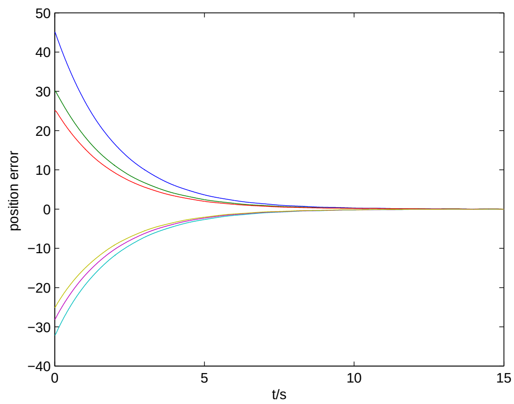

Fig. 1. Position tracking errors of followers

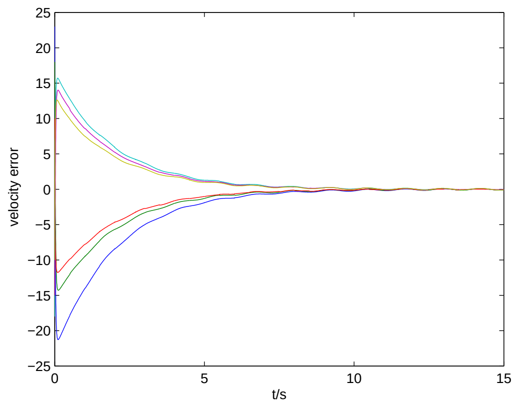

Fig. 2. Velocity tracking errors of followers

Consider a multi-agent system with six agents. The interconnection topology is also arbitrarily switched with switching period 1 among three graphs $\hat{\mathcal{G}}_i(i = 1, 2, 3)$. The Laplacian matrices $L_i(i = 1, 2, 3)$ for the three subgraphs $\hat{\mathcal{G}}_i(i = 1, 2, 3)$ are also defined as above. Consider time-varying time-delay case, we take $a = 1, b = 1, k = 10, r = 5, m = 1$, the disturbance model $\omega(t) = sin(5 * t)$ too. The initial positions and velocity of the all agents are randomly produced. At the same time, we can take the maximum time-delay is not greater than $d = 0.7$ and the maximum value of the derivation of time-delay is $d_1 = 0.92$. so we can get that the disturbance attenuation μ is not greater than $\mu_0 = 3.6766$. By using the Matlab LMI Control Toolbox, we can get $\bar{P}, Q_1, Q_2, R_1, R_2$ as follows, which satisfy the condition (32).

$$\bar{P} = 10^{-4}\times \begin{bmatrix} 7.037 & 0.665 & 0.308 & 0.264 & 0.113 & 0.467 \\ 0.665 & 6.311 & 0.848 & -0.084 & 0.767 & 0.680 \\ 0.308 & 0.848 & 9.117 & 0.375 & 0.421 & 1.499 \\ 0.264 & -0.084 & 0.375 & 6.057 & 0.631 & 0.427 \\ 0.113 & 0.767 & 0.421 & 0.631 & 8.075 & 0.741 \\ 0.467 & 0.680 & 1.499 & 0.427 & 0.741 & 7.537 \end{bmatrix}$$

$$Q1 = 10^{-4}\times \begin{bmatrix} 2.464 & 0.135 & 0.067 & 0.052 & 0.027 & 0.096 \\ 0.135 & 2.321 & 0.172 & -0.014 & 0.157 & 0.139 \\ 0.067 & 0.172 & 2.886 & 0.076 & 0.091 & 0.304 \\ 0.052 & -0.014 & 0.076 & 2.271 & 0.125 & 0.090 \\ 0.027 & 0.157 & 0.091 & 0.125 & 2.676 & 0.150 \\ 0.096 & 0.139 & 0.304 & 0.090 & 0.150 & 2.569 \end{bmatrix}$$

$$Q2 = 10^{-3}\times \begin{bmatrix} 2.2 & 0.3 & 0.1 & 0.1 & 0.0 & 0.2 \\ 0.3 & 1.9 & 0.4 & -0.0 & 0.4 & 0.3 \\ 0.1 & 0.4 & 3.2 & 0.2 & 0.2 & 0.7 \\ 0.1 & -0.0 & 0.2 & 1.8 & 0.3 & 0.2 \\ 0.0 & 0.4 & 0.2 & 0.3 & 2.7 & 0.3 \\ 0.2 & 0.3 & 0.7 & 0.2 & 0.3 & 2.5 \end{bmatrix}$$

$$R1 = 10^{-3}\times \begin{bmatrix} 3.2 & 0.5 & 0.2 & 0.2 & 0.1 & 0.3 \\ 0.5 & 2.6 & 0.6 & -0.1 & 0.5 & 0.5 \\ 0.2 & 0.6 & 4.7 & 0.2 & 0.2 & 1.1 \\ 0.2 & -0.1 & 0.2 & 2.5 & 0.4 & 0.3 \\ 0.1 & 0.5 & 0.2 & 0.4 & 3.9 & 0.5 \\ 0.3 & 0.5 & 1.1 & 0.3 & 0.5 & 3.5 \end{bmatrix}$$

$$R2 = 10^{-3}\times \begin{bmatrix} 1380 & -139.5 & 323.9 & -72.50 & 7.20 & -311.9 \\ -139.5 & 1215 & -190.2 & -51.5 & -183.7 & -110.2 \\ -323.9 & -190.2 & 1315 & -239 & -185.8 & 49.1 \\ -72.50 & -51.5 & -239 & 1284 & -145.3 & -277.2 \\ 7.20 & -183.7 & -185.8 & -45.3 & 1373 & -201.5 \\ -311.9 & -110.2 & 49.1 & -277.2 & -201.5 & 1443 \end{bmatrix}$$

The position errors in Fig. 3 are defined as $||x_i(t) - x_0(t)||$ and the velocity errors in Fig. 4 are defined as $||y_i(t) - y_0(t)||$. This two Figures show the multi-agent system in time-delay case can achieve consensus.

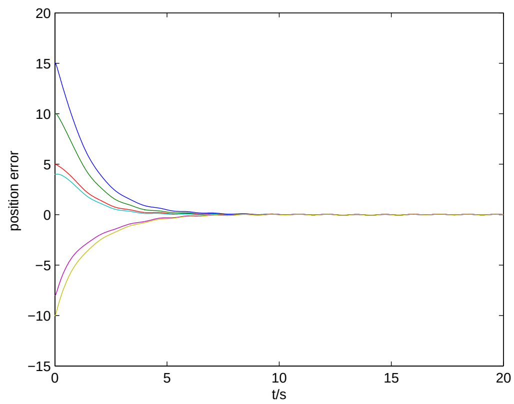

Fig. 3. Position errors of followers with time-delay network

V. CONCLUSION

This paper studied consensus problems for directed networks of agents with external disturbances on switching topologies. Each agent regulated its position and velocity based on its "neighbors" with the proposed consensus protocol on the premise that the systems satisfied the H_∞ performance index in the leader-following case. Both switching networks with and without time-delay are taken into consideration. In engineering applications, the

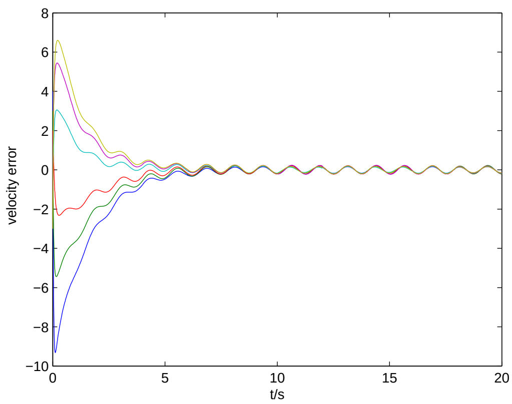

Fig. 4. velocity errors of followers with time-delay network

extreme value of the controlled output is confined and the confined ranges are different. The weighted coefficients are designed respectively according to the controlled output of the position and the velocity. A sufficient condition for the directed graph without time-delay is presented to ensure all agents to reach consensus and the system satisfies the H_∞ performance which provides a theoretical basis to design consensus protocol. Also, a sufficient condition is given for the directed graph with time-delay to make the system achieve the desire results. By using the similar method, it may be possible to probe H_∞ consensus problem of multi-agent systems under leaderless case, which will be our future work. Due to conservativeness of the common Lyapunov function method, we should probe less conservative method. In this paper, the assumption that agent's state are 1-dimension is for notational simplicity and will not lost generality. Although the dimension of agent considered in this paper is one, all results of this paper is also true for high dimension, and we can revise the expressions via kronecker product. Finally, the simulation examples also show the effectiveness of our theoretical results.

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