

Self-calibration of Varying Internal Camera Parameters Algorithm Based on Quasi-affine Reconstruction

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Abstract—This paper presents an method of self-calibration of varying internal camera parameters that based on quasi-affine reconstruction. In a stratified approach to self-calibration, a projective reconstruction is obtained first and this is successively refined first to an affine and then to a Euclidean reconstruction. It has been observed that the difficult step is to obtain the affine reconstruction, or equivalently to locate the plane at infinity in the projective coordinate frame. So, a quasi-affine reconstruction is obtained first by image sequences, then we can obtain the infinite plane in the quasi-affine space, and equivalently to affine reconstruction. Then the infinite homography matrix can be calculated through the affine reconstruction, and then using the infinite homography matrix and constraints of the image of absolute conic to calculate the camera internal parameters matrix, and further to measure the metric rreconstruction. This method does not require a special scene constraints(such as prapllel, perpendicular) information, and also the camera movement informations(such as pure translation or orthogonal movement), to achieve the goal of self-calibration. The theoretics analysis and experiments with real data demonstrate that this self-calibration method is available, stable and robust.

Index Terms—quasi-affine reconstruction, infinite plane, absolute conic, self-calibration

I. INTRODUCTION

Camera calibration is the key steps to obtain three-dimensional information from the two-dimensional image^[1]. In general, camera calibration can be divided into two categories^[2-5]:(1)Traditional camera calibration:Known scene of the space coordinates and the corresponding image points to calculate the camera projective matrix, this method has the advantage of high accuracy. But this method requires precise calibration block in the scene, which has the disadvantage of the small range of applications and real-time differential. (2)Self-calibration:Camera self-calibration can be divided into two subclasses:(a)The self-calibration based on active vision, the calibration method through an active

vision platform to control the camera to make some movement, such as translation, or a pure rotation orthogonal to get the camera motion parameters, so calculate the internal camera parameters linearly. This method is linear, high accuracy, but the drawback is the need to active vision platform, continues tio limit the application. (b)Self-calibration based on image sequences, the calibration method dose not need to know the coordinates of the space scene and the corresponding image points and external devices to control the camera movement , can do real-time calibration just need to know the scene information and feature informationfrom image sequences. The calibration method is the most convenient and most widely application, however , due to calibration process is not linear and need nonlinear optimization, so accuracy is not high. Since on accurate calibration block and do not accurately control the camera movement, the focus of self-calibration based on image sequences is to determine the constraints of camera internal parameters and image feature. There are three methods that determine the constraints of internal parameters:absolute dual quadric, absolute conic and kruppa equation. These three methods have different advantages and disadvantages of different image features, this article describes the use of self-calibration of the absolute conic.

The keystone of self-calibration based on absolute conic is to determine the infinite plane^[6-9], however, the difficulty of the calibration method is to determine the infinite plane. From the literature shows that methods of Determination of infinite plane as following:

Translational motion:For translational motion control camera, the affine reconstruction can be achieved by two images^[3]. Although the affine reconstruction by translational motion has been constructed for the demonstration, but the translational motion of the fundamental matrix is assumed to have a very restrictive form as follow:symmetric matrix. If there is no translational motion, or by the fundamental matrix is not symmetric matrices, affine reconstruction can not be achieved.

Scene constraints: Scene constraints on the image features such as parallelogram or parallelepiped can also be used to obtain affine reconstruction. As long as they identify three points coordinate of the plane at infinity, we can identify the plane at infinity, resulting in affine reconstruction. If the scene plane extraction patterns without rules, we can not achieve accurate calibration.

Parallel lines: The intersectant point of space parallel lines is the point at infinite plane, the image of the intersectant point is the vanishing point of this parallel lines. If there are three pairs of features like parallel lines, and each pair has a different direction, then we can find three different points of the plane at infinity, then obtain affine reconstruction. If we can not extract the features of 3 different parallel lines, we can not achieve accurate calibration.

Throughout the above methods to determine the plane at infinity, the basic needs to the outside world scene constraints. When the extracted image features can only obtain matching points can not be fitted or can not get the rules straight line parallel pattern, the above method to determine infinite plane will no longer be used. So this paper presents an method of self-calibration of varying internal camera parameters that based on quasi-affine reconstruction, we can obtain the infinite plane by using a quasi-affine reconstruction in the conditions of unknown scene information and camera motion information, then get the affine reconstruction. Then the infinite homography matrix can be calculated through the affine reconstruction, and then using the infinite homography matrix and constraints of the image of absolute conic to calculate the camera internal parameters matrix, and further to measure the metric reconstruction. The theoretics analysis and experiments with real data demonstrate that this self-calibration method is available, stable and robust.

II. CALCULATE QUASI-AFFINE RECONSTRUCTION

Quasi-affine reconstruction is a special projective reconstruction^[4], it is first introduced by Hartley^[5]. Any projective reconstruction that satisfies the equation $P_j X_i = \lambda_i x_i$ and let λ_i has the same symbols for all of i, will be a quasi-affine reconstruction. Given a set of image corresponding points $x_i \leftrightarrow x'_i, i=1,2,3...n$. Using Sturm's factorization can obtain the projective reconstruction of images and equation $x_i \approx P^j X_i$, making this implicit equation is given by a constant factor display:

$$\lambda_i^j x_i^j = P^j X_i \tag{1}$$

Projective reconstruction can be normalized as follow:

$$P^1 = (I | 0), P^j = (H_j | e_j) \quad j=2,3,\dots,n \tag{2}$$

According to the definition of quasi-affine reconstruction, if only we obtain the transformational

matrix H, and satisfy $P_{qa}^j = P^j H^{-1}$ and $X_{qai} = H X_i$ and let all of λ_i^j has same sign, $\{P_{qa}^j, X_{qai}\}$ is a quasi-affine reconstruction. Quasi-affine reconstruction of demand constraint is the constant factor λ_i^j has the same symbol, the constant factor λ_i^j has positive and negative direction, assume that we take forward as a standard (the fact that we can get both positive and negative quasi-affine reconstruction). If we get the constant factor λ_i^j is positive, then the quasi-affine reconstruction is obtained, or by changing the whole λ_i^j is positive, resulting in quasi-affine reconstruction. However, the symbol of constant factor λ_i^j in the equation (1) can not be reflected, so think of link the constant factor λ_i^j to the depth information of space point. By proving. A space point $X_i = (X, Y, Z, T)^T$ of relative depth of the camera is display:

$$depth(X_i; P^j) = \frac{sign(\det M^j) \lambda_i^j}{T \|m^3\|} \tag{3}$$

And M^j is the left 3×3 sub matrix of P^j , m^3 is third line of M^j . This means that with X_i or M^j unrelated to the specific homogeneous, that it is multiplied by a non-zero scale factor will not change. P^j is constant, so $sign(\det M^j)$ is constant, T can be set to 1 and $\|m^3\|$ also is constant, therefore, the symbol of constant factor λ_i^j is decided by symbol of depth information of space point. In this paper we focus only symbol instead of the size of the depth information. Therefore, we can rewrite equation (2):

$$depth(X_i; P^j) \cong \lambda_i^j T \det M^j \tag{4}$$

Here, \cong express the same symbol. $Sign(depth(X_i; P^j))$ is called cheirality of the space point X_i relative to the camera P^j .

The symbol of constant factor λ_i^j is decided by symbol of depth information of space point, so when the same spatial point of all the depth information of the symbols, the constant factor λ_i^j has the same symbols. Such as all of depth informations are positive, constant factor λ_i^j must all be the same symbols. Now the depth of information by translating the sign of the depth information of the formula can be written as:

$$depth(X_i; P^j) = (v^T X_i)(v^T C) \partial > 0 \tag{5}$$

Here $\partial = sign(\det H)$, valid values 1 or -1. v express a

4-dimensional vector that is mapped to infinite plane by H, define C_p^T is the vector: (c_1, c_2, c_3, c_4) , $c_i = (-1)^i \det \hat{P}^{(i)}$, $\hat{P}^{(i)}$ is a matrix of P that has been taken out the i th row. We can assume the center of the camera P^1 satisfy $(v^T C^1) \partial > 0$, then can be introduced the following inequality based on equation (5):

$$X_i^T v > 0 \quad \text{Established for all points } X_i$$

$$\partial C^{P_j^T} v > 0 \quad \text{Established for all camera } P^j \quad (6)$$

Equation (6) is the cheirality constraint. Every value of X_i , C and C' is known, they formed groups on the inequality of v . For the sake of the required transformation matrix H, using cheirality constraints to calculate the value of A firstly, then H is any matrix that v^T as its last line and meets the condition of $\det H \cong \partial$, if the last element is not zero, then H can choose the form of the first three lines: $[I \mid 0]$.

The following is to strike a quasi-affine reconstruction algorithm steps:

- (a) For each pair (i,j), let $P^j X_i = \omega_i^j x_i^j$ when point x_i^j is given.
- (b) With camera $-P^j$ instead of some camera P^j and with point $-X_i$ instead of some point X_i , so as to ensure that required: $\omega_i^j > 0$
- (c) Form the cheirality inequality:
 $X_i^T v > 0 \quad \text{Established for all points } X_i$
 $\partial C^{P_j^T} v > 0 \quad \text{Established for all camera } P^j$
- (d) For the two values of $\partial = \pm 1$, select a solution for this cheirality inequality. Set the solution is v_∂ . For any value of ∂ , the solution must exist, and sometimes the two values of solutions exist.
- (e) Define a matrix H_∂ , its last line is v_∂ and satisfies $\det(H_\partial) \cong \partial$. The matrix H_∂ is the required transpositional matrix. If H_+ and H_- are present, then they reverse the export of two quasi-affine reconstruction

III. DEFINE THE PLANE AT INFINITY

By the previous section, we can obtain the quasi-affine reconstruction of $\{X_i^{qa}, P_{qa}^j\}$. However, the quasi-affine reconstruction obtained is not unique, it is to be determined to a difference of one point and the camera centers on the quasi-affine transformation, therefore, we can choose the symbol of P_{qa}^j and X_i^{qa} to make the

last coordinate of X_i^{qa} and the determinant of every M_{qa}^j is positive to determine the unique quasi-affine reconstruction. After finding the quasi-affine reconstruction, we can define a infinite plane.

Assume π_∞ is the infinite plane that mapped to matrix H, the necessary and sufficient condition of π_∞ to meet $\{X_i^{qa}, P_{qa}^j\}$ is that the whole of π_∞ locate outside of the convex hull of point and the center of camera. Therefore, π_∞ does not pass through convex hull and origin point, using vector v^{qa} to instead of π_∞ , so the last coordinate of v^{qa} is not zero and can display: $v^{qa} = (v_1^{qa}, v_2^{qa}, v_3^{qa}, 1)^T$. Because coefficients of the projection equations of the quasi-affine reconstruction have the same cheirality, so cheirality inequality (6) becomes:

$$X_i^{qaT} v^{qa} > 0 \quad \text{Established for all points } X_i^{qa}$$

$$C^{P_{qa}^j} v^{qa} > 0 \quad \text{Established for all camera } P_{qa}^j \quad (7)$$

We can obtain v_i^{qa} by solving linear programming problems^[10-15], generally speaking, we can use Simplex Method to solve the cheirality inequality.

IV. Self-calibration

Through the above steps, we can obtain $\pi_\infty = (a^T \ 1)^T$ and $P^1 = [I \mid 0]$,

$$P_a^j = P_{qa}^j \begin{bmatrix} I & 0 \\ a^T & 1 \end{bmatrix}^{-1} = [H_\infty^j, e_j'] \quad , j=2,3,\dots,n \quad (8)$$

Here H_∞^j is the infinite homography between the first view and the j th view that induced by infinite plane.

Absolute conic Ω_∞ is a conic in π_∞ , according to the nature of the absolute conic, the projected images of which in two views as $\omega_1 = K_1^{-T} K_1^{-1}$ and $\omega_j = K_j^{-T} K_j^{-1}$, and H_∞^j can be displayed:

$$H_\infty^j = K_j R K_1^{-1} \quad (9)$$

Take a matrix transformation to formula (9):

$$H_\infty^j = K_j R K_1^{-1} \Leftrightarrow H_\infty^j K_1 = K_j R \Leftrightarrow$$

$$K_1^T H_\infty^{jT} = R^T K_j^T \quad , \text{multiply the same edge of the two equations after transformed:}$$

$$H_\infty^j K_1 K_1^T H_\infty^{jT} = K_j R R^T K_j^T = K_j K_j^T$$

$$\Leftrightarrow H_\infty^j \omega_1^{-1} H_\infty^{jT} = \omega_j^{-1} \quad (10)$$

Can obtain the equation:

$$H_\infty^{-jT} \omega_1 H_\infty^{-j} = \omega_j \quad (11)$$

Then draw the constraint equations of the image of absolute conic. Aspect ratio is assumed to be 1 and the main point is constant, we can obtain:

$$\begin{bmatrix} h_{11}^j & h_{12}^j & h_{13}^j \\ h_{21}^j & h_{22}^j & h_{23}^j \\ h_{31}^j & h_{32}^j & h_{33}^j \end{bmatrix}^{-T} \begin{bmatrix} f_1^2 + s_1^2 + u_0^2 & s_1 f_1 + u_0 v_0 & u_0 \\ s_1 f_1 + u_0 v_0 & f_1^2 + v_0^2 & v_0 \\ u_0 & v_0 & 1 \end{bmatrix} \begin{bmatrix} h_{11}^j & h_{12}^j & h_{13}^j \\ h_{21}^j & h_{22}^j & h_{23}^j \\ h_{31}^j & h_{32}^j & h_{33}^j \end{bmatrix}^{-1} = \begin{bmatrix} f_j^2 + s_j^2 + u_0^2 & s_j f_j + u_0 v_0 & u_0 \\ s_j f_j + u_0 v_0 & f_j^2 + v_0^2 & v_0 \\ u_0 & v_0 & 1 \end{bmatrix} \quad (12)$$

Through matrix transformation, the main point coordinates can coincide with the image center, it is $u_0 = v_0 = 0$, so can be obtained equation:

$$\begin{bmatrix} h_{11}^j & h_{12}^j & h_{13}^j \\ h_{21}^j & h_{22}^j & h_{23}^j \\ h_{31}^j & h_{32}^j & h_{33}^j \end{bmatrix}^{-T} \begin{bmatrix} f_1^2 + s_1^2 & s_1 f_1 & 0 \\ s_1 f_1 & f_1^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_{11}^j & h_{12}^j & h_{13}^j \\ h_{21}^j & h_{22}^j & h_{23}^j \\ h_{31}^j & h_{32}^j & h_{33}^j \end{bmatrix}^{-1} = \begin{bmatrix} f_j^2 + s_j^2 & s_j f_j & 0 \\ s_j f_j & f_j^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (13)$$

From formula(13) can get equations:

$$\omega_{j13} = \omega_{j23} = 0 \quad (14)$$

$$\omega_{j31} = \omega_{j32} = 0 \quad (15)$$

$$\omega_{j33} = 1 \quad (16)$$

The assumption that elements of matrix H_∞^{-j} is

$$H_\infty^{-j} = \begin{bmatrix} y_{11}^j & y_{12}^j & y_{13}^j \\ y_{21}^j & y_{22}^j & y_{23}^j \\ y_{31}^j & y_{32}^j & y_{33}^j \end{bmatrix}$$

indicated by y_{ij}^k , it is indicated by the left matrix of formula (13) multiplication and from the top of the equations (14), (15), (16), we can get the focal length and distorted factor equations

$$\begin{aligned} &(y_{11}^j y_{13}^j + y_{21}^j y_{23}^j) f_1^2 + (y_{11}^j y_{23}^j + y_{21}^j y_{13}^j) s_1 f_1 \\ &+ y_{11}^j y_{13}^j s_1^2 + y_{31}^j y_{33}^j = 0 \\ &(y_{12}^j y_{13}^j + y_{22}^j y_{23}^j) f_1^2 + (y_{12}^j y_{23}^j + y_{22}^j y_{13}^j) s_1 f_1 \\ &+ y_{12}^j y_{13}^j s_1^2 + y_{32}^j y_{33}^j = 0 \\ &(y_{13}^j y_{13}^j + y_{23}^j y_{23}^j) f_1^2 + (y_{13}^j y_{23}^j + y_{23}^j y_{13}^j) s_1 f_1 \\ &+ y_{13}^j y_{13}^j s_1^2 + y_{33}^j y_{33}^j = 1 \end{aligned} \quad (17)$$

Can be seen from above, when obtained the inverse matrix of infinity plane homography matrix is a nonsingular matrix, equations (17) have a unique solution but no solution when it is singular matrix.

In order to calculate the simple, this paper set s_1 for 0, means Zero Distortion, After calculating the f_1 , we can get the f_2 and s_2 with the formula (13). Using calibration matrix multiplying inverse transformation matrix, can get the final intrinsic parameter matrices.

IV. REAL EXPERIMENT

Experiments using real images verify the effectiveness of the method. The two images in the real experiment is taken by Canon DIGITAL IXUS 95 IS, the image size is 640×480 in figure one. focal length f changes in the process of shooting.

Using this method, can calculate the coordinate of infinite plane in the quasi-affine space is $\pi_\infty^{qa} = (0.0028 \quad -0.0011 \quad 3.2106 \quad 1)$, Infinite homography matrix :

$$H_\infty = \begin{bmatrix} 0.6952 & -0.2405 & 829.3428 \\ 0.1421 & 0.8914 & -65.7894 \\ -0.0003 & 0.0001 & 1 \end{bmatrix}$$



Figure 1 Two images of jardinière

Through (17), (12) we can obtain: $f_1=1625$, $s_1=0$, $u_0=320$, $v_0=240$; And then by (13), (12) we can obtain:

$f_2=1679$, $s_2=-0.254$, $u_0=318.6$, $v_0=213.1$

$$K_1 = \begin{bmatrix} 1625 & 0 & 320 \\ 0 & 1625 & 240 \\ 0 & 0 & 1 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 1675 & -0.254 & 318.6 \\ 0 & 1679 & 213.1 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 2 is a group of box-images, Using this method, can calculate the coordinate of infinite plane in the

quasi-affine space is
 $\pi_{\infty}^{qa} = (0.0046 \quad -0.0153 \quad 4.8302 \quad 1)$, Infinite
 homography matrix:

$$H_{\infty} = \begin{bmatrix} 0.5893 & -0.1305 & 634.3428 \\ 0.3021 & 0.6347 & -48.7054 \\ -0.0003 & 0.0001 & 1 \end{bmatrix}$$



Figure 2 Two images of a box

Through (17),(12)we can obtain:

$$f_1 = 1346.06, s_1 = 0, u_0 = 320, v_0 = 240 ;$$

And then by (13),(12) we can obtain:

$$f_2 = 1219.20, s_2 = -54.60, u_0 = 451.81, v_0 = 234.75 ;$$

and

$$K_1 = \begin{bmatrix} 1346.06 & 0 & 320 \\ 0 & 1346.06 & 240 \\ 0 & 0 & 1 \end{bmatrix} ,$$

$$K_2 = \begin{bmatrix} 1219.20 & -54.60 & 451.81 \\ 0 & 1219.20 & 234.75 \\ 0 & 0 & 1 \end{bmatrix}$$

V. CONCLUSION

In this paper, through the nature of points in the same side of camera to obtain quasi-affine reconstruction, and use nature of quasi-affine to search infinite plane and affine reconstruction. Simplex Method is used to linear programming chirality inequality to obtain the range of infinite plane, and get the infinite plane by taking the upper bound. After affine reconstruction obtained, using the nature of absolute conic and infinite homography to get the linear equation of calibration matrix K, then obtain the calibration matrix. This method do not need precise control of camera motion, and do not need to extract the image rules features (such as parallel, orthogonal), only the matching feature points can be obtained stable, robust and accurate calibration results.

ACKNOWLEDGMENT

Sponsored by: Nature Science Foundation of China(60973096) and Science Foundation of aviation in China (2010ZC56007)

REFERENCES

- [1] MENG Xiao-qiao, HU Zhan-yi. Recent Progress in Camera Self-Calibration[J]. ACTA AUTOMATICA SINICA, 2003, 29(1):110~124
- [2] Faugeras O, Luong Q T, Maybank S. Camera self-calibration : Theory and experiments. In : Proceedings of the 2nd European Conference on Computer Vision, Italy, 1992. 321~334
- [3] LI Hua, HU Zhan-yi. A Linear Camera Self-Calibration Technique Based on Projective Reconstruction[J]. Journal of Software, 2002, 13(12), 2286~2295
- [4] ZHANG Huai-Feng WU Fu-Chao HU Zhan-Yi. A Stratified Reconstruction Algorithm Based on Affine Point Correspondences[J]. Journal of Software, 2005, 8(28):1267~1276
- [5] Hartley R.I.. Chirality. International Journal of Computer Vision, 1998, 26(1):41~61
- [6] Sturm, P. Triggs, B. A factorization based algorithm for multi-image projective structure and motion. In: Proceedings of the European Conference on Computer Vision. LNCS 1064/5, Springer-Verlag, 1996. 709~720
- [7] Hartley R.I., de Agapite L. Haman E, Reid I. Camera calibration and the search for infinity. In: Proceedings of International Conference on Computer Vision, Kerkyra, Greece, 1999, 510~517
- [8] JIANG Zetao, WU Wenhuan. An Essential Matrix-based Camera Self-Calibration Method[J]. Journal of Image and Graphics, 2010, 15(4), 565~569
- [9] Huo Ju Yang Wei Yang Ming. A Self-Calibration Technique Based on the Geometry Property of the Vanish Point[J]. Acta Optica Sinica, 2010, 30(2), 465~472
- [10] YAMAMURA K. Interval solution of nonlinear equations using linear programming [A]. 1997 IEEE International Symposium Circuits and Systems[C]. Hong Kong: IEEE, 1997. 9-12.
- [11] TESSIER S R, BRENNECKE J, STADTHER M, et al. Reliable phase stability analysis for excess Gibbs energy models [J]. Computers Chem Engng, 2000, 55:1785-1796.
- [12] GAU Chao-yang, STADTHER M. New interval methodologies for reliable chemical process modeling[J]. Computers Chem Engng, 2001, 26:827-840.
- [13] YANG Bing. The practical optimization methods and computer programs[M]. Ha' erbing: Ha' erbing Ship-ping Engineering College Press, 1994.
- [14] SCHNEPPER C A, STADTHER M A. Robust process simulation using interval methods [J]. Computers Chem Engng, 1996, 20(2): 187-189.
- [15] KEARFOTT R B, NOVOA M. Algorithm 681 INT-BIS, a portable Interval newton/bisection package [J]. ACM Transactions on Mathematical Software, 1990, 16(2): 152-157. 287

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