

Adaptive Neural Network Tracking Control for a Class of SISO Affine Nonlinear Uncertain Systems

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Abstract—A direct adaptive neural network tracking control scheme is presented for a class of SISO affine nonlinear uncertain systems. Uncertainties meet the match conditions. Parameters in neural networks are updated using a gradient descent method which designed in order to minimize a quadratic cost function of the error between the unknown ideal implicit controller and the used neural networks controller. No robustifying control term is used in controller. The convergence of adaptive parameters and tracking error and the boundedness of all states in the corresponding closed-loop system are demonstrated by Lyapunov stability theorem. Simulation results illustrate the availability of this method .

Index Terms—uncertain nonlinear, neural network, Lyapunov stability theorem, tracking control

I. INTRODUCTION

There are some inevitable uncertainties in actual system which will cause instability and difficulties in dealing with system. Therefore, the study of uncertain nonlinear system is of vital importance. In recent years, control for uncertain nonlinear systems has aroused widespread interests about it [1-19]. Since neural networks and fuzzy logic are universal approximators, the adaptive control schemes of nonlinear systems that incorporate the techniques of fuzzy logic [4, 7, 8, 10, 13, 16, 17] or neural networks [1, 2, 3, 5, 9] have grown rapidly. The stability study in such schemes is performed by using the Lyapunov design approach. Conceptually, there are two distinct approaches that have been formulated in the design of adaptive control system: direct and indirect schemes. In the direct scheme, the fuzzy system or neural networks is used to approximate an unknown ideal controller. On the other hand, the indirect scheme uses fuzzy systems or neural networks to estimate the plant dynamics and then synthesizes a control law based on these estimates. In the above most methods the parameter adaptation laws are designed based on a Lyapunov approach , where an error signal between the desired output and the actual output is used to update the adjustable parameters and the control laws

are composed of three control terms: a linear control term , an adaptive neural network control term and a robustifying control term used to compensate for disturbances and approximation errors. In the paper , according to [4], we introduce a direct adaptive neural network control approach for a class of SISO affine nonlinear uncertain systems. The basic idea is to use neural network to adaptively construct an unknown ideal controller and the parameter adaptive laws is designed , based on the gradient descent method, to directly minimizing the error between the unknown ideal controller and the neural network controller And no robustifying control term is used in controller. This paper proves the availability of the method in both theory and simulation experiment.

The paper is organized as follows. First, the problem is formulated in Section II. Designing a control law with on-line tuning of neural network weighting factors is given in Section III. In Section IV, convergence and stability analysis of control system is given. In Section V, simulation results are presented to confirm the effectiveness and applicability of the proposed method. Finally, conclusions are included.

II. Problem Formulation

Consider the following SISO affine nonlinear uncertain system:

$$\begin{cases} \dot{x} = f(x) + \Delta f(x) + (g(x) + \Delta g(x))u \\ y = h(x) \end{cases} \quad (1)$$

Where $x \in R^n$ and $u, y \in R$ are system state, system input and output respectively. $\Omega_x \subset R^n$, $\Omega_u \subset R$ are two compact sets. $f(x)$ and $g(x)$ are smooth vector fields. $\Delta f(x)$ and $\Delta g(x)$ are uncertain terms. $h(x) \in R$ is smooth scalar function.

Assumption 1: Nominal system(1) possesses a strong relative degree n .

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \quad (2)$$

Assumption 2: Uncertainties meet the match conditions.

$$\Delta f(x) = g(x)\delta_1(x), \Delta g(x) = g(x)\delta_2(x) \quad (3)$$

According to differential geometry theory of nonlinear system, we know that there is a nonlinear transformation $\xi = T(x)$, which turns system(1) to

$$\begin{cases} \frac{d\xi_i}{dt} = \xi_{i+1} \quad i = 1, \dots, n-1 \\ \frac{d\xi_n}{dt} = \alpha(\xi) + \beta(\xi)\{\delta_1(x) + [1 + \delta_2(x)]u\} \\ y = \xi_1 \end{cases} \quad (4)$$

where $\xi_i = L_f^{i-1}h(x)$, $\alpha(\xi) = L_f^n h(x)$ and $\beta(\xi) = L_g L_f^{n-1}h(x) \neq 0$. The function $\beta(\xi)$ is nonzero and bounded for all $(x, u) \in \Omega_x \times \Omega_u$. This implies that $\beta(\xi)$ is strictly either positive or negative. Without loss of generality, it is assumed that it exists a positive constant c such that $\beta(\xi) \geq c > 0$ for all $(x, u) \in \Omega_x \times \Omega_u$.

Assumption 3: For all $x \in R^n$, we have

$$1 + \delta_2(x) \geq \eta(x) > 0 \quad (5)$$

Define the reference vector

$$y_d = (y_d \quad \dot{y}_d \quad \dots \quad y_d^{(n-1)})^T \in R^n$$

The reference signal y_d and its time derivative are assumed to be smooth and bounded. We also define the tracking error as

$$e = y_d - y$$

and corresponding error vector as

$$\underline{e} = (e, \dot{e}, \dots, e^{(n-1)})^T \in R^n$$

Then the system (4) can be transformed into the normal form in the new coordinate as follows:

$$\dot{\underline{e}} = A_0 \underline{e} + b \left[y_d^{(n)} - \alpha(\xi) - \beta(\xi)\{\delta_1(x) + [1 + \delta_2(x)]u\} \right] \quad (6)$$

where $A_0 = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{n \times n}$, $b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^{n \times 1}$.

Obviously, if (A_0, b) can be controllable, then there will exist a constant matrix $K = [k_0, k_1, \dots, k_{n-1}]^T$ which makes eigenvalues of matrix $A_c = A_0 - bK^T$ all have negative real part. Thus, for any given positive definite symmetric

matrix Q , there exists a unique positive definite symmetric solution P to the following Lyapunov algebraic equation:

$$A_c^T P + P A_c = -Q \quad (7)$$

The control objective is to design an adaptive neural network controller for system (1) such that the system output follows a desired trajectory while all signals in the closed-loop system remain bounded.

III. DESIGN OF CONTROLLER

Define a signal

$$v = y_d^{(n)} + K^T \underline{e} + \lambda \tanh\left(\frac{b^T P e}{\Xi}\right)$$

where $\tanh(\bullet)$ is the hyperbolic tangent function, Ξ, λ are the positive design parameters $\tanh(\bullet) \in (-1, 1)$, when error $e \rightarrow +\infty$, the value of $\tanh\left(\frac{e^T P b(\beta+l)}{\Xi}\right) \rightarrow +\infty$. And when error $e \rightarrow -\infty$, the value of $\tanh\left(\frac{e^T P b(\beta+l)}{\Xi}\right) \rightarrow -\infty$. When $e \rightarrow 0$,

$\tanh\left(\frac{e^T P b(\beta+l)}{\Xi}\right) \rightarrow 0$. The term $\lambda \tanh\left(\frac{b^T P e}{\Xi}\right)$ is a smooth approximation of the discontinuous term $\lambda \text{sign}(b^T P e)$ usually used in robust control. So,

λ is selected larger than the magnitude of the uncertainty and it will affect the convergence rate of the tracking error, and Ξ is chosen very small to best approximate the sign function and it will affect the size of the residual set to which the tracking error will converge. The sign function is not used here to avoid problems associated with it as chattering and solutions existence.

By adding and subtracting v in (6), we obtain

$$\begin{aligned} \dot{\underline{e}} &= (A_0 - bK^T) \underline{e} - b\lambda \tanh\left(\frac{b^T P e}{\Xi}\right) - \dots \\ &+ b \left[\alpha(\xi) + \beta(\xi)\{\delta_1(x) + (1 + \delta_2(x))u\} - v \right] \end{aligned} \quad (8)$$

if $\alpha(\xi), \beta(\xi), \delta_1(x), \delta_2(x)$ are known, there exists some ideal controller $u^*(z)$ satisfying the following equality :

$$u^*(z) = (\beta(\xi)[1 + \delta_2(x)])^{-1} (v - \alpha(\xi) - \beta(\xi)\delta_1(x)) \quad (9)$$

The closed-loop error dynamic is reduced to (10)

$$\dot{\underline{e}} = (A_0 - bK^T) \underline{e} - b\lambda \tanh\left(\frac{b^T P e}{\Xi}\right) \quad (10)$$

Let us consider the following positive function:

$$V = \underline{e}^T P \underline{e} \quad (11)$$

Using (7) and (10), the time derivative of (11) becomes

$$\dot{V} = -\underline{e}^T Q \underline{e} - 2\lambda b^T P \underline{e} \tanh\left(\frac{b^T P \underline{e}}{\Xi}\right) \quad (12)$$

Since the term $b^T P \underline{e} \tanh\left(\frac{b^T P \underline{e}}{\Xi}\right)$ is always positive, we conclude that $\dot{V} \leq 0$, and only when $e = 0$, $\dot{V} = 0$ which means $\lim_{t \rightarrow \infty} |e| = 0$.

However, when $\alpha(\xi), \beta(\xi), \delta_1(x), \delta_2(x)$ are unknown in ideal controller (9), $u^*(z)$ is not available. In what follows, a neural network will be used to construct the unknown ideal implicit controller.

In control engineering, radial basis function(RBF) NNs are usually used as a tool for modeling nonlinear functions because of their good capabilities in function approximation. In this paper, the following RBF NN based on GGAP-RBF^[20] algorithm is used to approximate the continuous function $u(z) = \phi^T(z)\theta$, where $z = [\xi^T, v]^T$, weight vector $\theta = (\theta_1, \dots, \theta_M)^T$, the NN node number $M > 1$; and $\phi(z) = (\phi_1(z) \dots \phi_M(z))^T$ with

$$\phi_i(z) = \exp\left[-\frac{(z - \mu_i)^T (z - \mu_i)}{\eta_i^2}\right], i = 1, 2, \dots, M$$

Where $\mu_i = [\mu_{i_1} \ \mu_{i_2} \ \dots \ \mu_{i_q}]^T$ is the center of the receptive field and μ_i is the width of the Gaussian function.

It has been proven that network can approximate any smooth function over a compact set $\Omega_z \subset R^q$ to arbitrarily any accuracy as

$$u^*(z) = \phi^T(z)\theta^* + \delta(z) \quad (13)$$

with bounded function approximation error $\delta(z)$ satisfying $|\delta(z)| \leq \bar{\delta}$. Where θ^* is an ideal parameter vector which minimizes the function $|\delta(z)|$. In this paper, we assume that the used neural network does not violate the universal approximation property on the compact set Ω_z , which is assumed large enough so that the variable z remains inside it under closed-loop control.

RBFNN represents a class of linearly parameterized approximators and can be replaced by any other linearly parameterized approximators such as spline functions[21] or fuzzy systems[22]. Moreover, nonlinearly parameterized approximators, such as multilayer neural network(MNN), can be linearized as linearly parameterized approximators, with the higher order terms of Taylor series expansions being taken as part of the modeling error, as shown in [23], [24].

Let us define the error between the controllers $u(z)$ and $u^*(z)$ as

$$e_u = u^*(z) - u(z)$$

Using (13), it becomes

$$e_u = u^*(z) - u(z) = \phi^T(z)\tilde{\theta} + \delta(z) \quad (14)$$

Where $\tilde{\theta} = \theta^* - \theta$ is the parameter estimation error vector.

By substituting $u^*(z)$ into the equation(8) and considering (9), we get

$$\begin{aligned} \dot{e} &= A_c \underline{e} - b\lambda \tanh\left(\frac{b^T P \underline{e}}{\Xi}\right) - b[\alpha(\xi) + \beta(\xi)\delta_1(x) - v] - \dots \\ &\quad - b[\beta(\xi)[1 + \delta_2(x)]u + \beta(\xi)[1 + \delta_2(x)]u^*(z) - \dots \\ &\quad - \beta(\xi)[1 + \delta_2(x)]u^*(z)] \\ &= A_c \underline{e} - b\lambda \tanh\left(\frac{b^T P \underline{e}}{\Xi}\right) - b\beta(\xi)[1 + \delta_2(x)](u(z) - u^*(z)) \end{aligned} \quad (15)$$

which can be rewritten as

$$e^{(n)} + K^T \underline{e} + \lambda \tanh\left(\frac{b^T P \underline{e}}{\Xi}\right) = \beta(\xi)[1 + \delta_2(x)]e_u \quad (16)$$

We notice here that $u^*(z)$ is an unknown quantity, so the signal e_u defined in (14) is not available. Eq.(16) will be used to overcome the difficulty. Indeed, from(16), we see that even if the signal e_u is not available for measurement, the quantity $\beta(\xi)[1 + \delta_2(x)]e_u$ is measurable. This fact will be exploited in the design of the parameters adaptive law.

Now, consider a quadratic cost function defined as

$$J_\theta = \frac{1}{2}[1 + \delta_2(x)]e_u^2 = \frac{1}{2}[1 + \delta_2(x)](u^*(z) - \phi^T(z)\theta)^2 \quad (17)$$

By applying the gradient descent method, we obtain as an adaptive law for the parameters θ

$$\dot{\theta} = -\gamma \nabla_\theta J(\theta) = \gamma[1 + \delta_2(x)]\phi(z)e_u \quad (18)$$

Since e_u and $\delta_2(x)$ are not available, the adaptive law (18) can not be implemented. In order to render (18) computable, from Eq.(16), we select the design parameter $\gamma = \gamma_\theta \beta(\xi)$, where γ_θ is a positive constant. At the same time, to improve the robustness of adaptive law in the presence of the approximation error, we modify it by introducing a σ -modification term as follows:

$$\begin{aligned} \dot{\theta} &= \gamma_\theta (\phi(z)\beta(\xi)[1 + \delta_2(x)]e_u - \sigma\theta) \\ &= \gamma_\theta \phi(z) \left\{ e^{(n)} + K^T \underline{e} + \lambda \tanh\left(\frac{b^T P \underline{e}}{\Xi}\right) \right\} - \gamma_\theta \sigma\theta \end{aligned} \quad (19)$$

where σ is a small positive constant

Because the aim of the σ -modification adaptive law is to avoid parameter drift, it does not need to be active when the estimated parameters are within some acceptable bound. The proposed adaptive controller is only composed of a neural network part without

additional control terms and the system stability relies entirely on the neural network. The term $\lambda \tanh\left(\frac{b^T P e}{\Xi}\right)$ in the parameter adaptive law(19)plays ,

in some way , the role of a robustifying control term. Thus , the robustness of the controller can be improved by selecting a large positive value for the design parameter λ and a small positive value for the parameter Ξ .

IV. CONVERGENCE AND STABILITY ANALYSIS OF CONTROL SYSTEM

Firstly, let us consider the convergence of neural network parameters. Considering the following positive function:

$$V_\theta = \frac{1}{2\gamma_\theta} \tilde{\theta}^T \tilde{\theta} \tag{20}$$

Using (14) and (19), the time derivative of (20) can be written as

$$\dot{V}_\theta = -\beta(\xi)(1+\delta_2(x))e_u^2 + \beta(\xi)(1+\delta_2(x))\delta(z)e_u + \sigma\tilde{\theta}^T\theta \tag{21}$$

Using the inequalities

$$\begin{aligned} \sigma\tilde{\theta}^T\theta &= -\frac{\sigma}{2}\|\tilde{\theta}\|^2 - \frac{\sigma}{2}\|\theta\|^2 + \frac{\sigma}{2}\|\tilde{\theta} + \theta\|^2 \\ &\leq -\frac{\sigma}{2}\|\tilde{\theta}\|^2 + \frac{\sigma}{2}\|\theta^*\|^2 \end{aligned} \tag{22}$$

$$\begin{aligned} -e_u^2 + \delta(z)e_u &= -\frac{1}{2}e_u^2 + \frac{1}{2}\delta^2(z) - \frac{1}{2}(e_u - \delta(z))^2 \\ &\leq -\frac{1}{2}e_u^2 + \frac{1}{2}\delta^2(z) \end{aligned} \tag{23}$$

Eq.(21) can be bounded as

$$\begin{aligned} \dot{V}_\theta &\leq -\frac{1}{2}\beta(\xi)(1+\delta_2(x))e_u^2 + \frac{1}{2}\beta(\xi)(1+\delta_2(x))\delta^2(z) - \dots \\ &\quad -\frac{\sigma}{2}\|\tilde{\theta}\|^2 + \frac{\sigma}{2}\|\theta^*\|^2 \end{aligned} \tag{24}$$

Since the parameters θ^* are constants, and the functions $\delta(z)$ and $\beta(\xi), \delta_2(x)$ are assumed bounded in this paper, so we can define a positive constant bound ψ as

$$\psi = \sup_i \left(\frac{1}{2}\beta(\xi)(1+\delta_2(x))\delta^2(z) \right) + \frac{\sigma}{2}\|\theta^*\|^2 \tag{25}$$

Then

$$\begin{aligned} \dot{V}_\theta &\leq -\frac{1}{2}\rho V_\theta + \psi - \frac{1}{2}\beta(\xi)(1+\delta_2(x))e_u^2 \\ &\leq -\rho V_\theta + \psi \end{aligned} \tag{26}$$

where $\rho = \sigma\gamma_\theta$.Eq.(26) implies that for $V_\theta > \frac{\psi}{\rho}$, $\dot{V}_\theta < 0$ and , therefore, $\tilde{\theta}$ is bounded. By integrating (26), we can establish that:

$$\|\tilde{\theta}\|^2 \leq \|\tilde{\theta}(0)\|^2 e^{-\rho t} + 2\gamma_\theta \frac{\psi}{\rho} \tag{27}$$

From (27) we have

$$\|\tilde{\theta}\| \leq \|\tilde{\theta}(0)\| e^{-0.5\rho t} + \sqrt{2\gamma_\theta\psi/\rho} \tag{28}$$

Using (28) and the fact that $\delta(z)$ and $\beta(\xi), \delta_2(x)$ are bounded, we can write

$$\begin{aligned} &|\beta(\xi, \eta)(1+\delta_2(x))(\phi^T(z)\tilde{\theta} + \delta(z))| \\ &\leq |\beta(\xi, \eta)(1+\delta_2(x))\phi^T(z)\tilde{\theta}| + \dots \\ &\quad + |\beta(\xi, \eta)(1+\delta_2(x))\delta(z)| \\ &\leq |\beta(\xi, \eta)(1+\delta_2(x))\|\phi^T(z)\|\|\tilde{\theta}\| + \dots \\ &\quad + |\beta(\xi, \eta)(1+\delta_2(x))\delta(z)| \\ &\leq |\beta(\xi, \eta)(1+\delta_2(x))\|\phi^T(z)\|\|\tilde{\theta}(0)\| e^{-0.5\rho t} + \dots \\ &\quad + |\beta(\xi, \eta)(1+\delta_2(x))\|\phi^T(z)\|\sqrt{2\gamma_\theta\psi/\rho} + \dots \\ &\quad + |\beta(\xi, \eta)(1+\delta_2(x))\delta(z)| \\ &\leq \psi_0 e^{-0.5\rho t} + \psi_1 \end{aligned} \tag{29}$$

where ψ_0, ψ_1 are some finite positive constants.

Theorem 1: Consider the system (1). Suppose that Assumption 1-3 are satisfied and the neural network approximation error in (14) is bounded , then the neural network controller and adaptation law given by (19) guarantees the convergence of the neural network parameters and the boundedness of all the signal in the closed-loop system, and the convergence of the tracking error to the residual set:

$$\Omega_e \leq \left\{ e \mid \|e\| \leq \sqrt{2\psi_1 K_c \Xi / (\lambda_{\min}(P)\alpha_e)} \right\} .$$

Proof. Consider the Lyapunov function candidate:

$$V(e) = e^T P e \tag{30}$$

Differentiating (30) with respect to time and using (7), (14), (29), and inequality

$$0 \leq |\zeta| - \zeta \cdot \tanh\left(\frac{\zeta}{\Xi}\right) \leq K_c \Xi$$

with $K_c = 0.2785$, we obtain

$$\begin{aligned} \dot{V}(e) &= \underline{e}^T (A_c^T P + P A_c) \underline{e} - 2b^T P e \lambda \tanh\left(\frac{b^T P e}{\Xi}\right) + \dots \\ &\quad + 2b^T P e \beta(\xi)(1 + \delta_2(x))(u^* - u) \\ &= -\underline{e}^T Q \underline{e} - 2b^T P e \lambda \tanh\left(\frac{b^T P e}{\Xi}\right) + \dots \\ &\quad + 2b^T P e \beta(\xi)(1 + \delta_2(x))(\phi^T(z)\tilde{\theta} + \delta(z)) \\ &\leq -\underline{e}^T Q \underline{e} - 2b^T P e \lambda \tanh\left(\frac{b^T P e}{\Xi}\right) + 2\|b^T P e\|(\psi_0 e^{-0.5\rho} + \psi_1) \\ &\leq -\underline{e}^T Q \underline{e} + 2\|b^T P e\|\psi_0 e^{-0.5\rho} + 2\psi_1 K_c \Xi \end{aligned} \tag{31}$$

Using the inequality

$$2\|b^T P e\|\psi_0 e^{-0.5\rho} \leq 0.5\|e\|^2 + 2\|b^T P\|^2 \psi_0^2 e^{-\rho}$$

Eq.(31) becomes

$$\dot{V}(e) \leq -(\lambda_{\min}(Q) - 0.5)\|e\|^2 + 2\|b^T P\|^2 \psi_0^2 e^{-\rho} + 2\psi_1 K_c \Xi \tag{32}$$

where $\lambda_{\min}(Q)$ denotes the minimum eigenvalue of the matrix Q and it is assumed chosen such that $\lambda_{\min}(Q) > 0.5$.

Eq. (32) can be written as :

$$\dot{V}(e) \leq -\alpha_e V(e) + 2\|b^T P\|^2 \psi_0^2 e^{-\rho} + 2\psi_1 K_c \Xi \tag{33}$$

where $\alpha_e = (\lambda_{\min}(Q) - 0.5) / \lambda_{\max}(P)$ with $\lambda_{\max}(P)$ is the maximum eigenvalue of the matrix P . Eq.(33) implies that for

$$V(e) \geq \left(2\|b^T P\|^2 \psi_0^2 e^{-\rho} + 2\psi_1 K_c \Xi\right) / \alpha_e$$

$\dot{V}(e) < 0$. Therefore, the tracking error vector is bounded, together with the boundedness of the desired trajectory and its derivatives, imply that the state vector x is bounded. Moreover, since the term $2\|b^T P\|^2 \psi_0^2 e^{-\rho} \rightarrow 0$, when $t \rightarrow \infty$, we can conclude that the function $V(e)$ will be asymptotically bounded, and therefore the tracking error will converge asymptotically to the residual set

$$\Omega_e \leq \left\{e \mid \|e\| \leq \sqrt{2\psi_1 K_c \Xi / (\lambda_{\min}(P)\alpha_e)}\right\} .$$

This completes the proof.

VI. SIMULATION STUDY

In this section, to illustrate the validity of the proposed adaptive neural network controller, the following SISO affine nonlinear uncertain system is simulated. The affine nonlinear system is described by the following differential equation:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} x_2 \\ -\sin x_1(t) \end{bmatrix} + \Delta f(x) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \Delta g(x) u \\ y &= x_1 \end{aligned} \tag{34}$$

where $\Delta f(x) = \begin{bmatrix} 0 \\ -\sin(t)\sin x_1(t) \end{bmatrix}$, $\Delta g(x) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. The control objective is to force the system output y to track the desired trajectory $y_d = 2\sin(0.5t)$. We know $\delta_1(x) = -\sin(t)\sin x_1(t)$, $\delta_2(x) = 0$, $L_g h(x) = 0$, $L_f h(x) = x_2$, $L_g L_f h(x) = 1 \neq 0$, $\alpha(x) = -\sin x_1(t)$, $\beta(x) = 1$. The system initial conditions are $x(0) = [0.2 \ 0.6]^T$. The design parameters used in this simulation are selected as follows $Q = \text{diag}[10, 10]$, $P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$, $K = [1, 2]^T$, $\Xi = 0.01$, $\gamma_\theta = 9$, $\sigma = 0.05$. The simulation result is shown in Fig1, 2, 3, 4.

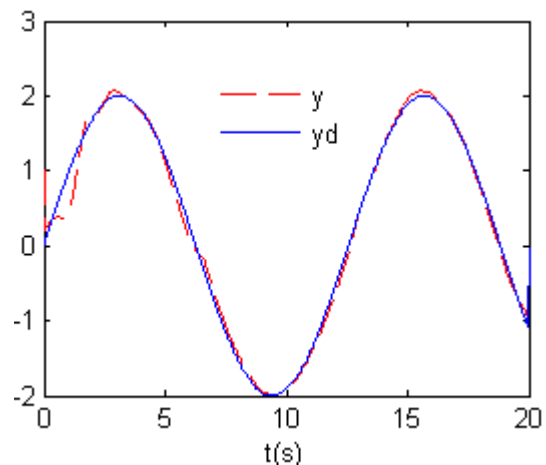


Figure 1. Plots of output tracking of system

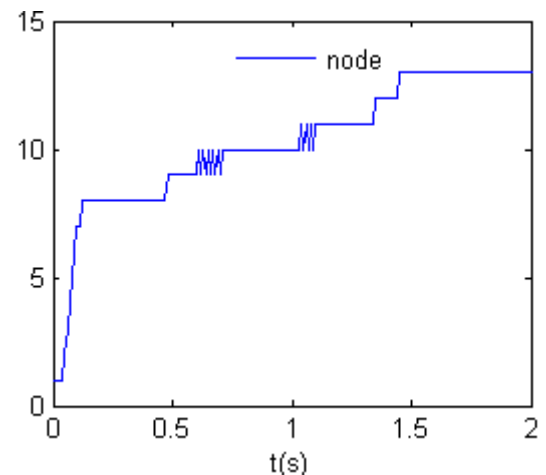


Figure 2. Node Number of Hidden Layer

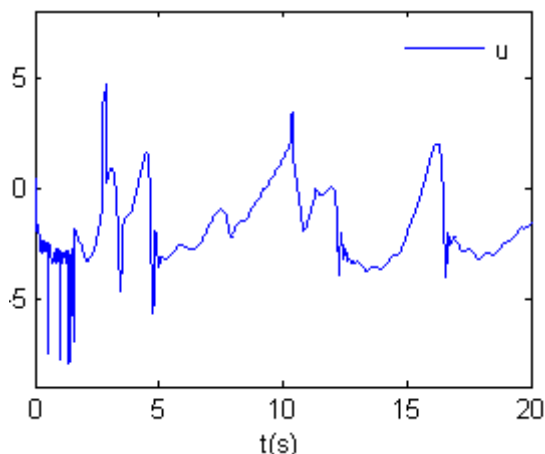
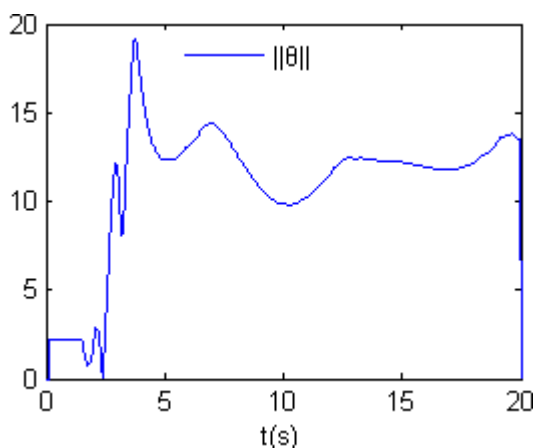


Fig.3 Plots of Control input

Fig.4. Norm of the weight vectors θ

The simulation result for the output is shown in Fig.1, the node changes are shown in Fig.2, and the control input signal is shown in Fig.3. Fig.4 shows the evolution of the Euclidian norm of the parameter estimates. It can be seen that the actual trajectories converge rapidly to the desired ones. The control signal and the estimated parameters are bounded. These simulation results demonstrate the tracking capability of the proposed controller and its effectiveness for control tracking of uncertain nonlinear systems.

V. CONCLUSIONS

In this paper, we proposed a new neural network adaptive control method for a class of SISO affine nonlinear uncertain systems. The scheme consists of an adaptive neural network control term with its adaptive law, and no robustifying control term is used in controller to compensate the influence of error between ideal controller and neural network controller which adjustable parameters are updated by using the gradient descent method. Simulation results demonstrate the feasibility of the proposed control scheme.

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