

Stability Analysis for Uncertain Stochastic Delayed Neural Networks of Neutral-Type with Discrete and Distributed Delays

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Abstract—The paper studies delay-dependent global robust problem for a class of uncertain stochastic delayed neural networks of neutral-type with discrete and distributed delays. Novel stability criteria are obtained in terms of linear matrix inequality (LMI) by employing the Lyapunov-Krasovskii functional method and using the free-weighting matrices technique. In addition, two examples are given to show the effectiveness of the obtained conditions.

Index Terms—Global robust stability; Delayed neural networks; Neutral-type; Lyapunov-Krasovskii functional; Discrete and distributed delays

I. INTRODUCTION

The problem of stability analysis for neural networks with time delays is of both practical and theoretical importance. The main reason for this is that time delays will affect stability of a neural network by creating oscillatory and instability characteristics. Therefore, a lot of reports about stability criteria for neural networks with time delays are reported in the literature, such as [1-10], and the references cited therein. It is worth noting that the existing stability criteria can be classified into two categories: delay-independent stability and delay-dependent stability, and the latter are less conservative than the former. Recently, there are some research issues about delayed neural networks of neutral-type with (or neutral type neural networks) time delays (see, e.g., [11-17]). In particular, there have been extensive results on the problem of the delay-dependent global stability of neutral-type with time delays in the literatures (see, e.g., [13, 15-16]).

In addition, the problem of global stability analysis for neural networks of neutral type with discrete and distributed delays has also received considerable attention; see for example, [18-21]. Based on the Lyapunov-Krasovskii functional method, the global exponential stability of neutral type neural networks with distributed delays was studied in [18]. The delay-dependent stability conditions for neutral type neural networks with both discrete and unbounded distributed delays were discussed in [19]. Moreover, for stochastic neural networks of neutral type, various stability results were obtained in [20] and [21]. However, there have been few studies devoted to the problem of uncertain stochastic delayed neural networks of neutral-type with discrete and distributed delays. The objective of this paper is to investigate the global robust stability criteria for uncertain stochastic delayed neural networks of neutral-type. For this purpose, an approach combining the free-weighting matrices technique together with the Lyapunov functional method is taken in our study. Finally, two numerical examples are given to demonstrate the applicability of the proposed stability criteria.

The remainder of the paper is structured as follows: In Section 2, the problem description, three assumptions and basic lemmas for the considered systems are presented. In Section 3, the delay-dependent global robust stability results of this paper are derived. Illustrative examples are given in Section 4, and the paper is concluded in Section 5.

Notations: The following notations will be used throughout this paper. For real symmetric matrices X and Y , the notation $X \geq Y$ (respectively, $X > Y$), means that $X - Y$ is positive semi-definite (respectively, positive definite); \mathfrak{R}^n and $\mathfrak{R}^{n \times n}$ denote the n -dimensional Euclidean space and the set of all $n \times n$ real matrices, respectively; The superscripts "T" and "-1" stand

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for matrix transposition and matrix inverse, respectively; (Ω, \mathcal{F}, P) is a complete probability space, where Ω is the sample space, \mathcal{F} is the σ -algebra of subsets of the sample space and P is the probability measure on \mathcal{F} ; The shorthand $\text{diag}\{M_1, M_2, \dots, M_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices M_1, M_2, \dots, M_n ; I is the identity matrix with appropriate dimensions. The symmetric terms in a symmetric matrix are denoted by $(*)$.

II. PROBLEM DESCRIPTION AND PRELIMINARIES

This section considers the following class of neural networks with discrete and unbounded distributed delays described by a non-linear neutral delay differential equation:

$$\begin{aligned} \dot{v}_i(t) = & -c_i v_i(t) + \sum_{j=1}^n w_{ij1} g_j(v_j(t)) \\ & + \sum_{j=1}^n w_{ij2} g_j(v_j(t - \tau(t))) \\ & + \sum_{j=1}^n a_{ij} \int_{-\infty}^t k_j(t-s) g_j(v_j(s)) ds \\ & + \sum_{j=1}^n b_{ij} \dot{v}_j(t - h(t)) + I_i, i = 1, 2, \dots, n, \end{aligned} \tag{1}$$

where $\dot{v}_i(t)$ is the state of the i th neuron at time t , $c_i > 0$ denotes the passive decay rate, w_{ij1}, w_{ij2}, a_{ij} and b_{ij} are the interconnection matrices representing the weight coefficients of the neurons, $g_j(\cdot)$ is the activation function; I_i is a external constant inputs; The delay k_j is a real valued continuous nonnegative function defined on $[0, +\infty]$, which is assumed to satisfy $\int_0^\infty k_j(s) ds = 1, j = 1, 2, \dots, n$.

For system (1), the following assumptions are given:

Assumption 1. Each neuron activation function $g_i(\cdot) (i = 1, 2, \dots, n)$ is bound and satisfies the following condition:

$$l_i^- \leq \frac{g_i(x_1) - g_i(x_2)}{x_1 - x_2} \leq l_i^+, i = 1, 2, \dots, n, \tag{2}$$

$$x_1, x_2 \in \mathcal{R}^n, x_1 \neq x_2,$$

where l_i^-, l_i^+ are some constants.

Remark 1: l_i^- and l_i^+ can be positive, negative, and zero.

Assumption 2. The time-varying delays $\tau(t)$ and $h(t)$ satisfy

$$\begin{aligned} 0 \leq \tau(t) \leq \tau, \dot{\tau}(t) \leq \tau_d < 1, \\ 0 < h(t) \leq h, \dot{h}(t) \leq h_d < 1, \end{aligned} \tag{3}$$

where τ, τ_d, h , and h_d are constants.

Assume $v^* = [v_1^*, v_2^*, \dots, v_n^*]^T$ is an equilibrium point of system (1). It can be easily derive that the transformation $x_i = v_i - v_i^*$ puts system (1) into the following system:

$$\begin{aligned} \dot{x}(t) = & -Cx(t) + W_1 f(x(t)) + W_2 f(x(t - \tau(t))) \\ & + A \int_{-\infty}^t K(t-s) f(x(s)) ds \\ & + B\dot{x}(t - h(t)), \end{aligned} \tag{4}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathcal{R}^n$ is the neural state vector, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t))]^T \in \mathcal{R}^n$ is the neuron activation function vector with $f(0) = 0$. $C = \text{diag}\{c_1, c_2, \dots, c_n\} > 0$, $W_1 \in \mathcal{R}^{n \times n}$, $W_2 \in \mathcal{R}^{n \times n}$, $A \in \mathcal{R}^{n \times n}$ and $B \in \mathcal{R}^{n \times n}$ are the connection weight matrices.

Note that since each function $g_j(\cdot)$ satisfies the hypotheses Assumption 1, hence $f_j(\cdot)$ satisfies

$$l_i^- \leq \frac{f_i(x_1) - f_i(x_2)}{x_1 - x_2} \leq l_i^+, i = 1, 2, \dots, n, \tag{5}$$

$$x_1, x_2 \in \mathcal{R}^n, x_1 \neq x_2,$$

where l_i^-, l_i^+ are some constants.

In the following, we propose the uncertain stochastic neural networks of neutral-type with discrete and distributed delays is described by

$$\begin{aligned} d[x(t) - B(t)x(t - h(t))] = & [-C(t)x(t) + W_1(t)f(x(t)) \\ & + W_2(t)f(x(t - \tau(t))) \\ & + A(t) \int_{-\infty}^t K(t-s)f(x(s)) ds] dt \\ & + [H_0(t)x(t) + H_1(t)x(t - \tau(t)) \\ & + H_2(t)x(t - h(t))] d\omega(t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} B(t) = & B + \Delta B(t), C(t) = C + \Delta C(t), \\ W_1(t) = & W_1 + \Delta W_1(t), W_2(t) = W_2 + \Delta W_2(t), \\ A(t) = & A + \Delta A(t), H_0(t) = H_0 + \Delta H_0(t), \\ H_1(t) = & H_1 + \Delta H_1(t), H_2(t) = H_2 + \Delta H_2(t). \end{aligned}$$

Assumption 3. The parameter uncertainties $\Delta B(t), \Delta C(t), \Delta W_1(t), \Delta W_2(t), \Delta A(t), \Delta H_0(t), \Delta H_1(t)$ and $\Delta H_2(t)$ are of the form

$$\begin{aligned} [\Delta B(t), \Delta C(t), \Delta W_1(t), \Delta W_2(t), \\ \Delta A(t), \Delta H_0(t), \Delta H_1(t), \Delta H_2(t)] \\ = HF(t)[Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8], \end{aligned} \tag{7}$$

where $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7$, and Q_8 are known constant matrices with appropriate dimensions. The uncertain matrices $\Delta(t)$ that satisfy $\Delta(t) = [I - F(t)J]^{-1} F(t)$, where J is also a known matrix satisfying $I - JJ^T > 0$ and $F(t)$ is uncertain matrix satisfying

$$F(t)^T F(t) \leq I, \forall t \geq 0.$$

For the sake of simplicity, the following notations are adopted

$$\begin{aligned} \rho_1(t) = & -C(t)x(t) + W_1(t)f(x(t)) + W_2(t)f(x(t - \tau(t))) \\ & + A(t) \int_{-\infty}^t K(t-s)f(x(s)) ds, \end{aligned} \tag{8}$$

$$\rho_2(t) = H_0(t)x(t) + H_1(t)x(t - \tau(t)) + H_2(t)x(t - h(t)). \tag{9}$$

System (6) then reads as

$$d[x(t) - B(t)x(t-h(t))] = \rho_1(t)dt + \rho_2(t)d\omega(t). \quad (10)$$

In the following section, propose a new stability criterion for the system described by (6). The following lemmas are useful in deriving the criterion:

Lemma 1(Schur complement [22]). Given constant matrices Υ_1, Υ_2 and Υ_3 with appropriate dimensions, where $\Upsilon_1^T = \Upsilon_1$ and $\Upsilon_2^T = \Upsilon_2 > 0$, then $\Upsilon_1 + \Upsilon_3^T \Upsilon_2^{-1} \Upsilon_3 < 0$ if and only if

$$\begin{bmatrix} \Upsilon_1 & \Upsilon_3^T \\ * & -\Upsilon_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Upsilon_2 & \Upsilon_3 \\ * & \Upsilon_1 \end{bmatrix} < 0. \quad (11)$$

Lemma 2[23]. For any constant matrix $\Upsilon > 0$, any scalars a and b with $a < b$, and a vector function $x(t): [a, b] \rightarrow \mathfrak{R}^n$ such that the integrals concerned as well defined, the following holds

$$\left[\int_a^b x(s)ds \right]^T \Upsilon \left[\int_a^b x(s)ds \right] \leq (b-a) \int_a^b x^T(s) \Upsilon x(s)ds. \quad (12)$$

Lemma 3[24]. For any real vectors a, b and any matrix $Q > 0$ with appropriate dimensions, it follows that $\pm 2a^T b \leq a^T Q a + b^T Q^{-1} b$.

$$(13)$$

Lemma 4[25]. Let M, E and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ satisfying

$F(t)^T F(t) \leq I$, then, the following inequality holds for any $\varepsilon > 0$,

$$MF(t)E + E^T F^T(t)M^T \leq \varepsilon^{-1}MM^T + \varepsilon E^T E. \quad (14)$$

III. ROBUST STABILITY ANALYSIS

In the following theorem, a novel robust delay-dependent criterion for global asymptotic stability of system (6) is derived in terms of LMI.

Theorem 1. For given scalars τ, τ_d, h , and h_d satisfy (3), system (6) is globally robustly stochastically asymptotically stable in the mean square, if there exist matrices $P > 0, Z_j = Z_j^T > 0, j = 1, 2, \dots, 5, R_j = R_j^T > 0, j = 1, 2, 3, 4, M_j, N_j, U_j, V_j, j = 1, 2, \dots, 10$, and diagonal matrices $E > 0, T_j > 0, j = 1, 2$, and two positive scalars $\varepsilon_1, \varepsilon_2$ such that the following LMI holds:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & MH & NH \\ * & \Pi_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \quad (15)$$

where

$$\Pi_{11} = [\phi_{ij}]_{10 \times 10}, \Pi_{12} = \Pi_{14} = [U_i]_{10 \times 1},$$

$$\Pi_{13} = \Pi_{15} = [V_i]_{10 \times 1}, i, j = 1, 2, \dots, 10,$$

$$\Pi_{22} = -(1 - \tau_d)\tau^{-1}R_1, \Pi_{33} = -(1 - h_d)h^{-1}R_2,$$

$$\Pi_{44} = -(1 - \tau_d)R_3, \Pi_{55} = -(1 - h_d)R_4,$$

with

$$\begin{aligned} \phi_{1,1} = & -PC - CP^T + Z_1 + Z_3 + Z_4 \\ & + Z_5 - 2L_1T_1 - M_1C - CM_1^T \\ & + N_1H_0 + H_0^T N_1^T + U_1 + U_1^T \\ & + V_1 + V_1^T + \varepsilon_1 Q_1^T Q_1, \end{aligned}$$

$$\phi_{1,2} = -CM_2^T + H_0^T N_2^T + N_1H_1 + U_2^T - U_1 + V_2^T,$$

$$\begin{aligned} \phi_{1,3} = & PW_1 + L_2T_1 - CM_3^T + M_1W_1 + H_0^T N_3^T \\ & - N_1 + U_3^T + V_3^T, \end{aligned}$$

$$\begin{aligned} \phi_{1,4} = & PW_2 + L_2T_2 - CM_4^T + M_1W_2 + H_0^T N_4^T \\ & + U_4^T + V_4^T, \end{aligned}$$

$$\phi_{1,5} = PA - CM_5^T + M_1A + H_0^T N_5^T + U_5^T + V_5^T,$$

$$\begin{aligned} \phi_{1,6} = & CP^T B - CM_6^T + H_0^T N_6^T + N_1H_2 + U_6^T + V_6^T \\ & - U_1B - V_1B - V_1, \end{aligned}$$

$$\phi_{1,7} = -CM_7^T - M_1 + H_0^T N_7^T + U_7^T + V_7^T,$$

$$\phi_{1,8} = -CM_8^T + H_0^T N_8^T + U_8^T + V_8^T,$$

$$\phi_{1,9} = -CM_9^T + H_0^T N_9^T + U_9^T + V_9^T + V_1B,$$

$$\phi_{1,10} = -CM_{10}^T + H_0^T N_{10}^T + U_{10}^T + U_1B + V_{10}^T,$$

$$\begin{aligned} \phi_{2,2} = & -(1 - \tau_d)Z_1 + 2L_1T_2 + N_2H_1 + H_1^T N_2^T - U_2 \\ & - U_2^T + \varepsilon_2 Q_5^T Q_5, \end{aligned}$$

$$\phi_{2,3} = M_2W_1 + H_1^T N_3^T - U_3^T,$$

$$\phi_{2,4} = M_2W_2 + H_1^T N_4^T - U_4^T,$$

$$\phi_{2,5} = M_2A + H_1^T N_5^T - U_5^T,$$

$$\phi_{2,6} = H_1^T N_6^T + N_2H_2 - U_6^T - U_2B - V_2B - V_2,$$

$$\phi_{2,7} = -M_2 + H_1^T N_7^T - U_7^T,$$

$$\phi_{2,8} = H_1^T N_8^T - N_2 - U_8^T,$$

$$\phi_{2,9} = H_1^T N_9^T - U_9^T + V_2B,$$

$$\phi_{2,10} = H_1^T N_{10}^T - U_{10}^T + U_2B,$$

$$\phi_{3,3} = Z_2 + E - 2T_1 + M_3W_1 + W_1^T M_3^T + \varepsilon_1 Q_2^T Q_2,$$

$$\phi_{3,4} = W_1^T M_4^T + M_3W_2,$$

$$\phi_{3,5} = W_1^T M_5^T + M_3A,$$

$$\phi_{3,6} = -W_1^T P^T B + W_1^T M_6^T + N_3H_2 - U_3B - V_3B - V_3,$$

$$\phi_{3,7} = W_1^T M_7^T - M_3,$$

$$\phi_{3,8} = W_1^T M_8^T - N_3,$$

$$\phi_{3,9} = W_1^T M_9^T + V_3B,$$

$$\phi_{3,10} = W_1^T M_{10}^T + U_3B,$$

$$\phi_{4,4} = -(1 - \tau_d)Z_2 - 2T_2 + M_4W_2 + W_2^T M_4^T + \varepsilon_1 Q_3^T Q_3,$$

$$\phi_{4,5} = W_2^T M_5^T + M_4A,$$

$$\phi_{4,6} = -W_2^T P^T B + W_2^T M_6^T + N_4H_2 - U_4B - V_4B - V_4,$$

$$\phi_{4,7} = W_2^T M_7^T - M_4,$$

$$\phi_{4,8} = W_2^T M_8^T - N_4,$$

$$\begin{aligned}
 \phi_{4,9} &= W_2^T M_9^T + V_4 B, \\
 \phi_{4,10} &= W_2^T M_{10}^T + U_4 B, \\
 \phi_{5,5} &= -E + AZ_1 A^T + M_5 A + A^T M_5^T + \varepsilon_2 Q_6^T Q_6, \\
 \phi_{5,6} &= -A^T P^T B + A^T M_6^T + N_5 H_2 - U_5 B - V_5 B - V_5, \\
 \phi_{5,7} &= A^T M_7^T - M_5, \\
 \phi_{5,8} &= A^T M_8^T - N_5, \\
 \phi_{5,9} &= A^T M_9^T + V_5 B, \\
 \phi_{5,10} &= A^T M_{10}^T + U_5 B, \\
 \phi_{6,6} &= -(1-h_d)Z_3 + N_6 H_2 + H_2^T N_6^T - U_6 B - B^T U_6^T \\
 &\quad - V_6 B - B^T V_6^T - V_6 - V_6^T + \varepsilon_1 Q_4^T Q_4, \\
 \phi_{6,7} &= -M_6 + H_2^T N_7^T - B^T U_7^T - B^T V_7^T - V_7^T, \\
 \phi_{6,8} &= H_2^T N_8^T - N_6 - B^T U_8^T - B^T V_8^T - V_8^T, \\
 \phi_{6,9} &= H_2^T N_9^T - B^T U_9^T - B^T V_9^T - V_9^T + V_6 B, \\
 \phi_{6,10} &= H_2^T N_{10}^T - B^T U_{10}^T - B^T V_{10}^T - V_{10}^T, \\
 \phi_{7,7} &= P + \tau R_1 + h R_2 - M_7 - M_7^T, \\
 \phi_{7,8} &= -M_8^T - N_7, \phi_{7,9} = -M_9^T + V_7 B, \\
 \phi_{7,10} &= -M_{10}^T + U_7 B, \\
 \phi_{8,8} &= P + \tau R_3 + h R_4 - N_8 - N_8^T, \\
 \phi_{8,9} &= -N_9^T + V_8 B, \\
 \phi_{8,10} &= -N_{10}^T + U_8 B, \\
 \phi_{9,9} &= -(1-2h_d)Z_4 + V_9 B + B^T V_9^T + \varepsilon_2 Q_7^T Q_7, \\
 \phi_{9,10} &= U_9 B + B^T V_{10}^T, \\
 \phi_{10,10} &= -(1-\tau_d - h_d)Z_5 + U_{10} B + B^T U_{10}^T + \varepsilon_2 Q_8^T Q_8.
 \end{aligned}$$

Proof: Using Lemma 1 (Schur complement), $\Pi < 0$ implies that

$$\begin{bmatrix}
 \bar{\Pi}_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} \\
 * & \Pi_{22} & 0 & 0 & 0 \\
 * & * & \Pi_{33} & 0 & 0 \\
 * & * & * & \Pi_{44} & 0 \\
 * & * & * & * & \Pi_{55}
 \end{bmatrix} \tag{16}$$

$$+\varepsilon_1^{-1} \Gamma_1 \Gamma_1^T + \varepsilon_1 \vartheta_1^T \vartheta_1 + \varepsilon_2^{-1} \Gamma_2 \Gamma_2^T + \varepsilon_2 \vartheta_2^T \vartheta_2 < 0,$$

where

$$\begin{aligned}
 \bar{\Pi}_{11} &= [\bar{\phi}_{i,j}]_{10 \times 10} = [\phi_{i,j}]_{10 \times 10}, i, j = 1, 2, \dots, 10, \\
 (i, j) &\neq (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (9, 9), (10, 10), \\
 \Gamma_1 &= [M_i H]_{10 \times 1}, \\
 \vartheta_1 &= [-Q_1 \ 0 \ Q_2 \ Q_3 \ 0 \ Q_4 \ 0 \ 0 \ 0 \ 0], \\
 \Gamma_2 &= [N_i H]_{10 \times 1}, \\
 \vartheta_2 &= [0 \ Q_5 \ 0 \ 0 \ Q_6 \ 0 \ 0 \ 0 \ Q_7 \ Q_8],
 \end{aligned}$$

$$\Gamma_1 = [M_i H]_{10 \times 1},$$

$$\vartheta_1 = [-Q_1 \ 0 \ Q_2 \ Q_3 \ 0 \ Q_4 \ 0 \ 0 \ 0 \ 0],$$

$$\Gamma_2 = [N_i H]_{10 \times 1},$$

$$\vartheta_2 = [0 \ Q_5 \ 0 \ 0 \ Q_6 \ 0 \ 0 \ 0 \ Q_7 \ Q_8],$$

with

$$\begin{aligned}
 \bar{\phi}_{1,1} &= -PC - CP^T + Z_1 + Z_3 + Z_4 + Z_5 - 2L_1 T_1 - M_1 C \\
 &\quad - CM_1^T + N_1 H_0 + H_0^T N_1^T + U_1 + U_1^T + V_1 + V_1^T, \\
 \bar{\phi}_{2,2} &= -(1-\tau_d)Q_2 + L_1 T_2 + N_2 H_1 + H_1^T N_2^T - U_2 B \\
 &\quad - B^T U_2^T - U_2 - U_2^T, \\
 \bar{\phi}_{3,3} &= Q_2 + E - 2T_1 + M_3 W_1 + W_1^T M_3^T, \\
 \bar{\phi}_{4,4} &= -(1-\tau_d)Q_2 - 2T_2 + M_4 W_2 + W_2^T M_4^T, \\
 \bar{\phi}_{5,5} &= -E + AZ_1 A^T + M_5 A + A^T M_5^T, \\
 \bar{\phi}_{6,6} &= -(1-h_d)Z_3 + N_6 H_2 + H_2^T N_6^T - V_6 B \\
 &\quad - B^T V_6^T - V_6 - V_6^T, \\
 \bar{\phi}_{9,9} &= -(1-2h_d)Z_4 + V_9 B + B^T V_9^T, \\
 \bar{\phi}_{10,10} &= -Q_3(1-\tau_d - h_d) + U_{10} B + B^T U_{10}^T.
 \end{aligned}$$

Then, noting that (7), by using Lemma 4, one can get

$$\begin{bmatrix}
 \Delta \Pi_{11} & 0 & 0 & 0 & 0 \\
 * & 0 & 0 & 0 & 0 \\
 * & * & 0 & 0 & 0 \\
 * & * & * & 0 & 0 \\
 * & * & * & * & 0
 \end{bmatrix}$$

$$\begin{aligned}
 &= \Gamma_1 F(t) \vartheta_1 + \vartheta_1^T F^T(t) \Gamma_1^T + \Gamma_2 F(t) \vartheta_2 + \vartheta_2^T F^T(t) \Gamma_2^T \tag{17} \\
 &\leq \varepsilon_1^{-1} \Gamma_1 \Gamma_1^T + \varepsilon_1 \vartheta_1^T \vartheta_1 + \varepsilon_2^{-1} \Gamma_2 \Gamma_2^T + \varepsilon_2 \vartheta_2^T \vartheta_2 < 0,
 \end{aligned}$$

Therefore, one has

$$\Upsilon = \begin{bmatrix}
 \bar{\Pi}_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} \\
 * & \Pi_{22} & 0 & 0 & 0 \\
 * & * & \Pi_{33} & 0 & 0 \\
 * & * & * & \Pi_{44} & 0 \\
 * & * & * & * & \Pi_{55}
 \end{bmatrix} < 0. \tag{18}$$

Consider the following lyapunov-krasoskill functional for system (6) as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{19}$$

where

$$V_1(t) = [x(t) - Bx(t-h(t))]^T P [x(t) - Bx(t-h(t))],$$

$$\begin{aligned}
 V_2(t) &= \int_{t-\tau(t)}^t x^T(s) Z_1 x(s) ds + \int_{t-\tau(t)}^t f^T(x(s)) Z_2 f(x(s)) ds \\
 &\quad + \int_{t-h(t)}^t x^T(s) Z_3 x(s) ds + \int_{t-2h(t)}^t x^T(s) Z_4 x(s) ds \\
 &\quad + \int_{t-\tau(t)-h(t)}^t x^T(s) Z_5 x(s) ds,
 \end{aligned}$$

$$\begin{aligned}
 V_3(t) &= \int_{t-\tau(t)}^t [s - (t - \tau(t))] \rho_1^T(s) R_1 \rho_1(s) ds \\
 &\quad + \int_{t-h(t)}^t [s - (t - h(t))] \rho_1^T(s) R_2 \rho_1(s) ds \\
 &\quad + \int_{t-\tau(t)}^t [s - (t - \tau(t))] \rho_2^T(s) R_3 \rho_2(s) d\omega(s) \\
 &\quad + \int_{t-h(t)}^t [s - (t - h(t))] \rho_2^T(s) R_4 \rho_2(s) d\omega(s),
 \end{aligned}$$

$$V_4(t) = \sum_{i=0}^n e_i \int_0^t \int_{t-\sigma}^t k_i(\sigma) f_i^2(x_i(s)) ds d\sigma.$$

By it $\hat{\circ}$'s differential formula, the stochastic derivative of $V(t)$ along the trajectory of system (6) is

$$dV(t) = \{LV(t)\}dt + \left\{ [x(t) - Bx(t-h(t))]^T \right. \\ \left. \times P\rho_2(t) \right\} d\omega(t), \tag{20}$$

where

$$LV_1(t) = 2[x(t) - Bx(t-h(t))]^T P[-Cx(t) + W_1f(x(t)) \\ + W_2f(x(t-\tau(t))) + A \int_{-\infty}^t K(t-s)f(x(s))ds] \\ + \rho_2^T(t)P\rho_2(t), \tag{21}$$

$$LV_2(t) = x^T(t)Z_1x(t) - (1-\dot{\tau}(t))x^T(t-\tau(t))Z_1x(t-\tau(t)) \\ + f^T(x(t))Z_2f(x(t)) - (1-\dot{\tau}(t))f^T(x(t-\tau(t))) \\ \times Z_2f(x(t-\tau(t))) + x^T(t)Z_3x(t) \\ - (1-\dot{h}(t))x^T(t-h(t))Z_3x(t-h(t)) \\ + x^T(t)Z_4x(t) - (1-2\dot{h}(t))x^T(t-2h(t))Z_4 \\ \times x(t-2h(t)) + x^T(t)Z_5x(t) - (1-\dot{\tau}(t)-\dot{h}(t)) \\ \times x^T(t-\tau(t)-h(t))Z_5x(t-\tau(t)-h(t)) \\ \leq x^T(t)Z_1x(t) - (1-\tau_d)x^T(t-\tau(t))Z_1x(t-\tau(t)) \\ + f^T(x(t))Z_2f(x(t)) - (1-\tau_d)f^T(x(t-\tau(t))) \\ \times Z_2f(x(t-\tau(t))) + x^T(t)Z_3x(t) \\ - (1-\dot{h}(t))x^T(t-h(t))Z_3x(t-h(t)) \\ + x^T(t)Z_4x(t) - (1-2h_d)x^T(t-2h(t))Z_4 \\ \times x(t-2h(t)) + x^T(t)Z_5x(t) - (1-\tau_d-h_d) \\ \times x^T(t-\tau(t)-h(t))Z_5x(t-\tau(t)-h(t)), \tag{22}$$

$$LV_3(t) = \tau(t)\rho_1^T(t)R_1\rho_1(t) - (1-\dot{\tau}(t)) \int_{t-\tau(t)}^t \rho_1^T(s)R_1 \\ \times \rho_1(s)ds + h(t)\rho_1^T(t)R_2\rho_1(t) \\ - (1-\dot{h}(t)) \int_{t-h(t)}^t \rho_1^T(s)R_2\rho_1(s)ds \\ + \tau(t)\rho_2^T(t)R_3\rho_2(t) \\ - (1-\dot{\tau}(t)) \int_{t-\tau(t)}^t \rho_2^T(s)R_3\rho_2(s)d\omega(s) \\ + h(t)\rho_2^T(t)R_4\rho_2(t) \\ - (1-\dot{h}(t)) \int_{t-h(t)}^t \rho_2^T(s)R_4\rho_2(s)d\omega(s) \\ \leq \tau\rho_1^T(t)R_1\rho_1(t) - (1-\tau_d) \int_{t-\tau(t)}^t \rho_1^T(s)R_1 \\ \times \rho_1(s)ds + h\rho_1^T(t)R_2\rho_1(t) \\ - (1-h_d) \int_{t-h(t)}^t \rho_1^T(s)R_2\rho_1(s)ds \\ + \tau\rho_2^T(t)R_3\rho_2(t) \\ - (1-\tau_d) \int_{t-\tau(t)}^t \rho_2^T(s)R_3\rho_2(s)d\omega(s) \\ + h\rho_2^T(t)R_4\rho_2(t) \\ - (1-h_d) \int_{t-h(t)}^t \rho_2^T(s)R_4\rho_2(s)d\omega(s), \tag{23}$$

$$LV_4(t) = \sum_{j=1}^n e_j \int_0^\infty k_j(\delta)f_j^2(x_i(t))d\delta \\ - \sum_{j=1}^n e_j \int_0^\infty k_j(\delta)f_j^2(x_j(t-\delta))d\delta \\ = f^T(x(t))Ef(x(t)) \\ - \sum_{j=1}^n e_j \int_0^\infty k_j(\delta)d\delta \int_0^\infty k_j(\delta)f_j^2(x_j(t-\delta))d\delta \\ \leq f^T(x(t))Ef(x(t)) \\ - \sum_{j=1}^n e_j \left(\int_0^\infty k_j(\delta)f(x_j(t-\delta))d\delta \right)^2 \\ \leq f^T(x(t))Ef(x(t)) - \left[\int_{-\infty}^t K(t-s)f(x(s))ds \right]^T \\ \times E \left[\int_{-\infty}^t K(t-s)f(x(s))ds \right]. \tag{24}$$

By using Lemma 2, one can get

$$-(1-\tau_d) \int_{t-\tau(t)}^t \rho_1^T(s)R_1\rho_1(s)ds \\ \leq -(1-\tau_d)\tau^{-1} \left[\int_{t-\tau(t)}^t \rho_1(s)ds \right]^T R_1 \left[\int_{t-\tau(t)}^t \rho_1(s)ds \right], \tag{25}$$

$$-(1-h_d) \int_{t-h(t)}^t \rho_1^T(s)R_2\rho_1(s)ds \\ \leq -(1-h_d)h^{-1} \left[\int_{t-\tau(t)}^t \rho_1(s)ds \right]^T R_2 \left[\int_{t-\tau(t)}^t \rho_1(s)ds \right], \tag{26}$$

From (5), one can know that

$$\left[f_i(x_i(t)) - l_i^- x_i(t) \right] \left[f_i(x_i(t)) - l_i^+ x_i(t) \right] \leq 0, \\ \left[f_i(x_i(t-\tau(t))) - l_i^- x_i(t-\tau(t)) \right] \\ \times \left[f_i(x_i(t-\tau(t))) - l_i^+ x_i(t-\tau(t)) \right] \leq 0.$$

Then, for any $T_j = \text{diag}\{t_{1j}, t_{2j}, \dots, t_{mj}\} \geq 0, j = 1, 2$, it follows that

$$0 \leq -2 \sum_{i=1}^n t_{i1} \left[f_i(x_i(t)) - l_i^- x_i(t) \right] \left[f_i(x_i(t)) - l_i^+ x_i(t) \right] \\ = -2f^T(x(t))T_1f(x(t)) + 2x^T(t)L_2T_1f(x(t)) \\ - 2x^T(t)L_1T_1x(t), \tag{27}$$

$$0 \leq -2 \sum_{i=1}^n t_{i2} \left[f_i(x_i(t-\tau(t))) - l_i^- x_i(t-\tau(t)) \right] \\ \times \left[f_i(x_i(t-\tau(t))) - l_i^+ x_i(t-\tau(t)) \right] \\ = -2f^T(x(t-\tau(t)))T_2f(x(t-\tau(t))) \\ + 2x^T(t-\tau(t))L_2T_2f(x(t-\tau(t))) \\ - 2x^T(t-\tau(t))L_1T_2x(t-\tau(t)), \tag{28}$$

where

$$L_1 = \text{diag}\{l_1^-, l_2^-, \dots, l_n^-\},$$

$$L_2 = \text{diag}\{l_1^+, l_2^+, \dots, l_n^+\}.$$

From (8), (9) and the Newton-Leibniz formula, the following equations hold

$$0 = 2\xi^T(t)M[-Cx(t) + W_1f(x(t)) + W_2f(x(t-\tau(t))) \\ + A \int_{-\infty}^t K(t-s)f(x(s))ds - \rho_1(t)], \tag{29}$$

$$0 = 2\xi^T(t)N \times [H_0x(t) + H_1x(t - \tau(t)) + H_2x(t - h(t)) - \rho_2(t)], \quad (30)$$

$$0 = 2\xi^T(t)U [x(t) - Bx(t - h(t)) - x(t - \tau(t)) + Bx(t - \tau(t) - h(t)) - \int_{t-\tau(t)}^t \rho_1(s)ds - \int_{t-\tau(t)}^t \rho_2(s)d\omega(s)], \quad (31)$$

$$0 = 2\xi^T(t)V [x(t) - Bx(t - h(t)) - x(t - h(t)) + Bx(t - 2h(t)) - \int_{t-h(t)}^t \rho_1(s)ds - \int_{t-h(t)}^t \rho_2(s)d\omega(s)], \quad (32)$$

where

$$\xi^T(t) = [x^T(t), x^T(t - \tau(t)), f^T(x(t)), f^T(x(t - \tau(t))), \left(\int_{-\infty}^t K(t-s)f(x(s))ds\right)^T, x^T(t - h(t)), \rho_1^T(t), \rho_2^T(t), x^T(t - 2h(t)), x^T(t - \tau(t) - h(t))],$$

From Lemmas 2 and 3, the following inequalities can be obtained

$$-2\xi^T(t)U \int_{t-\tau(t)}^t \rho_1(s)ds \leq (1 - \tau_d)^{-1} \xi^T(t)U \tau R_1^{-1}U^T \xi(t) + (1 - \tau_d)\tau^{-1} \left[\int_{t-\tau(t)}^t \rho_1(s)ds\right]^T R_1 \left[\int_{t-\tau(t)}^t \rho_1(s)ds\right], \quad (33)$$

$$-2\xi^T(t)V \int_{t-h(t)}^t \rho_1(s)ds \leq (1 - \tau_d)^{-1} \xi^T(t)V h R_2^{-1}V^T \xi(t) + (1 - h_d)h^{-1} \left[\int_{t-h(t)}^t \rho_1(s)ds\right]^T R_2 \left[\int_{t-h(t)}^t \rho_1(s)ds\right], \quad (34)$$

$$-2\xi^T(t)U \int_{t-\tau(t)}^t \rho_2(s)d\omega(s) \leq (1 - \tau_d)^{-1} \xi^T(t)U R_3^{-1}U^T \xi(t) + (1 - \tau_d) \left[\int_{t-\tau(t)}^t \rho_2(s)d\omega(s)\right]^T R_3 \left[\int_{t-\tau(t)}^t \rho_2(s)d\omega(s)\right], \quad (35)$$

$$-2\xi^T(t)V \int_{t-h(t)}^t \rho_2(s)d\omega(s) \leq (1 - \tau_d)^{-1} \xi^T(t)V R_4^{-1}V^T \xi(t) + (1 - h_d) \left[\int_{t-h(t)}^t \rho_2(s)d\omega(s)\right]^T R_4 \left[\int_{t-h(t)}^t \rho_2(s)d\omega(s)\right]. \quad (36)$$

Then, combining (21)-(36) and using (20), one can obtain that

$$dV(t) \leq \{\xi^T(t)Y\xi(t) + \Pi\} dt + [x(t) - Bx(t - h(t))]P\rho_2(t)d\omega(t), \quad (37)$$

where Y is given in (15) and

$$\xi^T(t) = [x^T(t), x^T(t - \tau(t)), f^T(x(t)), f^T(x(t - \tau(t))), \left(\int_{-\infty}^t K(t-s)f(x(s))ds\right)^T, x^T(t - h(t)), \rho_1^T(t), \rho_2^T(t), x^T(t - 2h(t)), x^T(t - \tau(t) - h(t))],$$

$$\begin{aligned} \Pi = & -(1 - \tau_d) \int_{t-\tau(t)}^t \rho_2^T(s)R_3\rho_2(s)d\omega(s) \\ & + (1 - \tau_d) \left[\int_{t-\tau(t)}^t \rho_2(s)d\omega(s)\right]^T R_3 \left[\int_{t-\tau(t)}^t \rho_2(s)d\omega(s)\right] \\ & - (1 - h_d) \int_{t-h(t)}^t \rho_2^T(s)R_4\rho_2(s)d\omega(s) \\ & + (1 - h_d) \left[\int_{t-h(t)}^t \rho_2(s)d\omega(s)\right]^T R_4 \left[\int_{t-h(t)}^t \rho_2(s)d\omega(s)\right]. \end{aligned}$$

It is obvious that for $\Upsilon < 0$, and there exists a scalar $\delta > 0$ such that

$$\Upsilon + \text{diag}\{\delta I, 0, 0, 0, 0, 0, 0, 0, 0\} < 0, \quad (38)$$

By isometry, we can obtain

$$\begin{aligned} & E\left\{(1 - \tau_d) \int_{t-\tau(t)}^t \rho_2^T(s)R_3\rho_2(s)d\omega(s)\right\} \\ & = \left\{(1 - \tau_d) \int_{t-\tau(t)}^t E(\rho_2^T(s)R_3\rho_2(s))d\omega(s)\right\} \\ & = E\left\{(1 - \tau_d) \left[\int_{t-\tau(t)}^t \rho_2(s)d\omega(s)\right]^T R_3 \left[\int_{t-\tau(t)}^t \rho_2(s)d\omega(s)\right]\right\}, \\ & E\left\{(1 - h_d) \int_{t-h(t)}^t \rho_2^T(s)R_4\rho_2(s)d\omega(s)\right\} \\ & = \left\{(1 - h_d) \int_{t-h(t)}^t E(\rho_2^T(s)R_4\rho_2(s))d\omega(s)\right\} \\ & = E\left\{(1 - h_d) \left[\int_{t-h(t)}^t \rho_2(s)d\omega(s)\right]^T R_4 \left[\int_{t-h(t)}^t \rho_2(s)d\omega(s)\right]\right\}. \end{aligned}$$

Tacking the mathematical expectation of both sides of (37) and considering (38), there exists

$$\frac{dEV(x(t), t)}{dt} \leq E\{\xi^T(t)Y\xi(t)\} \leq -\delta E\|x(t)\|^2, \quad (39)$$

which indicates from Lyapunov stability theory that system (6) is globally robustly asymptotically stable in the mean square. This completes the proof.

Remark 1: The criterion given in Theorem 1 is delay-dependent. It is well known that the delay-dependent criteria are less conservative than delay-independent criteria when the time delay is small.

Remark 2: Note that the condition (15) is given as a LMI. Therefore, by using the MATLAB LMI Toolbox, it is straightforward to check the feasibility of (15) without turning any parameters.

IV. NUMERICAL EXAMPLES

To illustrate the effectiveness of the theory developed in this paper, two numerical examples are proposed as follows.

Example 1. Consider a two-neuron uncertain stochastic delayed neural network of neutral-type (6) with the following parameters

$$C = \begin{bmatrix} 13 & 0 \\ 0 & 13 \end{bmatrix}, W_1 = \begin{bmatrix} 2.1 & -1.10 \\ -1.1 & 3.2 \end{bmatrix}, W_2 = \begin{bmatrix} -0.1 & -1 \\ 0.15 & 0.07 \end{bmatrix},$$

$$A = \begin{bmatrix} 0.32 & 0.02 \\ -0.17 & 0.24 \end{bmatrix}, B = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.3 \end{bmatrix},$$

$$Q_1 = Q_2 = Q_3 = Q_4 = 0.3I, Q_5 = Q_6 = Q_7 = Q_8 = I, H = I, H_0 = H_1 = H_2 = 0.5I, l_1^- = 0.2, l_1^+ = 0.7, l_2^- = 0.2, l_2^+ = 0.7,$$

$$\tau(t) = h(t) = 0.3 - 0.3 \sin(t).$$

One can assume that $\tau = 0.6, \tau_d = 0.3, h = 0.6, h_d = 0.3, L_1 = 0.14, L_2 = 0.9I$. Now, solving the LMI (15) in Theorem 1, by using Matlab LMI Control toolbox, one can find that system (6) described by Example 1 is globally robustly stochastically asymptotically stable in the mean square. Then, one gets a part of the feasible solution as follows:

$$P = 10^3 \times \begin{bmatrix} 6.7514 & -0.5402 \\ -0.5402 & 6.7514 \end{bmatrix},$$

$$Z_1 = 10^4 \times \begin{bmatrix} 7.4017 & -1.1079 \\ -1.1079 & 8.2406 \end{bmatrix},$$

$$Z_2 = 10^5 \times \begin{bmatrix} 2.0169 & 0.9377 \\ 0.9377 & 2.2579 \end{bmatrix},$$

$$Z_3 = 10^4 \times \begin{bmatrix} 5.6052 & -0.0161 \\ -0.0161 & 5.4425 \end{bmatrix},$$

$$Z_4 = 10^4 \times \begin{bmatrix} 7.8647 & 0.0850 \\ 0.0850 & 7.5987 \end{bmatrix},$$

$$Z_5 = 10^4 \times \begin{bmatrix} 7.8723 & 0.0667 \\ 0.0667 & 7.6868 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 204.7661 & 57.0021 \\ 57.0021 & 161.5664 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 209.0609 & 60.0534 \\ 60.0534 & 165.1786 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 253.7132 & 65.3119 \\ 65.3119 & 203.9602 \end{bmatrix},$$

$$R_4 = \begin{bmatrix} 260.9183 & 70.4665 \\ 70.4665 & 210.0610 \end{bmatrix},$$

$$E = 10^5 \times \text{diag}\{1.6206 \ 1.6206\},$$

$$T_1 = 10^6 \times \text{diag}\{2.1220 \ 2.1220\},$$

$$T_2 = 10^4 \times \text{diag}\{5.7895 \ 5.7895\},$$

$$\varepsilon_1 = 1.3698e+005, \varepsilon_2 = 1.0110e+004.$$

Example 2. Consider the following three-neuron uncertain stochastic delayed neural network of neutral-type given in (6) with the following parameters

$$C = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 40 \end{bmatrix}, W_1 = \begin{bmatrix} 2 & -0.42 & -0.31 \\ -0.12 & -0.81 & -0.17 \\ 0.27 & 0.92 & -0.30 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 1.71 & 0.10 & -0.50 \\ 0.25 & 1.92 & 1.12 \\ -0.10 & 0.65 & 1.2 \end{bmatrix}, A = \begin{bmatrix} -0.7 & 0.63 & -0.89 \\ -0.1 & 2.13 & 1.12 \\ 1.11 & 0.63 & 1.77 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}, H_0 = H_1 = 0.5I, H_2 = I,$$

$$Q_1 = Q_2 = Q_3 = Q_4 = 0.5I, Q_5 = Q_6 = Q_7 = Q_8 = I,$$

$$l_1^- = 0.1, l_1^+ = 0.9, H = I,$$

$$l_2^- = 0.1, l_2^+ = 0.9, l_3^- = 0.1, l_3^+ = 0.9.$$

Let $\tau = 0.3, \tau_d = 0.1, h = 0.3, h_d = 0.1, L_1 = 0.09I, L_2 = I$. By applying Theorem 1, there exists a feasible solution which guarantees the globally robustly stochastically asymptotically of system (6). Limited the length of the paper, one only show a part of the feasible solution as follows:

$$P = 10^3 \times \begin{bmatrix} 3.6988 & -0.0212 & -0.0462 \\ -0.0212 & 1.4745 & 0.0278 \\ -0.0462 & 0.0278 & 0.8878 \end{bmatrix},$$

$$Z_1 = 10^4 \times \begin{bmatrix} 3.3174 & 0.0248 & -0.0588 \\ 0.0248 & 3.0363 & 0.0046 \\ -0.0588 & 0.0046 & 3.1449 \end{bmatrix},$$

$$Z_2 = 10^5 \times \begin{bmatrix} 2.7981 & 0.1107 & -0.2359 \\ 0.1107 & 3.5955 & 0.0443 \\ -0.2359 & 0.0443 & 3.9982 \end{bmatrix},$$

$$Z_3 = 10^4 \times \begin{bmatrix} 3.7978 & -0.0094 & 0.0107 \\ -0.0094 & 3.3512 & -0.0121 \\ 0.0107 & -0.0121 & 3.4363 \end{bmatrix},$$

$$Z_4 = 10^4 \times \begin{bmatrix} 2.6961 & 0.0175 & 0.0161 \\ 0.0175 & 3.2382 & -0.0235 \\ 0.0161 & -0.0235 & 3.4106 \end{bmatrix},$$

$$Z_5 = 10^4 \times \begin{bmatrix} 2.7107 & 0.0178 & 0.0134 \\ 0.0178 & 3.2393 & -0.0236 \\ 0.0134 & -0.0236 & 3.4114 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 117.0222 & 0.5465 & 2.7203 \\ 0.5465 & 93.9421 & -0.7805 \\ 2.7203 & -0.7805 & 54.0367 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 116.7599 & 0.4887 & 2.6319 \\ 0.4887 & 94.0604 & -0.8046 \\ 2.6319 & -0.8046 & 54.0288 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 130.5592 & 0.8793 & 2.9740 \\ 0.8793 & 99.4150 & -0.9430 \\ 2.9740 & -0.9430 & 53.3031 \end{bmatrix},$$

$$R_4 = \begin{bmatrix} 130.9855 & 0.7539 & 2.9546 \\ 0.7539 & 99.7877 & -0.9392 \\ 2.9546 & -0.9392 & 53.3622 \end{bmatrix},$$

$$E = 10^5 \times \text{diag}\{2.5062 \ 2.5062 \ 2.5062\},$$

$$T_1 = 10^6 \times \text{diag}\{2.0088 \ 2.0088 \ 2.0088\},$$

$$T_2 = 10^4 \times \text{diag}\{5.1283 \ 5.1283 \ 5.1283\},$$

$$\varepsilon_1 = 5.0687e+004, \varepsilon_2 = 6.3903e+003.$$

V. CONCLUSIONS

A novel delay-dependent global robust stability criterion for neural networks of neutral-type with discrete and distributed has been provided. The new sufficient criterion has been presented in terms of LMI. The result is obtained based on the Lyapunov-Krasovskii method in combination with the free-weighting matrices technique. The validity of the proposed approach has been demonstrated by numerical examples.

ACKNOWLEDGMENT

This work is supported by the Fundamental Research Funds for the Central Universities (No. CDJXS11172237).

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