Train Coordinated Optimization Operation with Regenerative Braking

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Abstract—Reducing the traction energy consumption plays an important role in railway energy saving. The traditional train optimization operation theories, taking the coasting as the means of energy saving, can’t fit complicated lines and is suitable for the single train only. Meanwhile, the highly complicated line will cause the risk of train collision. On the background of the “11th Five-Year” State Science and Technology Support project, a multi-train coordinated optimization operation method based on regenerative braking and safety constraint is studied, and then its application is researched relied on the experimental line. Firstly, a multi-train and multi-object dispatch method for energy saving with safety constraints is researched, and the flexible time constraint model is established. Secondly, on this basis, the multi-train coordinated optimization operation algorithm with the constraints of flexible time and safety is got. At last, the application research is done with the experimental line as the background. The results show that the multi-train coordinated optimization operation method based on regenerative braking, with flexible time and safety constraints, can decrease the energy consumption greatly and improve the traffic volume dramatically in the condition of complicated lines.

Index Terms—train optimization operation, multi-train coordination, safety constraints, regenerative braking, the steep ramp with gradient 100%

I. INTRODUCTION

Railway traction energy consumption accounts for 60%-70% of the railway energy consumption in all the railway departments. Reducing the railway traction energy consumption plays an important role in railway energy saving. The key factors in the railway traction energy consumption: 1, Energy consumption as a result of the train running against the resistance; 2, Kinetic energy loss caused by the train braking.

In the coal mine areas in northwest China, the coal transport lines, from the mine to the railway station or pithead power plant, are very complicated. Thus, the traditional railway can’t be used, and the road transportation has severe disadvantages of high cost and heavy pollution. So, a major project of “11th Five-Year” State Science and Technology Support Plan was set up by the Ministry of Science and Technology to research the innovative rail transportation system fitted to such complicated lines, and with the characteristics of energy saving and environment-friendly.

The traditional train optimization operation theories, taking the coasting as the means of energy saving, can’t fit such complicated lines and is suitable for the single train only. Meanwhile, the highly complicated line will cause the risk of train collision.

Therefore, to research the train optimization operation theories, which take the regenerative braking and multi-train coordination as the major technical means of energy saving and meet safety constraints, has theoretical significance and application value.

II. RELATED WORK

The train optimization operation research can be divided into three classes:

1. The first class is from the standpoint of train control strategy for energy saving-optimized control strategy based on the fixed train structure and fixed operation environment. The research object is the train without energy-feedback. From the standpoint of different constrains, the first class can be divided into two classes: 1, the train optimized control strategy without energy-feedback on the condition of no-safety constraints. 2, the train optimized control strategy without energy-feedback on the condition of safety constraints. At present the method of the research class 1 is comprised of optimal control method [1,2] and intelligent control method [3,4]. In order to ensure the train running safety, the train should limit the speed in the curve and steep gradient section. The method of the research class 1 can’t adapt
such complicated line, so the method of the research class 2 becomes an important research field [5,6].

The second class is from the standpoint of train dispatch for energy saving the influence of train formation for energy saving in train operation. The research object is the train group. From the standpoint of different constrains, the second class can be divided into three classes: 1, the train optimized dispatch without energy-feedback on the condition of no-safety constraints [7,8]. 2, the train optimized dispatch without energy-feedback on the condition of safety constraints [9,10]. 3, the train optimized dispatch with energy-feedback on the condition of safety constraints [11,12].

The third class is from the standpoint of train dynamics system, such as energy storage equipment. The research on the train operation for energy saving which takes the train dynamics system as the research object starts on the 60's of 20th century, so far some technology is relatively mature and abundant[13,14].

The problems of the research on the train optimization operation research are as follows in conclusion:

- The traditional train optimization operation theories, taking the coasting as the means of energy saving can only fit general ramp (the gradient is below 15%), the train optimization operation theories fitted the complicated lines (the gradient is above 100%) are in the beginning stage.
- Energy-feedback is not taken into consideration in the traditional train dispatch for energy saving, so the train dispatch for energy saving with energy-feedback is in the beginning stage.
- The traditional train optimization operation theory research is in the view of separated train, and the safety constraint contains only line speed limit, the train tracking is not taken into consideration.
- The traditional theory research on the train operation for energy saving doesn’t combine the train optimization operation with the train dispatch for energy saving.

According to the present research situation above, on the background of the “11th Five-Year” State Science and Technology Support project, a multi-train coordinated optimization operation method based on regenerative braking and safety constraint is studied, and then its application is researched relied on the experimental line. Firstly, a multi-train and multi-object dispatch method for energy saving with safety constraints is researched, and the flexible time constraint model is established. Secondly, on this basis, the multi-train coordinated optimization operation algorithm with the constraints of flexible time and safety is got. At last, the application research is done with the experimental line as the background.

III. TRAIN DISPATCH FOR ENERGY SAVING WITH REGENERATIVE BRAKING

The train dispatch method for energy saving with regenerative braking is researched, so that the model of the train dispatch is got.

Firstly, the object function for the model of train dispatch method for energy saving is researched. Secondly, the constraint condition is studied.

A. Object function

The object function for the model of train dispatch method for energy saving is based on two objects: 1, the minimum energy consumed in a unit time $t$. 2, the maximum traffic volume in a unit time $I$. However, the two objects are contradictory: in a limiting case, if the trains are not moved in a unit time, the energy consumed is zero, but the traffic volume is zero too; if the traffic volume is large, the energy consumed is high too. So we have to balance the two objects to optimize the train dispatch method for energy saving.

Firstly, the model of train dispatch for energy saving is established. One section is cut from the whole line, as is shown in figure 1, section $p - 2$ is a double section, and section $p$ is a single section. Suppose $P$ is the set of the track sections, $P_e \in P$ is a single section. $P_r \in P$ is a double section. So $P_e \cup P_r = P$, $P_e \cap P_r = \emptyset$. $d_t$ is the length of the section $p$, $I$ is the set of the up trains, $J$ is the set of the down trains, $i \in I$ is the train going up, $j \in J$ is the train going down.

$$\mathbf{R}^p = \mathbf{a}^p + \beta^p \mathbf{v}^p + \gamma^p (\mathbf{v}^p)^2$$  \hspace{1cm} (1)

Where

$\mathbf{a}^p = a + (G_p + 0.8 r_p) W_i$

$\beta^p = b W_i$

$\gamma^p = c A_i$

$\mathbf{v}^p$ : the average resistance due to the train going on the ramp or the curve

$a,b,c$ : the adjusted parameter

$G_p$ : the gradient of the section $p \in P$

$r_p$ : the curvature of the section $p \in P$

$W_i$ : the weight of the train $i \in I$

The factors that influence the energy consumption in the train operation: 1, the energy consumption for the train overcome the resistance; 2, the energy consumption for the accelerating or braking [8,15].

Aiming at the first factor, according to the Davis formula [16], the unit energy consumption for the train $i \in I$ running at the average speed of $\mathbf{v}^p$ on section $p \in P$

$$\mathbf{R}^p = \mathbf{a}^p + \beta^p \mathbf{v}^p + \gamma^p (\mathbf{v}^p)^2$$  \hspace{1cm} (1)

Where

$\mathbf{a}^p = a + (G_p + 0.8 r_p) W_i$

$\beta^p = b W_i$

$\gamma^p = c A_i$

$\mathbf{v}^p$ : the average resistance due to the train going on the ramp or the curve

$a,b,c$ : the adjusted parameter

$G_p$ : the gradient of the section $p \in P$

$r_p$ : the curvature of the section $p \in P$

$W_i$ : the weight of the train $i \in I$
Suppose $i_p$ is the moment that the train going up arrives at the destination, $x^m_p$ and $x^u_p$ are the lower bound and the upper bound. Suppose $y^m_p$ is the moment that the train going down arrives at the destination, $x^m_p$ and $x^u_p$ are the lower bound and the upper bound. $x^e_p \in [0,1]$ is the weight of energy consumption and operation time of the train going up. $x^e_p \in [0,1]$ is the weight of energy consumption and operation time of the train going down.

According to (7)-(10), the general objective function is

$$ f(x, y) = \lambda_m \sum_{i \in I} f^e_i(x) + (1 - \lambda_m) x^m_p f_j + \lambda_m \sum_{j \in J} f^w_j(y) + (1 - \lambda_m) x^m_p f_j $$

$$ = x^e \sum_{i \in I} \sum_{p \in P} \psi_p^i + (1 - x^u_p) x^m_p f_j $$

$$ + x^w \sum_{j \in J} \sum_{p \in P} \Phi^j_p + (1 - x^u_p) x^m_p f_j $$

### B. Constraint Conditions

In order to ensure the safety, the speed limits of the different lines are distinct. $P_1 \in P$ is a single section. $P_2 \in P$ is a double section, $P_2 \in P$ can be divided into the main line and the branch line. Suppose

- $\nu^i_p$: the average speed of the train $i \in I$ on section $P_1 \in P$,
- $\psi^i_p$: the lower bound and $\nu^i_p$ is the upper bound.
- $\nu^j_p$: the average speed of the train $j \in J$ on section $P_1 \in P$ , where $\psi^j_p$ is the lower bound and $\nu^j_p$ is the upper bound.
- $\psi^j_{pm}$: the average speed of the train $i \in I$ on the main line of section $P_2 \in P$ , where $\psi^j_{pm}$ is the lower bound and $\nu^j_{pm}$ is the upper bound.

According to formula (1)-(4), the energy consumption due to the train $i \in I$ entering the section $p \in P$, so the average speed that the train is

$$ \nu^i_p = \frac{d_p}{x^p_{p} - x^p_{p-1}} \quad \nu^i_p \mid p \in P \quad \nu^i_p \mid p \in P $$

$$ U^i_p > 0 \quad \psi^i_p < \nu^i_p < \nu^i_{p+1} \quad \psi^i_p < \nu^i_p < \nu^i_{p+1} $$

According to formula (5), the energy consumption due to the train $i \in I$ accelerating or braking on the section $p \in P$ is $\psi^i_p$.

If $\nu^i_p > \nu^i_{p-1}$, that is

$$ \frac{d_p}{x^p_{p} - x^p_{p-1}} > \frac{d_{p-1}}{x^p_{p-1} - x^p_{p-2}} $$

$$ U^i_p > 0 \quad \psi^i_p < \nu^i_p < \nu^i_{p-1} \quad \psi^i_p < \nu^i_p < \nu^i_{p-1} $$

At the moment the train brakes and produces the renewable energy.

According to formula (1)-(4), the energy consumption due to the train $i \in I$ running on the section $p$ is

$$ \psi^i_p = \nu^i_p + U^i_p $$

Similarly, the energy consumption due to the train $j \in J$ running on the section $p$ is

$$ \Phi^j_p = \nu^j_p + U^j_p $$

$$ $$
The average speed of the train $i$ on the branch line of section $P_2 \in P$, where $\nu_p^i$ is the lower bound and $\overline{\nu}_p^i$ is the upper bound.

The average speed of the train $j \in J$ on the main line of section $P_2 \in P$, where $\nu_p^j$ is the lower bound and $\overline{\nu}_p^j$ is the upper bound.

The average speed of the train $j \in J$ on the branch line of section $P_2 \in P$, where $\nu_p^j$ is the lower bound and $\overline{\nu}_p^j$ is the upper bound.

So,

$$\frac{d_p}{\overline{\nu}_p^j} \leq x_p^{j+1} - x_p^j \leq \frac{d_p}{\nu_p^j}$$

Suppose

$$D_p = \begin{cases} 
1, & \text{(the train } i \in I \text{ enters the branch of the section } p \in P_2) \\
0, & \text{(the train } i \in I \text{ enters the main of the section } p \in P_2) 
\end{cases}$$

$$D_p^j = \begin{cases} 
1, & \text{(the train } j \in J \text{ enters the branch of the section } p \in P_2) \\
0, & \text{(the train } j \in J \text{ enters the main of the section } p \in P_2) 
\end{cases}$$

$M$: a positive number that is infinitely great, according to (12).

$$-MD_p + \frac{d_p}{\overline{\nu}_p^j} \leq x_p^{j+1} - x_p^j \leq \frac{d_p}{\nu_p^j} + MD_p$$

(13) is the constraint condition 1.

Constraint Condition 2: the safety constraint on the no switch section

Suppose $s$ is the safety buffer time of the no switch section. In order to ensure the safety, the time between the successive two trains which enter the same section is $s$. As is shown in figure 1, $\forall k,l \in I, p \in P_1$, if the train $k$ enters the section $p \in P_1$ earlier than the train $l$ , the train $l$ can’t enter the section $p \in P_1$ until the time $s$ after the train $k$ enters the section $p \in P_1$, that is

$$s + x_p^k \leq x_p^l$$

(14)

Suppose

$$B_{w} = \begin{cases} 
1, & \text{(the train } k \in I \text{ enters the section } p \in P_1 \text{ earlier than the train } l \in I) \\
0, & \text{(the train } l \in I \text{ enters the section } p \in P_1 \text{ earlier than the train } k \in I) 
\end{cases}$$

$$B_{w}^j = \begin{cases} 
1, & \text{(the train } k \in J \text{ enters the section } p \in P_1 \text{ earlier than the train } l \in J) \\
0, & \text{(the train } l \in J \text{ enters the section } p \in P_1 \text{ earlier than the train } k \in J) 
\end{cases}$$

According to (14),

$$s + x_p^k \leq x_p^l + M(1-B_{w}^j)(\forall k,l \in I, p \in P_1)$$

(15) is the constraint condition 2.

Constraint Condition 3: the safety constraint on the switch section

Suppose $s_2$ is the safety buffer time of the switch section. In order to ensure the safety, the other train can’t enter the switch section until the train going ahead leave, and the buffer time is $s_2$.

$$C_{w} = \begin{cases} 
1, & \text{(after the train } k \in I \cup J \text{ leaves the section } p \in P_2) \\
0, & \text{其它) (其它)} 
\end{cases}$$

So,

$$s + x_p^k \leq x_p^l + M(1-C_{kl}(\forall k,l \in I, p \in P_2)$$

(16)

As is shown in figure 1, if the train $l$ doesn’t enter the section $p \in P_2$ until the train $k$ leaves the section $p \in P_2$ and the time $s_2$ passes,

$$s_2 + x_p^k \leq x_p^l + M(1-C_{kl}(\forall k,l \in I, p \in P_2)$$

(17) is the constraint condition 3.

Constraint Condition 4: the traffic volume in a unit time

Suppose the whole traffic volume is at least $mq$ ($m$ is the train number, $q$ is the single train volume), so the time that the $m$th train arrives the last section $p_f$ can’t be more than $T$, that is

$$s_{pf} \leq T$$

(18) is the constraint condition 4.
Now we get the object function and the constraint conditions of the model of the train dispatch method for energy saving, then the solution which is the flexible time can be easily got by branch and bound algorithm.

IV. TRAIN COORDINATED OPTIMIZATION OPERATION WITH THE CONSTRAINTS OF FLEXIBLE TIME AND SAFETY

The automatic train operation (ATO) does the traction calculation on the basis of train group real-time status and the train group predicting operation condition, and then the train group predicting status was got. According to the train group predicting status, train group real-time status and external status, ATO gets the fuzzy object evaluation and then does fuzzy reasoning, and gets the satisfactory optimization, then does reinforcement learning, and gets the most energy efficient, the safest train coordinated optimization operation through coordinating the relationship among trains for energy saving, as is shown in figure 2.

The function of the on schedule evaluation is applied to connect the flexible time in the section III with the model of train coordinated optimization operation, so that the flexible time is considered as the constraint condition of the method of the train coordinated optimization operation.

FIGURE 2. The model of train coordinated optimization operation

A. Traction calculation

Based on the traction calculation theory, we can get the predicting current $I$ , the predicting speed $v$ , the predicting acceleration $a$ , the predicting maximum overrunning distance $s_{\text{max}}$ , and the predicting running distance $s$ , and the predicting running time $t$ .

B. Predicting fuzzy control

Some rules should be followed to make the train run safely and energy-efficiently [17]:

1. certain rules should be followed when the train group condition changes, as is shown in table 1. For example, when the train is in traction status, it can’t change to decelerating brake status or stopping brake status immediately.

<table>
<thead>
<tr>
<th>Status</th>
<th>Priority</th>
<th>Status to be changed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting</td>
<td>1</td>
<td>Traction</td>
</tr>
<tr>
<td>Traction</td>
<td>1</td>
<td>Coasting</td>
</tr>
<tr>
<td>Coasting</td>
<td>1</td>
<td>Traction</td>
</tr>
<tr>
<td>Restraining speed</td>
<td>1</td>
<td>Coasting</td>
</tr>
</tbody>
</table>

TABLE 1: RULES FOR THE CHANGE OF OPERATION CONDITION

2, the operation condition of the train should be changed smoothly from one status to another. For example, when the train operation condition changes from coasting to traction, the traction force can’t be too big at once, and should be increased gradually.

1) Fuzzy object evaluation

The procedure for energy-efficient train operation: 1, train starting. 2, uniform velocity. 3, accelerating or decelerating brake before entering the speed limit section. 4, decelerating brake. 5, stopping brake. According to the process, we evaluate safety, speed following, stopping accuracy, energy conservation, on schedule of the train operation.

Safety evaluation

a. Evaluation function of the acceleration safety

The acceleration safety depends on the acceleration value and the rate of acceleration changing. If the acceleration is too big or the rate of acceleration changing is too fast, it may lead to the damage of the vehicle structure. So the acceleration value and the rate of acceleration changing should be controlled to make sure that the train running steady.

Suppose $a_{\text{max}}$ : maximum acceleration for safe operation. $a(t)$ : predicting acceleration. So the evaluation function of the acceleration safety is

$$
\mu_q(a(t)) = \begin{cases} 
0, & a(t) \geq a_{\text{max}} \\
1 - \frac{a(t)}{a_{\text{max}}}, & 0 \leq a(t) < a_{\text{max}} 
\end{cases} \quad (19)
$$

Suppose $\Delta a_{\text{max}}$ : maximum rate of acceleration changing for safe train operation. $\Delta a$ : predicting rate of acceleration changing. $\mu_q(\Delta a) \in [0,1]$ : the rate of acceleration changing safety evaluation function. In formula (19) we replace $a(t)$ , $a_{\text{max}}$ with $\Delta a(t)$ , $\Delta a_{\text{max}}$ then we can get $\mu_q(\Delta a)$ .

b. Evaluation function of the stopping position safety

The stopping position safety displays when the train stops. Suppose $s_{\text{mb}}(t)$ : target stopping position. So the evaluation function of the stopping position safety is

$$
\mu_q(s(t)) = \begin{cases} 
0, & s(t) \geq s_{\text{max}}(t) \\
\frac{s_{\text{max}}(t) - s(t)}{s_{\text{max}}(t) - s_{\text{mb}}(t)}, & s_{\text{mb}}(t) < s(t) < s_{\text{max}}(t) \\
1, & 0 \leq s(t) \leq s_{\text{mb}}(t)
\end{cases} \quad (20)
$$

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c. Evaluation function of the speed safety

The train speed should be less than the speed limit when the train is running. Suppose \( \mu_Y(x(t)) \) : evaluation function of the speed safety, \( v_{\text{max}}(x(t)) \) : limiting speed. \( v_{mb}(x(t)) \) : target speed. We define \( v_{mb}(x(t)) = 0.9v_{\text{max}}(x(t)) \). In formula (20) we replace \( s(t) \cdot r_{\text{max}}(t) \) \( s_{mb}(t) \) with \( v(x(t)) \), \( v_{\text{max}}(x(t)) \), \( v_{mb}(x(t)) \), then we can get \( \mu_Y(x(t)) \).

Speed following evaluation

Speed following evaluation depends on the consistency between the predicting speed \( v(x(t)) \) and the target speed \( v_{mb}(x(t)) \). Suppose \( \Delta v(x(t)) = v(x(t)) - v_{mb}(x(t)) \), the evaluation function of the speed following is

\[
\mu_1(\Delta v(x(t))) = \begin{cases} 
0, & \Delta v(x(t)) \leq -10 \\
10 + \Delta v(x(t)), & -10 < \Delta v(x(t)) < 0 \\
8.5, & \Delta v(x(t)) \geq 0 \\
1, & \Delta v(x(t)) = 0 \\
1 - \Delta v(x(t)), & 0 < \Delta v(x(t)) < 3 \\
0, & \Delta v(x(t)) \geq 3 
\end{cases}
\] (21)

Stopping accuracy evaluation

Stopping accuracy function depends on the deviation of the predicting stopping position \( s(t) \) and the target stopping position \( s_{mb}(t) \). Suppose \( \Delta s(t) = s(t) - s_{mb}(t) \), the evaluation function of the stopping accuracy is

\[
\mu_1(\Delta s(t)) = \begin{cases} 
1, & \Delta s(t) \leq -A \\
e^{-k(\Delta s(t) + A)}, & -A < \Delta s(t) < A \\
e^{-k(\Delta s(t) - A)}, & \Delta s(t) \geq A 
\end{cases}
\] (22)

Energy conservation evaluation

Suppose \( I(t) \): predicting current consumed. \( I_n(t) \): the regenerative current from the power-supply network. If \( I(t) > I_n(t) \), the regenerative current will be consumed by the train. If \( 0 < I(t) < I_n(t) \), only a part of the regenerative current will be consumed by the train. If \( I(t) < 0 \), the train produces regenerative current. The energy conservation evaluation depends on the deviation of \( I(t) \) and \( I_n(t) \).

\[
\mu_e(I(t)) = \begin{cases} 
1, & I(t) > I_n(t) \\
I(t), & 0 < I(t) < I_n(t) \\
0, & I(t) < 0 
\end{cases}
\] (23)

On schedule evaluation

On schedule evaluation depends on the deviation of the predicting running time \( t \) and specified running time \( t_{mb} \).

Suppose \( \Delta t = t - t_{mb} \). The degree of the flexibility \( \Delta t \) is evaluated. The evaluation function of the schedule is

\[
\mu_z(\Delta t) = \begin{cases} 
0, & \Delta t \leq -\Delta t_m \\
1 + \frac{\Delta t}{\Delta t_m}, & -\Delta t_m < \Delta t \leq 0 \\
1 - \frac{\Delta t}{\Delta t_m}, & 0 < \Delta t < \Delta t_m \\
0, & \Delta t \geq \Delta t_m 
\end{cases}
\] (24)

2) Fuzzy reasoning

\( m_1, m_2, m_3, m_4, m_5 \) respectively stand for safety, speed following, stopping accuracy, energy conservation, on schedule. \( m_1 \) includes acceleration safety \( m_1 \), speed safety \( m_2 \), position safety \( m_3 \). The factors that influence the train section operation are \( m_1, m_2, m_4, m_5 \). The factors that influence the train automatic stopping are \( m_1, m_3, m_4, m_5 \). Suppose \( \mu_{mn}(u) \) : the nth safety satisfactory optimization value. \( w_n \) is the weight of the fuzzy object. Under the prerequisite of safety \( \mu_{mn}(u) > 0, n \in Z, 1 \leq n \leq 3 \), the weights of the train section operation from high to low are \( w_1, w_4, w_5, w_2 \) and \( w_1 + w_2 + w_4 + w_5 = 1 \). The weights of the train automatic stopping from high to low are \( w_1, w_4, w_5, w_3 \), and \( w_1 + w_3 + w_4 + w_5 = 1 \).

Suppose \( K(t) \): the train operation condition in time \( t \). According to table 1, if \( K(t) > 0 \), \( K(t+1) > 0 \) or \( K(t+1) = 0 \). If \( K(t) < 0 \), \( K(t+1) < 0 \) or \( K(t+1) = 0 \), so the satisfactory optimization function of the train section operation control:

\[
\mu(u) = (\mu_{m_1}(u))w_1 + (\mu_{m_2}(u))w_2 + (\mu_{m_3}(u))w_3 + (\mu_{m_4}(u))w_4 + (\mu_{m_5}(u))w_5 \\
\mu_{mn}(u) > 0, n \in Z, 1 \leq n \leq 3 \\
w_1 + w_2 + w_4 + w_5 = 1 \\
\text{if} : K(t) > 0, \text{then} : K(t+1) > 0 \text{or} K(t+1) = 0 \\
\text{if} : K(t) < 0, \text{then} : K(t+1) < 0 \text{or} K(t+1) = 0
\] (25)

We can get the satisfactory optimization function of the train automatic stopping control in a similar way. As is shown in figure 3, we get the satisfactory optimization value of the train operation, and the forward \( p \) optimal values of the traction operation mode are the optimum operation value.

![Figure 3. Satisfactory optimization model](image-url)
3) Reinforcement learning

If the relationship of the trains is well coordinated, the energy will be adequately used. We make use of the reinforcement learning to get the most energy efficient control strategy from the p recommended traction operating mode.

Every train is regarded as an intelligent agent, whose state \( s \) contains the velocity and location of the train. Setting \( S = \{v, s\} \), where \( v \) stands for the set of all the possible speed of the train, \( s \) stands for the set of all the possible positions of the train. Therefore, the state of the \( k \)th train is \( s_k \in S \); the action set \( A \) is made up of \( p \) recommended traction working conditions from predictive fuzzy controlling, the action of the \( k \)th train is \( a_k \in A \); the reward of the \( k \)th train is determined by the value of regenerative energy which is gained by the \( k \)th train. For example, if the value of regenerative energy gained by the \( k \)th train is 1 kwh, then \( r_k = 1 \). If regenerative energy generated by the \( k \)th train is 1 kwh, the \( r_k = -1 \).

Determination of action depends on both its own state and the state and action of other trains in the process of the trains’ study. Usually, the reinforcement learning of multi-agent system is distributed, concurrent and fault-tolerant [18], as shown in figure 4.

We replace the action of a single train with combined motions of all trains when the single train system is expanded to multi-train system, then \( Q^k_{t+1}(s_t, a^{1}_{t}, \ldots, a^{n}_{t}) = (1 - \alpha_t)Q^k_t(s_t, a^{1}_{t}, \ldots, a^{n}_{t}) + \alpha_t[\gamma V^k_t(s_{t+1}) + \beta \pi^k(s_{t+1})] \)

But there is still one question that is updating state-value function \( V^k_t(s_{t+1}) \). We adopt the Friend-or-Foe Q learning. At time \( t \), the relationship among the trains which use regeneration energy each other is called Friend, and

\[
\text{Nash}_Q^{k}(s_{t}) = \max_{a^{1}_{t} \in A^{1}} \min_{a^{2}_{t} \in A^{2}} \cdots \min_{a^{n}_{t} \in A^{n}} \sum_{i = 1}^{n} \pi^{i}(a^{i}_{t})Q^{k}_{i}(s_{t}, a^{1}_{t}, \ldots, a^{n}_{t})
\]

\[
\text{Nash}_Q^{k}(s_{t}) = \max_{a^{1}_{t} \in A^{1}} \min_{a^{2}_{t} \in A^{2}} \cdots \min_{a^{n}_{t} \in A^{n}} \sum_{i = 1}^{n} \pi^{i}(a^{i}_{t})Q^{k}_{i}(s_{t}, a^{1}_{t}, \ldots, a^{n}_{t})
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\]
From figure 8, we got that: 1, when the number of iterations is 20042, the traction energy converges to 993.297 kW.h. 2, the traction energy shakes slighter as the number of iterations increases after 20042. It is the phenomenon that the trains lingered between exploitation and exploration. After the convergence, the probability that the trains choose the return $r_k$ is higher than before, because the trains have found the optimal strategy.

In the figure 8, the result is much better than the method without train dispatch for energy saving in which traction energy converges to 1173.844 kW.h in [19].

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In the figure 8, the result is much better than the method without train dispatch for energy saving in which traction energy converges to 1173.844 kW.h in [19].
According to the experimental data, if it works for 5 hours, the traffic volume of the coal is 996.483 tons before optimization, and the traffic volume of the coal is 1244.831 tons after optimization, which is almost the same as the result in [19].

VI. CONCLUSION

The experiment results demonstrate that the train group energy control strategy based on the complete security constraints not only decreases the traction energy consumption heavily, but also improves the traffic volume largely in unit time, and the results are much better than that in [19].

REFERENCES


ACKNOWLEDGMENT

This work is supported by National Key Technology R&D Program “Key Technology Research and the Core Equipment Developing of the innovative linear electric motors transport system” and partially supported by the National High Technology Research and Development Program (863 Program) “Key Technology Research and the Core Equipment Developing of the High-Speed Maglev Train Decentralized Operation Control System”.

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