An Improve Genetic Algorithm Based on Fixed Point Algorithms

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***Abstract*—An improved genetic algorithm is proposed to solve optimal problems, which is based on fixed point algorithms of continuous self-mapping in Euclidean space. The algorithm operates on a simplicial subdivision of searching space and generates the integer labels at the vertices, then, applied crossover operators and increasing dimension operators according to these labels. In this case, it is used as an objective convergence criterion and termination criterion that the labels of every individual are completely labeled simplexes. The algorithm combines genetic algorithms with fixed point algorithms and triangulation theory to maintain the proper diversity, stability and convergence of the population. Several numerical examples are provided to be examined and the numerical results illustrate that the proposed algorithm has higher global optimization capability, computing efficiency and stronger stability than traditional numerical optimization methods and the standard genetic algorithm.**

***Index Terms*—genetic algorithm; fixed point; simplicial subdivision; integer label; completely labeled simplex**

I. INTRODUCTION

The basic principles of traditional numerical optimization methods(NUM) is that the optimization process starts from a given initial point of the solution space, then carries on the single point iteration along the definite search direction and approaches optimal solution gradually[1-3]. This process, affected by the initial point, is a steady iteration and has a single track, and difficult to obtain the global optimal solution, especially for optimization problems of non-convex functions and multimodal functions.

For the shortcomings, stochastic optimization methods have been developed rapidly[4-5]. Genetic Algorithms (GAs) are the most popular implementation and have been successfully applied to a variety of complex real world problems[6-8]. The GAs are iterative procedures evolving a population of individuals. The individuals are candidate solutions to a specific domain. During each generation, the individuals in the current population are rated for their effective evaluations, and a new population of candidate solutions is formed using reproduction operators, such as selection, crossover, and mutation[9]. GAs has an influence on the population diversity directly. The majority genetic algorithms keep the diversity of population through niche technology. Though this technology, the super-individual can be prevented by using the fitness information of individuals within niche, and the diversity of population can also be kept, so the solutions distribute evenly in the solution space in order to avoid premature convergence[10-14]. However, niche technology can not fully guarantee the probabilistic stability and ergodicity of population distribution, and these shortcomings lead to premature convergence, slow convergence rate in latter period and also influent the validity and practicality. In theory, GAs can converge to the global optimum, but actually, there are no mature principle and methods to design the convergence criteria, in contrary, the actual design relies on experiences and understanding degree of the question which is to be solved. The common method is taken the max iteration as convergence criteria, or when average fitness has not been enhanced significantly of consecutive m iterations, the algorithm will stops. Therefore, the objective design of convergence criteria has great significance.

In this paper, an improved GA based on fixed point algorithm and subdivision theory is proposed, joining the thought of simplicial subdivision to keep diversity of population. Crossover operators and increasing dimension operators relying on the integer labels are designed to guide the optimization process of genetic algorithm. Whether every individual of the population is a completely labeled simplex can be used as an objective convergence criterion and that determined whether the algorithm will be terminated. The contrast optimization results have indicated that the improved GA is highly valid.

II. FIXED POINT ALGORITHM

A. Basic Concepts

Definition 1 (Fixed Point): Suppose that X is a subset of the vector of R^n, for a random point x of X, there is always a corresponding point $f(x) \in X$, then we say it is a self-mapping f, crediting to $f: X \to X$. For a point x of X, satisfying an equation of $f(x) = x$, then we

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call that x is a fixed point of f.

Definition 2 (Simplex): Suppose that $x^0,\cdots,x^p$ of R^m are the points which are independent of affine, subject to R^m is a subset of R^n and $m<n$. At the circumstance, we call that $<x^0,\cdots,x^p>$ is a simplex, subject to:

$$< x^0,\cdots,x^p >= \{x = \sum_{i=0}^{p}\lambda_i x^i \,|\lambda_i > 0, i=0,\cdots,p; \sum_{i=0}^{p}\lambda_i = 1\}$$

and each point of $x^0,\cdots,x^p$ is called the vertex of the simplex.

Definition 3 (Simplicial Subdivision): Suppose that C is a convex set of R^n; then call that G is a simplicial subdivision of C, if G satisfies the three following conditions.

(1) It is the set of an n-dimensional simplex.

(2) All surfaces of simplexes of G constitute a segmentation of C.

(3) Each point of C has a domain which intersects with limited simplex in G.

Definition 4 (K_1 Subdivision of R^n): Each coordinate component of the point in the set of K_1^0 is integer, and N is a vector denoted by $N=\{1,\cdots,n\}$. Each basic vector such as $u^1,\cdots,u^n$ of R^n is respectively column vector of the unit matrix π is a substitution of N and an n-dimension simplex of $<y^0,\cdots,y^n>$ denoted by $k_1(y^0,\pi)$, with condition of $y^i = y^{i-1}+u^{\pi(i)}$. All the simplexes like $k_1(y^0,\pi)$ constitute a set denoted as K_1, that is K_1 subdivision,

Definition 5 (Completely Labeled Simplex): Taken σ as a completely labeled simplex satisfying that the integer labels of its vertices are $n+1$ kind of different labels such as $0,\cdots,n$.

B. Fixed Point Algorithm

Fixed point theory is one of the most famous results in topology and it has applied widely in many fields. With respect to the following optimal problems: Suppose that f is a self-mapping, also is a convex function, that is $f: R^n \to R^n$, looking for a point x that is the minimum of f. The necessary and sufficient condition of being an extreme point is that its gradient is zero, that is $\nabla f(x^*)=0$. Suppose that g is a self-mapping, that is $g: R^n \to R^n$, we can converse the solution of zero point problems to solution of fixed point problems though the function of $g(x)=x-\nabla f(x)$, subject to $x \in R^n$.

The fixed point algorithm carries on proper simplicial subdivision to the solution space firstly, then computes the integer labels at the vertices, and at last, generates the finite sequences composed of adjacent nearly-complete simplexes, though the pivotal operations among the simplexes. According to the degree of a simplex, we can judge whether the simplex is a completely labeled simplex. The vertices of the completely labeled simplex are ε fixed points. The two-dimension optimal problems carry on K_1 subdivision, integer label and pivotal operations; begin from an artificial initial point outside the domain; then generates the finite sequences composed of adjacent nearly-complete simplexes, eventually arrive at a completely simplex which contains the optimization solution of f. To solve the two-dimension optimal problems, carry on K_1 subdivision to the Euclidean Space R^2, the goal is to find such a triangle under the self-mapping of g. The first coordinate component of one vertex of the triangle dropped, another vertex's second coordinate component decreased, and two coordinate components of the third vertex remained unabated. Because of continuity of g, if this triangle's diameter is small enough, the changes of three vertices will not differ too far. In this case, each vertex is an approximate fixed point.

III. THE IGA BASED ON FIXED POINT ALGORITHM AND SUBDISION THEORY

The premise of solving optimal problems with fixed point algorithm is that the optimal function is convex function. The algorithm finds the completely labeled simplex which contains the global optimum along with the sequence of nearly-complete labeled simplex, however, the actual optimization problems are often non-convex functions and multimodal functions, which corresponds lots of completely labeled simplexes, and among which, there must be a completely labeled simplex containing global optimum at least. The simplex sequence which embarks from the artificial beginning point can determine only one completely labeled simplex; therefore, it is very difficult to find the completely labeled simplex which contains the globally optimal solution. So how to find all the completely labeled simplexes is the key problem, genetic algorithm has a strong ability of searching global optimal solution and can find all completely labeled simplexes.

In the fixed point algorithm, the integer label of vertex in subdivision is decided by the relation between function f and its various components. With this information, the algorithm can converge to a nearby completely labeled simplex quickly, finding the ε fixed point. The genetic algorithm searches the optimal solution by using bearing simplex and information of its vertices` integer labels to enhance the principle necessity and keep balance between randomness and necessity.

The computation only implements among the nearly-complete labeled simplexes. As a node, the degree of a nearly-complete labeled simplex does not exceed 2 and the number of simplex of the bounded function $f(R^n)$ is limited. If the times going though each nearly-complete is finite, the computation process can achieve a completely labeled simplex within finite steps. By using the information, genetic algorithm can design the objective convergence criteria.

In this paper, the performance of a genetic algorithm is improved by joining the fixed point algorithm into the genetic algorithm.

A. Encode Scheme

Given that the shortcomings of binary-code such as low accuracy, occupation of storage space and the poor efficiency, the algorithm uses real-code to generate high-precision individual. The real-code can produce individuals with high accuracy and is suitable for the computation of continuous variables. However, its scope of application is relatively broad, which enhances the randomness of genetic algorithm with poor stability and premature convergence. In this section, the information of vertices and the integer label are introduced to real-code. The code is as follows:

$$\{ x_1 \cdots x_n, y^0, \cdots, y^n, z^0 \cdots z^n, f_X \}$$

Subject to $x_i, i \in (1, \cdots, n)$ is the design variable of individual; $y^i, i \in (0, \cdots, n)$ is the vertex of bearing simplex; $z^i, i \in (0, \cdots, n)$ is the integer label of y^i; f_X is the objective function value corresponding to x_i.

B. Fitness Function

The searching process of genetic algorithm is based on the fitness information of individual and the fitness is obtained according to the objective function for the optimization problem. The design goal of the IGA proposed in this paper is to find the completely labeled simplexes and evaluate the individuals on the basis of the integer label information of bearing simplex of the individual.

C. The Initialization of the IGA

Firstly, the optimization problems are transformed into fixed point problems. Secondly, carries on simplicial subdivision to the self-mapping f denoted by $f: R^n \to R^n$; set the population size and generate the initial generation; calculates the bearing simplex of each individual and the integer labels of vertices; at last, calculates the function value corresponding to each individual.

D. Crossover Operator

The crossover operator generates the new individual though the recombination of two father individuals, so this operator is an important means of generating new individual. The operating process of crossover operator of the IGA presented in this paper is divided into two steps:

(1) Classy the bearing simplexes by the information of integer label of nearly-complete labeled edge.

(2) Carry on the crossover operation between the individuals belonging to different class or different bearing simplexes.

E. Mutation Operator

This algorithm adapts the uniform mutation, and generates the individuals firstly in the simplexes which do not contain any individual, and secondly mutate the individual in non-completely labeled simplex.

F. Increasing Dimension Operator

When the dimension of vertex label of an individual generated randomly or by genetic operation is less than n (the dimension is denoted by m, that is $m < n$), the search process begins from this individual and carries on searching along a sequence of adjacent simplexes with same dimension. Accordingly, the searching can find a simplex with dimension of $m+1$, and carrying on search successively can find the simplex with dimension of n.

Given the above reasons, this algorithm presented in this paper adds a new genetic operator based on the three traditional ones. The reason is as follows:

The integer labels of vertices of bearing simplex may be the same. At the same time, the edges of these bearing simplexes are not nearly-complete labeled edges. However, in theory, each individual converges in a completely labeled simplex. To enable the individual with the same integer labels achieve the nearly-complete labeled simplex though a certain route, the increasing dimension operator is designed. Taken two-dimension Euclidean Space for example, the process of increasing dimension operator is as follows:

Process of increasing dimension: Finds the vertex whose objective function value is max by comparing the vertices with same integer label in a simplex, and then denote the vertex as y^0, and the other two vertices are denoted by y^1 and y^2. The substitution of vector N has two forms when *N={1,2}*, respectively are*{1,2}*and*{2, 1}*. $\tau(0)$ is a edge which is determined by y^1 and y^2. Replace the original simplex with an adjacent simplex which shares $\tau(0)$ with the original one. And then find a new point randomly in it as a new individual and calculate its integer labels of vertices. If the three labels are still the same, then repeat the above process until finding a simplex with different labels. If a simplex has been searched, then marking it with a flag, and at this time, replace the original individual with the current one. The current one must be superior to the original.

G. Selection Operator

Select the individuals from the population composed of new individuals generated by genetic operators and parent individuals according to the strategy of selection excellent one from parent and offspring. This strategy ensures that the excellent genes can inherit into its descendant and make sure that the population can converge into the global optimization solution. All the completely labeled simplexes in each generation are selected into the next iteration.

H. Diversity of Population

The diversity of population affects the performance of algorithm directly. The algorithm proposed in this paper adapts two strategies to maintain population diversity and to avoid emerging the super individual. The two strategies are as follows:

(1) The individuals in a bearing simplex are similar, and so the crossover operation is prohibited between individuals in a same bearing simplex, avoiding the same individual appearing.

(2) When carrying on the increasing dimension operation, replace the original individual with the searching individual.

I. Convergence Criteria

When the bearing simplex of all individuals in population are the completely labeled simplex, the algorithm stops. At last, output the ε fixed point. To elaborate the application of this convergence criterion used in this paper, here, this paper takes the dual multimodal functions as example.

J. Flow of improvement genetic algorithm

Step1: The optimized problem will be transformed into the fixed point problem.

Step2: Carry on K_1 subdivision to the solution space.

Step3: Generate initial population.

Step4: Calculate the bearing simplex of the individual and compute integer label of its vertices.

Step5: While (not satisfying convergence criteria).

Do {Carrying on crossover operator
Carrying on mutation operator
Carrying on increasing operator
Carrying on selection operator
Judgment of convergence criteria}

Step6: Output the information of completely labeled simplex.

IV. EXPERIMENTS AND ANALYSIS

A. Test Functions:

In this paper, the optimization experiment is carried on by using three functions which respectively belong to typical convex functions and non-convex functions, and the experiment adapts three methods such as NUM, SGA and the IGA proposed in this paper. The test functions are as follows:

$$\theta_1 : \min \theta(X) = \sum_{i=1}^{2} x_i^2 \qquad -5 \le x_i \le 5$$

$$\theta_2 : \min \theta(X) = 100(x_1^2 - x_2)^2 + (1 - x_1)^2$$

$$-20 \le x_i \le 20$$

Among the above equations, the mapping of θ_1 is a simple convex function and it reaches the minimum at the point of (0, 0), as in figure 1; θ_2 is a non-convex function and the minimum point of it is (1, 1), as in figure 2.

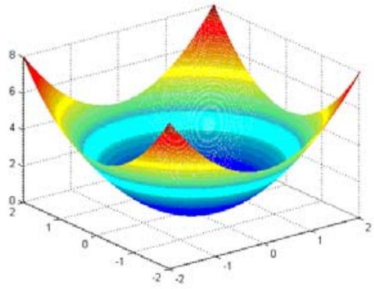

Figure 1 Convex function of θ_1

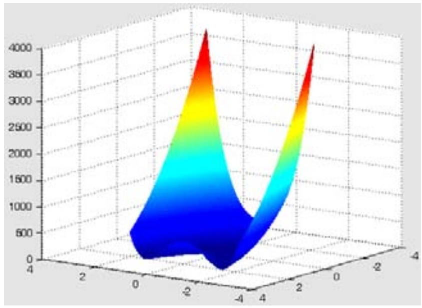

Figure 2 Non-convex function of θ_2

According to the flow of IGA, the optimized problem will be transformed into the fixed point problem firstly and the process is as follows:

$$F_1 : f(X) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 2x_1 \\ 2x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$-5 \le x_i \le 5$$

$$F_2 : f(X) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} (400x_1^2 - x_2)x_1 + 2x_1 - 2 \\ -200x_1^2 + 200x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$-20 \le x_i \le 20$$

Then carry on K_1 subdivision to the solution space in two-dimension Euclidean Space. Label the vertices of simplexes according to $l(x)$. As F_1 is a convex function, the global optimization solution is contained in the completely labeled simplex. θ_2 is a non-convex function. The gobal optimization solution is contained in the completely labeled simplex.

B.Optimization Results and Analysis:

The above analysis shows that the global optimization solution is closely related to the completely labeled simplex. Under the given subdivision accuracy, the optimization solution which meets the requirement of accuracy is in completely labeled simplex.

(1) The process of NUM: Carry on the optimization by using the command of "fminunc" which is in the toolbox of the Matlab software. Firstly, create the m file of objective function, and then appoint the initial point and input the optimization command to optimize objective

functions. Different initial points obtain different optimization results, the results are shown in table 1.

From the data analysis, it can be seen that NUM obtain different results when the initial points are different for non-convex functions and multimodal functions, especially for the latter, this algorithm often falls into local optimal solution.

(2) The parameters of the SGA are set as follows: the chromosome length of binary-code is 10; the population size is 200; the crossover probability is 0.6 and the mutation probability is 0.02; the max generation is 100. the population distribution in the 50th generation is described in figure 3, figure 4 describe the population distribution when the iteration is 100.

From these figures, it can be seen that the results of the SGA are different each time. Besides, it also has shortcomings of poor stability and obvious difference between convergence effect and theoretical results.

(3) The parameters of the IGA are set as follows: the population size is 200. For the convex function of θ_1, the IGA runs 4 iterations at most. In figure 5, the distribution of initial population is described; figure 6 describe the population distribution corresponding to 4th generation. The simplex in which the individuals are crowded is a completely labeled simplex containing the global optimization solutions. For the non-convex function of θ_2, the improved algorithm runs 6 iterations at most. In figure 7, the distribution of initial population is described; figure 8 to figure 9 describe the population distribution in the evolution process. Combining with figure 1, it can be seen that the algorithm converges into the extreme value rapidly in the 6th generation. In figure 10, the intersection of the three colors corresponds to the global optimization solution.

V. CONCLUSION

In this paper, an IGA for the optimization of convex functions and non-convex functions has been presented. The IGA is based on fixed point algorithm and subdivision theory; the crossover operator and increasing dimension operator are designed with information of integer labels of vertices. Along with the sequence of nearly-complete labeled simplexes, all the completely labeled simplexes can be found quickly. From the experimental results, it can be seen that the validity, stability and convergence of the IGA are also guaranteed. The algorithm has been tested on several standard test problems and which proves that the algorithm has a higher valid and effectiveness.

TABLE 1.
OPTIMIZATION RESULTS OF NUM

Optimization function	Initial points		Optimization results		
	x_1	x_2	x_1^*	x_2^*	$\theta(x_1^*, x_2^*)$
F_1	-5	-5	-0.1907* e-006	-0.1907* e-006	7.2760e-014
	-5	5	-0.1907* e-006	-0.1907* e-006	7.2760e-014
	5	-5	-0.1907* e-006	-0.1907* e-006	7.2760e-014
	5	5	-0.1907* e-006	-0.1907* e-006	7.2760e-014
	0	0	0	0	0
F_2	-2	-2	0.9983	0.9967	2.7813e-006
	-16	-16	1.2954	1.6821	0.0889
	-12	-16	2.6036	6.7820	2.5726
	12	5	2.3900	5.7149	1.9329
	0	0	1.0000	1.0000	2.4997e-011

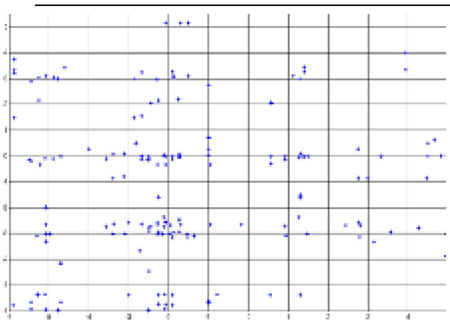
Figure 3 50th generation

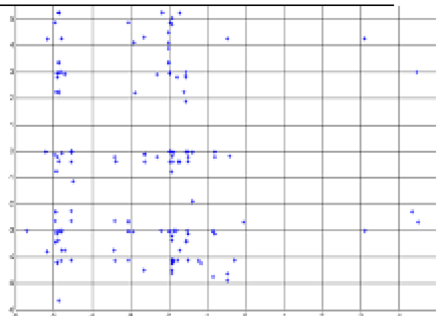
Figure 4 100th generation

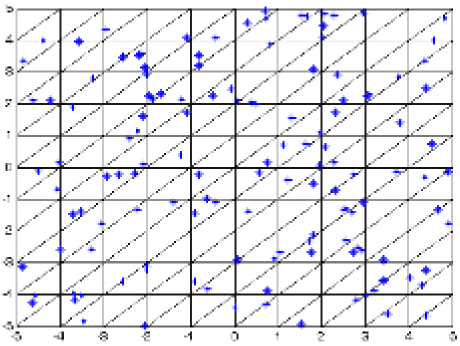

Figure 5 initial generation

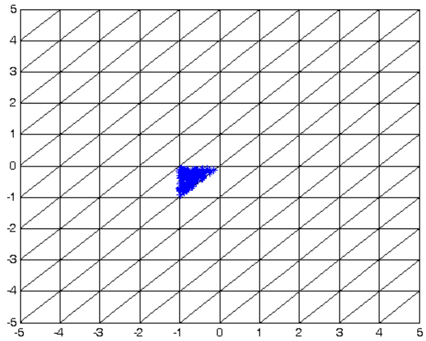

Figure 6 4th generation

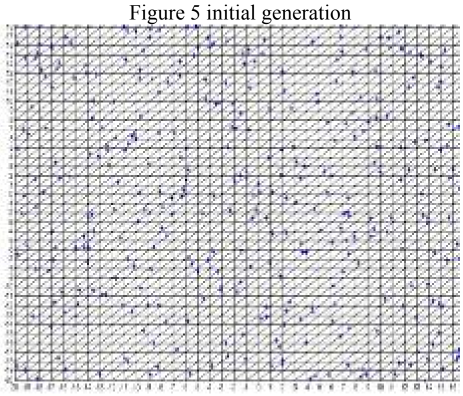

Figure 7 initial generation

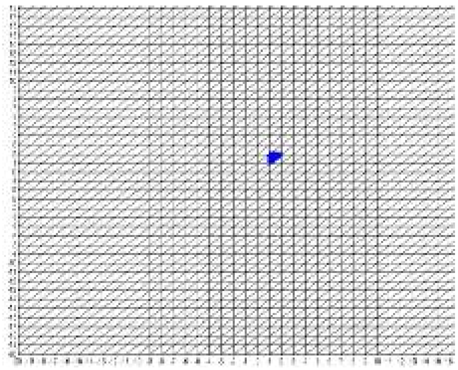

Figure 8 6th generation

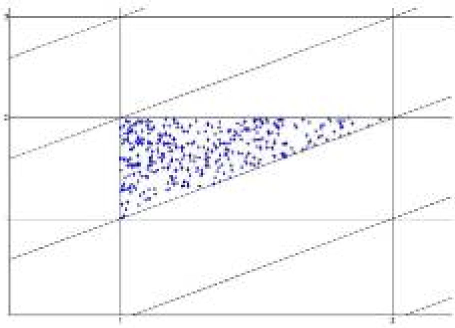

Figure 9 6th generation

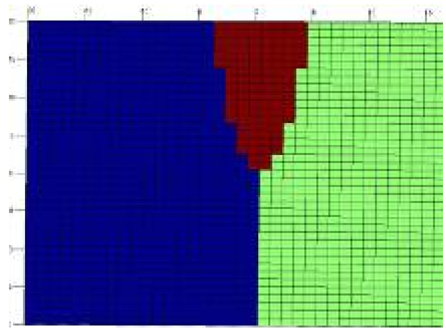

Figure 10 labeled simplex distribution of F_2

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