A new Approach based on Ant Colony Optimization (ACO) to Determine the Supply Chain (SC) Design for a Product Mix

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Abstract—**Manufacturing supply chain(SC) faces changing business environment and various customer demands. Pareto Ant Colony Optimisation (P-ACO) in order to obtain the non-dominated set of different SC designs was utilized as the guidance for designing manufacturing SC. P-ACO explores the solution space on the basis of applying the Ant Colony Optimisation algorithm and implementing more than one pheromone matrix, one for every objective. The SC design problem has been addressed by using Pareto Ant Colony Optimisation in which two objectives are minimised simultaneously. There were tested two ways in which the quantity of pheromones in the PM is incremented. In the SPM, the pheromone increment is a function of the two objectives, cost and time, while in MPM the pheromone matrix is divided into two pheromones, one for the cost and another one for the time. It could be concluded that the number of solutions do not depend on if the pheromone is split or is a function of the two variables because both method explore the same solution space. Although both methods explore the same solution space, the POS generated by every one is different. The POS that is generated when the pheromone matrix is split got solutions with lower time and cost than SMP because in the probabilistic decision rule a value of $\lambda = 0.2$ is used. It means that the ants preferred solution with a low cost instead of solutions with low time. The strategy of letting the best-so-far ant deposit pheromone over the PM accelerates the algorithm to get the optimal POS although the number of ants in the colony is small.**
 An experimental example is used to test the algorithm and show the benefits of utilising two pheromone matrices and multiple ant colonies in SC optimisation problem.

Index Terms—**supply chain design, multi-objective optimisation, ant colony, meta-heuristics**

I. INTRODUCTION

Today's rapidly changing business environment requires corporations to continuously evaluate and configure their supply chains (SCs) to provide customers with high quality products/services at the lowest possible cost and within the shortest possible time[1][2]. A supply chain is a network of optional resources through which materials (raw materials, work in progress, and finished products) flow along one direction while information (demand data, due date, delivery and assembly cost and time) along both directions in order to meet demands from customers[3]. The process of finding the best flow patterns (i.e., choices of resources) for every product in a product mix is known as the optimisation of the SC design[4][5]. When a manufacturer decides from which supplier to get each of the required components and in which manufacturing plant each of the sub-assemblies and final products must be assembled, the who-serves-whom relationships for the supply chain are established. As a consequence, the flow patterns for every product are determined. There may be multiple suppliers that could supply the same component as well as optional manufacturing plants that could assemble the same sub-assembly or product, each differentiated by a lead-time and cost. Given all the possible options for resources, the supply chain configuration problem is to select the options that minimise the total cost while keeping the total time as short as possible (or within what customers are prepared to accept).

This paper utilises Pareto Ant Colony Optimisation (P-ACO) in order to obtain the non-dominated set of different SC designs. P-ACO explores the solution space on the basis of applying the Ant Colony Optimisation

algorithm and implementing more than one pheromone matrix, one for every objective. This paper is organized as follows. In section 2, the theory of P-ACO is explained; section 3 described the proposed problem and solution method for designing the SC by means of P-ACO. In order to test the proposed method an experimental application is depicted in section 4. The results are shown in section 5, and the paper concluded in section 6.

II. PARETO ANT COLONY OPTIMISATION

With ant colony optimisation (ACO) meta-heuristics, colonies of artificial ants cooperate to find solutions to difficult discrete optimisation problems[6][7]. Real ants have the capability of smelling and depositing a chemical substance, referred to as pheromones (τ), as a way of communicating with each other. Ants move randomly when they leave the nest to forage for food but when ants find a pheromone trail, they decide whether or not to follow it. If they decide to do so, they deposit their own pheromones over the trail. The probability that an ant selects one path over another is based on the strengths of pheromones smelt over the paths. The stronger the pheromone smelt over a path, the more likely the ant will select the path. Over time, the amount of pheromone on a path also evaporates. Before the colony finds the shortest path between the nest and the food, they use all the potential paths in equal numbers, depositing pheromones as they travel but the ant that takes the shorter path will return to the nest first with food. The shorter path will have the higher pheromone strength because the path has "fresh" pheromone and has not yet evaporated and will be more attractive to other ants that look for the food source.

With artificial ants (A), ants travel in a network in which pheromones are deposited over either the vertices or edges. The nest is represented by an initial condition and the food by a terminal condition. Ants select a vertex or a node for stepping forward based on a probabilistic decision rule that is a function of the strength of pheromone deposited over the node or edge. The way in which an ant deposits and smells pheromone is by means of a pheromone matrix (PM). The problem is represented by a set of constraints. Every time an ant selects a vertex or edge, it has to evaluate the set of constraints.

The goal of Pareto Optimisation (PO) is to obtain a complete Pareto Optimal Set of solutions (POS) for a problem that has more than one objective[8][9] . Every solution in the Pareto set is called a non-dominated solution. In the case of P-ACO, the set of non-dominated solutions is computed by a number of ant colonies that have the same number of ants. When every ant has generated a solution all the solutions are compared and only the non-dominated solutions are permitted to deposit pheromone over the pheromone matrix.

In the case of the supply chain design problem, there are two objectives to be minimised, the total cost (c) and the total lead time (t). A solution, i.e., a supply chain design, SCD^u generated by an ant u will have a total cost

and total time associated, c^{SCD^u} and t^{SCD^u} respectively. The Pareto set contains all the non-dominated designs $\{SCD^1, SCD^2,...,SCD^m\}$. SCD^u is said to dominate SCD^v, if $\quad (t^{SCD^U} \leq t^{SCD^v}) \wedge (c^{SCD^u} \leq c^{SCD^v}) \quad$ and $(t^{SCD^U} < t^{SCD^v}) \vee (c^{SCD^u} < c^{SCD^v})$. This is written as $SCD^u \succ SCD^v$. The set of dominated and non-dominated $SCDs$ is denoted by Z; SCD^u belongs to the non-dominated set, Z^*, if $\{SCD^u \in Z \mid \exists SCD^u \succ SCD^v\}$.

III. THE PROPOSED METHOD

A. Problem Definition

The problem being addressed involves the optimal choice of resources across a supply chain to minimise the total cost and lead-time simultaneously for a product (or a product mix). Considering a supply chain network with N nodes (e.g., corresponding to a product with N stages of operations involving the sourcing/supply of each of the components, the assembling of each of the sub-assemblies and the final product, and the delivering of the product to customer), each with a number of resource options, the total supply chain cost is determined by,

$$TC = \sum_{i=1}^{N} \sum_{j=1}^{N_i} C_{ij} y_{ij} \qquad (1)$$

Where N_i is the number of resource options available to node i, C_{ij} is the cost added by j^{th} resource option for node i and y_{ij} is a decision variable which equals 1 if the jth option is selected for node i and 0 otherwise.

Due to the demand and supply relationship between nodes of the supply network, the activity at certain nodes can not start until all inputs are available. An example of such is at an assembly node where processing can not start until all input components have arrived. Therefore the cumulative lead-time at such a node will be the sum of the processing lead-time of the node and the maximum delivery lead-time of all input components, that is

$$LT_i = \sum_{j=1}^{N_i} LT_{ij} y_{ij} + \max_{k \in S_i}(LT_k) \qquad (2)$$

Where LT_i is the cumulative lead-time for node i, LT_{ij} is the processing lead-time of the jth resource option for node i, S_i is the set of nodes (corresponding to parts/operations) that input to node i, and LT_k is the cumulative lead-time at node k.

The cumulative lead-time at the final node of the network corresponds to the lead-time of the entire network, which can be written as:

$$TT = LT_N = \sum_{j=1}^{N_N} LT_{Nj} \cdot y_{Nj} + \max_{i \in S_N}(LT_i) \qquad (3)$$

Assuming unlimited capacity of resources, the problem is therefore defined by two objective functions (1) and (3) and a number of constraints:

$$\sum_{j=1}^{N_i} LT_{ij} y_{ij} + \max_{k \in S_i}(LT_k) - LT_i = 0 \quad \text{for } i \in N \quad (4)$$

$$y_{ij} = \begin{cases} 1 \text{ if } j \text{ is selected} \\ 0 \text{ otherwise} \end{cases} \quad \text{for } i \in N \quad (5)$$

$$\sum_{j=1}^{N_i} y_{ij} = 1 \quad \text{for } i \in N \quad (6)$$

B. P-ACO approach to solving the problem

In order to solve the problem with P-ACO, it is necessary to represent the problem as a graph in which ants travel to find solutions, to represent the pheromone in terms of optimisation objectives, and to determine how the pheromone is to be updated and used to guide decision making by ants.

A supply chain network, with optional resources at each of the nodes, represents a perfect graph that can be used by ants in P-ACO. Ants can travel between resources in the network from node 1 until node N, picking up a choice of optional resources at each of the nodes. This is as if they pick up components involved in a product, assign components to assembly plants to produce the sub-assemblies and the final product, and deliver the final product to customer.

In P-ACO, pheromone represents the desirability of ants to follow a particular path. In the problem considered, it represents the desirability of ants to choose each of the resource options at each of the network nodes. It is represented by a pheromone matrix ($PM = \{\tau_{ij}\}$) whose values are updated, either through evaporation over time, or through enhancement by an ant when the ant completes a tour, based on the performance of the resulting path (i.e., resource combination). In the case of former, an evaporation factor ρ is applied after a time interval where $\tau_{ij} \leftarrow (1 - \rho)\tau_{ij}$. In the case of latter, there are two ways in which *PM* could be updated. The first is called single-pheromone multi-objective method (*SPM*) in which the increment of τ_{ij}, $\Delta\tau_{ij}$, corresponding to a selected resource r_{ij}, on a resulting path (resource combination) *P*, are calculated through an equation that is a function of the two objectives.

$$\Delta\tau_{ij} = \frac{1}{S}\left[\left(\frac{TC_p}{\theta}\right)^2 + \left(\frac{TT_p}{\theta}\right)^2\right] \quad \forall r_{ij} \in P \quad (7)$$

Where *m* is the number of ants in the colony, θ is a parameter that balances the value of the cost and time, TC_p and TT_p represent the resulting total cost and time for path *P*. When an ant travels through the network, the probabilistic decision rule, p_{ij}, that the ant will choose the jth resource at node *i, i.e., r_{ij},* is represented by (8).

$$p_{ij} = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{Nr_{ij}} [\tau_{i'j'}]^\alpha [\eta_{i'j'}]^\beta} \quad (8)$$

Where η_{ij} is the heuristic value that is calculated by $\eta_{ij} = \frac{1}{C_{ij}}$, α and β are parameters that balance the relative importance between the ant's skill to follow a path with the highest pheromone strength and the ant's ability to look for new paths respectively with values between (0, 1), and Nr_{ij} is the neighbourhood of choices while the ant is in r_{ij} and it is defined as $Nr_{ij} = \{r_{i'j'} \mid i = i', 1 \leq j' \leq N_i\}$.

The second way in which *PM* could be updated is to split it into two pheromone matrix, one corresponding to each of the objectives, PM^{TC} and PM^{LT}. This is called multi-pheromone multi-objective method (MPM). The increments in the fist matrix are calculated by

$$\Delta\tau_{ij}^{TC} = \frac{1}{TC_p} \quad (9)$$

and in the second by

$$\Delta\tau_{ij}^{LT} = \frac{1}{TT_p} \quad (10)$$

The heuristic value is split as well,

$$\Delta\eta_{ij}^{TC} = \frac{1}{C_{ij}} \quad (11)$$

$$\Delta\eta_{ij}^{LT} = \frac{1}{LT_{ij}} \quad (12)$$

for cost and time, respectively. The probabilistic decision rule is calculated by (13).

$$p_{ij} = \frac{\left[(\tau_{ij}^{TC})^{1-\lambda}(\tau_{ij}^{LT})^\lambda\right]^\alpha \left[(\eta_{ij}^{TC})^{1-\lambda}(\eta_{ij}^{LT})^\lambda\right]^\beta}{\sum_{Nr_{ij}} \left[(\tau_{i'j'}^{TC})^{1-\lambda}(\tau_{i'j'}^{LT})^\lambda\right]^\alpha \left[(\eta_{i'j'}^{TC})^{1-\lambda}(\eta_{i'j'}^{LT})^\lambda\right]^\beta} \quad (13)$$

Where λ regulates the relative importance of different objectives, $\lambda \in (0, 1)$[10].

To evaluate the convergence rate, the global and local explore ability of the modified P-ACO algorithm and the implementation of the decline disturbance index on the performance of the improved algorithm, this paper tests six typical Benchmark functions as in table I to compare P-ACO with SACO.

Table II shows the results about the average optimization and variance of the optimization functions simulated by P-ACO and SACO.

IV. EXPERIMENTAL TEST

A supply chain test case is adapted from literature [11]. The example involves a supply chain producing two types of products namely: Gray Notebook and Blue Notebook. The former satisfies both US and export

TABLE II.
TEST FUNCTIONS SELECTED FOR EXPERIMENT

Name	Function	Search space	Optimal/position
Ackley	$f_1(x,y) = 20 + e^{-20} e^{-\frac{1}{5}\sqrt{\frac{1}{n}(x^2+y^2)}} - e^{\frac{1}{n}(\cos(2\pi x)+\cos(2\pi y))}$	$(-30, 30)^n$	22.2956 $(\pm 29.5008, \pm 29.5008)$
Rastrigin	$f_2(x) = \sum_i^n [x_i^2 - 10\cos(2\pi x_i) + 10]$	$(-5.12, 5.12)^n$	80.7066 $(\pm 4.52, \pm 4.52)$
Rosenbrock	$f_3(x) = \sum_i^n [100(x_{i-1} - x_i^2)^2 + (1 - x_i)^2]$	$(-5.12, 5.12)^n$	$0(1,\dots,1)$
Griewank	$f_4(x) = \frac{1}{4000}\sum_{i=1}^n (x_i)^2 - \prod_{i=1}^n \cos\left(\frac{(x_i)}{\sqrt{i}}\right) + 1$	$(-300, 300)^n$	$0(0,\dots,0)$
Schaffer's f6	$f_5(x,y) = 0.5 + \frac{(\sin\sqrt{x_i^2 + y_i^2})^2 - 0.5}{(1 + 0.01(x_i^2 + y_i^2)^2)^2}$	$(-10, 10)^n$	$0(0,\dots,0)$
Schaffer's f7	$f_6(x,y) = \sum_{i=1}^n (x_i^2 + y_i^2)^{0.25} \times [\sin(50 \times (x_i^2 + y_i^2)^{0.1}) + 1.0]$	$(-100, 100)^n$	$0(0,\dots,0)$

TABLE I.
PERFORMANCE COMPARISON BETWEEN P-ACO AND STANDARD ACO

Functions	Dimension	Gmax	SACO		P_ACO	
			Average Optimization	Variance	Average Optimization	variance
Ackley	2	1000	22.23	-6.5	22.2777	-18.5
Rastrigin	2	1000	80.6975	5.1e-005	80.7055	0.3e-007
Rosenbrock	2	1000	5.3e-013	-0.9e-026	3.7e-016	1.4e-033
Griewank	2	1000	8.1e-003	0.2e-005	3.1e-013	1.2e-025
Shere's f6	2	1000	9.2e-017	2.8e-033	1.7e-016	1.7e-032
Shere's f7	2	1000	2.7e-013	1.8e-022	5.7e-015	1.2e-029

demand while the latter is sold to the US market only. The two products require similar components and sub-assemblies until a point of differentiation where either a grey or blue cover is included. The tree structure in Figure 1 shows the components and operations that are required to manufacture the products and deliver them to markets. For each product, four types of components are required to produce a circuit board assembly, which is then assembled with an LCD display, a metal housing, a battery, and various other components to produce a notebook subassembly. The notebook subassembly is then integrated with either a grey or a blue cover to make up the final products.

The supply chain network with all the resource options is shown in Figure 1. It consists of 6 tiers of resources. As in previous description, operational stages in the network, i.e., parts and operations required to fulfil customer demands for the two products, are sequenced with numbers from 1 to 17, and each resource is indexed with two subscript numbers, the first representing operational stages and the second the resource options. Therefore r_{ij} represents the jth resource option for the ith

operation. Thus for the first operational stage, the sourcing of parts w/8, there are four optional resources (r_{11}, r_{12}, r_{13}, r_{14}) representing different suppliers. For the fifth stage, the assembly of circuit board, there are two optional resources (r_{51}, r_{52}) representing two different plants. In the current tests, variations of costs and lead-times of individual resources in performing different operations, over time, were not considered. Local planning at resources was not considered thus cost and lead-time data of a resource performing a particular operation are assumed to be fixed and be constants regardless of which downstream resource performs the next operation. The tests were run once for every customer-product combination using SPM and MPM. The parameters used in the tests were: 100 ant colonies each with 200 ants (Q =100 and S = 200). The values of α, β and ρ are set as 1.0, 2.0, 0.02 respectively as proposed by literature[12]. The value of λ is set at 0.2.

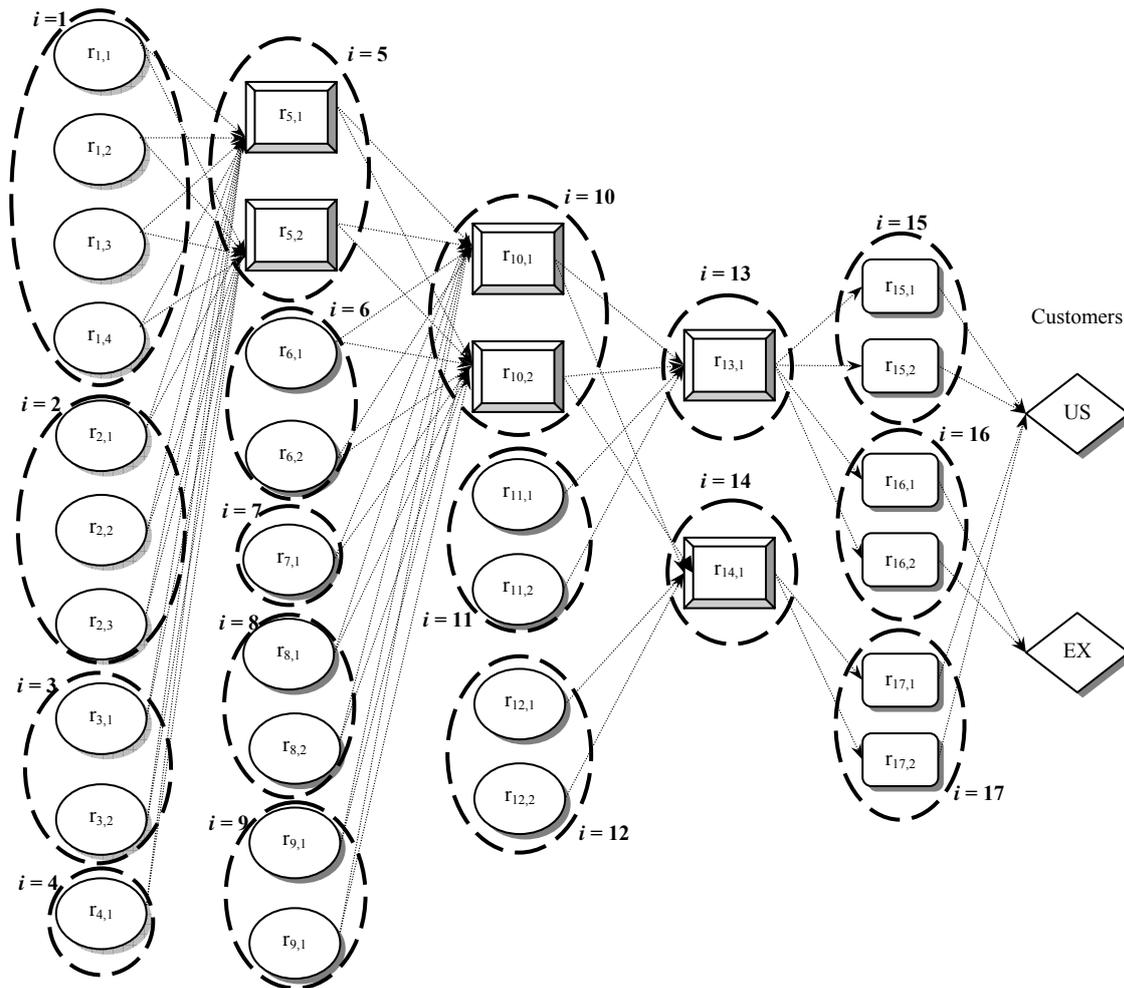The example has 24276 possible solutions $\prod_{i=1}^{N} N_i$.

Figure 1.   Supply chain network showing resource options

Figure 2 (a), (c), and (f) show all the solutions generated by the algorithm using SPM and MPM for US demand Gray notebook, Export Demand Gray notebook, and US demand Blue notebook, respectively. The corresponding POSs are depicted in Figure 2 (b), (d) and (f). From the results, it can be observed that the numbers of solutions derived with SPM and MPM are almost the same. However, the POS generated by MPM contain solutions with lower cost and time than that generated by SPM. This suggests that the use of multiple pheromone matrices can help achieve better results with the same number of colonies and ants. Once the POS for very customer-product combination has been generated, the next step is to select one solution from each POS and use the selected solutions to determine a final SCD which could be used as a common supply chain structure for the mix of customer-product combinations.

V. CONCLUSION

The use of social insects to assist with combinatorial optimisation problems has tremendous potential as it is shown in the experimental application solved in this paper. This paper proposes a new approach to determining the supply chain (SC) design for a product mix comprising complex hierarchies of subassembly and components. For the supply chains considered, there may be multiple suppliers that could supply the same components as well as optional manufacturing plants that could assemble the subassemblies and the products. Each of these options is differentiated by its lead time and cost. Given all the possible options the supply chain design problem is to select the options that minimise the total cost while keeping the total lead time within required delivery due dates. This work introduces Pareto Ant Colony Optimisation as an especially effective meta-heuristic for solving the problem of SC Design. A number of ant colonies generate a Pareto Optimal Set of
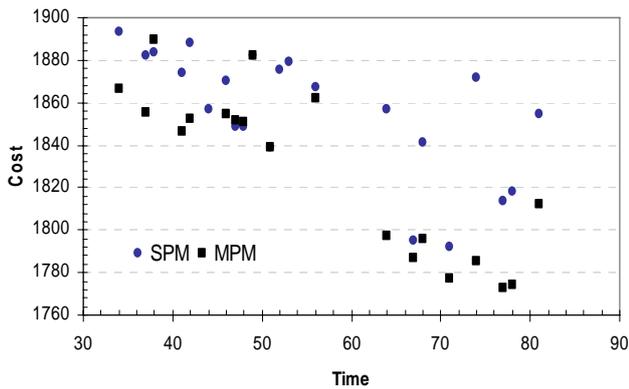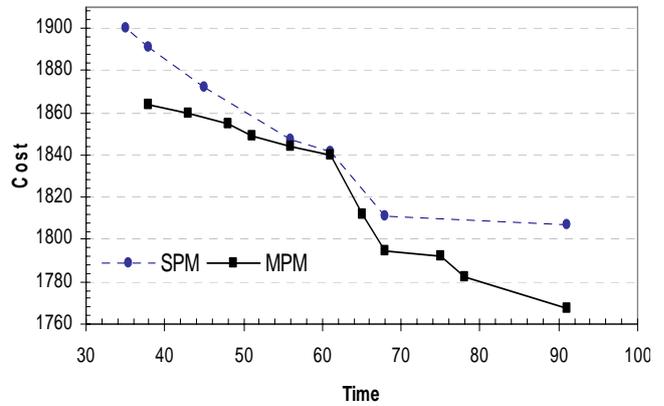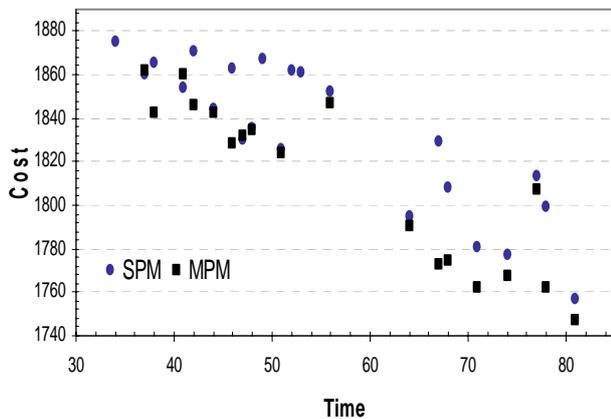
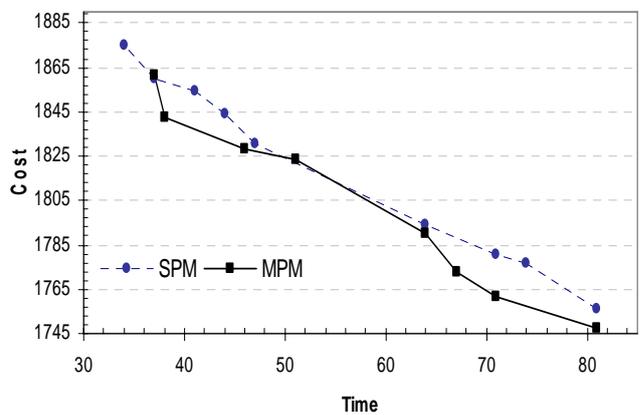(a) All solutions (US demand, Gray)
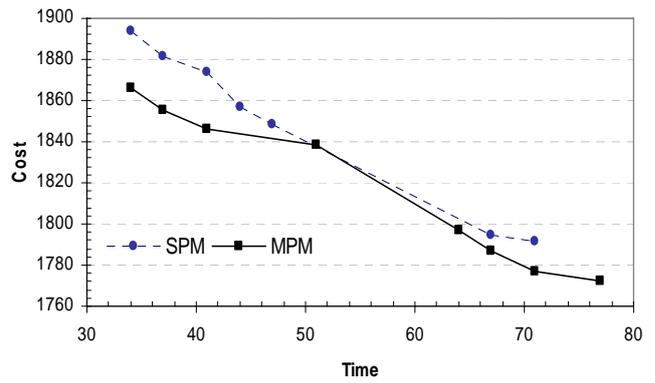
(b) POS (US demand, Gray)
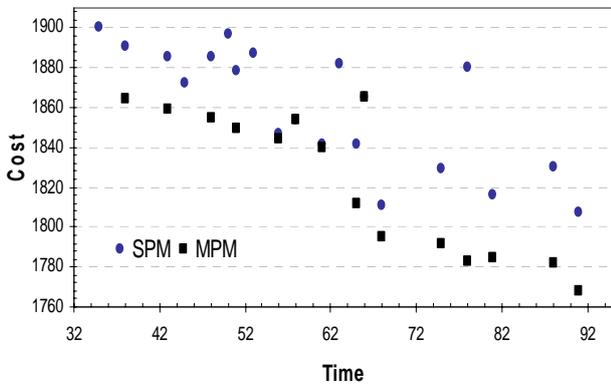
(c) All solutions (Export demand, Gray)

(d) POS (Export demand, Gray)

(e) All solutions (US demand, Blue)

(f) POS (US demand, Blue)

Figure 2.   Test results

SC Designs in which only the non dominated SC designs allow ants to deposit pheromones over the time and cost pheromone matrices. Although both methods explore the same solution space, the POS generated by every one is different. The POS that is generated when the pheromone matrix is split got solutions with lower time and cost than SMP because in the probabilistic decision rule a value of λ = 0.2 is used. It means that the ants preferred solution with a low cost instead of solutions with low time. The strategy of letting the best-

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