A Novel Model for the Optimization of Interplanetary Trajectory Using Evolutionary Algorithm

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Abstract—A novel model, called evolutionary patched model, based on the patched conic approximation is applied to the optimization of space missions with engineering constraints in this paper. The interplanetary trajectory consists of geocentric escape orbit, heliocentric transfer orbit and target capture orbit. The model, firstly, gets the escape orbit and capture orbit by optimizing the elements of orbits with evolutionary algorithm, and then calculate the heliocentric transfer orbit by solving the Lambert’s problem, which is just opposite to the procedure of patched conic method. A module based on orthogonal test is introduced in the initialization process to produce a good starting population for the evolutionary algorithm. Earth-Mars mission has been considered. The results show that the good performance obtained with this model and the improvement in terms of efficiency and computational cost.

Index Terms—patched conic approximation, interplanetary trajectory, optimization model, evolutionary algorithm

I. INTRODUCTION

A very important problem in mission analysis is the planning of interplanetary trajectories, which consists in launching a spacecraft from a given astronomical body (usually the Earth) along a trajectory which leads to some other astronomical body. The aim of the mission can be to land on the body or to put the spacecraft into the planet’s orbit and the objective of a model is to help trajectory planners in taking the best decision, e.g., on the starting date and other relevant parameters, in order to obtain a “low-cost” mission.

As is known to all, a Hohmann transfer is a particular type of minimum energy transfer orbit for an interplanetary mission. In fact, the Hohmann transfer requires a minimum initial fuel in order to reach the foreign planet. The Hohmann is commonly used to transfer from one circular orbit to another. Thus, it is an attractive option for designing future missions from Earth to Mars. But the Hohmann transfer requires a change in true anomaly of 180 degrees and only provides a rough estimate of how to reach Mars.

The patched-conic approximation is a more accurate solution to interplanetary transfer description [1], [2]. It involves partitioning the overall transfer into several two-body problems. This approximation provides a much better understanding of the relation between the departure orbit and the overall transfer than the analytic Hohmann solution. However, the patched-conic approximation is still limited in that the engineering constrains of escape orbit and capture orbit are not fully taken into account.

We desire a higher fidelity method for estimating the required initial fuel and braking fuel. In addition, we seek a method which allows us to find a minimum energy transfer orbit for an interplanetary mission from Earth to Mars.

In the field of space trajectory optimization, evolutionary algorithms (EAs) were widely applied to solve these problems [3]-[7]. These studies usually agree in stating that EAs are suitable means for the optimization of this kind of missions, but they are almost only consider the minimum total fuels as the optimization objective without engineering constraints. It is not convenient for trajectory planners to design a particular trajectory that meets the engineering requirements.

A novel model, we call it evolutionary patched model, was introduced in the present paper. In this model, we consider the engineering constrains as the constraints conditions, and get the escape orbit and capture orbit by optimizing the elements of orbits with evolutionary algorithm, then calculate the heliocentric transfer orbit. Finally, three orbits are patched if they meet some prescribed conditions.

II. PATCHED CONIC APPROXIMATION

The patched conic approximation offers an efficient method for describing interplanetary trajectories [1]. By partitioning the overall orbits into a series of two-body orbits, as shown in Fig. 1, it greatly simplifies mission analysis. For each of the portions of an orbit, one
The gravitational force is assumed to be acting upon the spacecraft at a time.

\[ \Delta v = f(t_0, t) \]

Equation (1) is a simple problem without any other constraints. It is easy to solve by Global Optimization method, but it is not practical for the trajectory planners to design an expected mission orbit, as mentioned above.

**III. EVOLUTIONARY PATCHED MODEL**

Although the patched conic method gives a good approximation of trajectories for interplanetary spacecraft missions, there are some defects for trajectory planners to design a mission orbit, especially for optimization. Because the escape orbit and capture orbit are obtained from hyperbolic excess velocities, in other words, it is not easy to design an orbit which meets the engineering constraints, such as argument of perigee of escape orbit must be within a certain interval and so on. In most cases, we need do repetitive computation again and again to find a suitable orbit.

For these reasons, evolutionary patched model is proposed to overcome these defects. As shown in Figure 1, if we have known that altitude of perigee, \( v_\text{eV} \), velocity of perigee, \( v_\text{ep} \), inclination, \( i_\text{e} \), argument of perigee, \( \omega_\text{e} \) and right ascension of satellite node, \( \Omega_\text{e} \), then

\[
{\bar{R}}_\text{ep} = R_\text{e}(-\Omega_\text{e})R_\text{e}(-i_\text{e})R_\text{e}(-\omega_\text{e})[R_\text{e} + r_\text{ep}, 0, 0]^T
\]

\[
{\bar{V}}_\text{ep} = R_\text{e}(-\Omega_\text{e})R_\text{e}(-i_\text{e})R_\text{e}(-\omega_\text{e})[0, v_\text{ep}, 0]^T
\]

Where the vectors \( {\bar{R}}_\text{ep} \) and \( {\bar{V}}_\text{ep} \) are the position and velocity of spacecraft in the equator reference frame centered into the Earth respectively. \( R_\text{e} \) is the mean radius of Earth, and \( T \) indicates matrix transposition.

\[
\sigma = \sigma(\bar{R}, \bar{V}, t)
\]

\[
\sigma \Leftrightarrow (a, e, i, \omega, \Omega, M)
\]

Consequently, we can determine the escape orbit and get the time \( t_\text{e\infty} \) MJD2000, the position (\( \bar{R}_\text{e\infty} \)) and velocity (\( \bar{V}_\text{e\infty} \)) at the bound of Earth’s sphere of influence according to the time of perigee passing, \( \tau_\text{e} \) MJD2000.

\[
{\bar{R}} = {\bar{R}}(\sigma, t)
\]

\[
{\bar{V}} = {\bar{V}}(\sigma, t)
\]

Then, we can calculate \( \bar{R}_1 \) and \( \bar{V}_1 \), which are the spacecraft’s position and velocity in the equator reference frame centered into the Sun at the time \( t_\text{e\infty} \) MJD2000, by coordinate conversion:

\[
\bar{R}_1 = \bar{R}_\text{e\infty} + \bar{R}_\text{E}
\]

\[
\bar{V}_1 = \bar{V}_\text{e\infty} + \bar{V}_\text{E}
\]

Where the vectors \( \bar{R}_\text{E} \) and \( \bar{V}_\text{E} \) are the position and velocity of Earth in the equator reference frame centered into the Sun respectively.

We can deal with capture orbit in much the same way as escape orbit, and get the time \( t_\text{m\infty} \) MJD2000, the position(\( \bar{R}_\text{m\infty} \)) and velocity(\( \bar{V}_\text{m\infty} \)) at the bound of Mars’s sphere of influence, also \( \bar{R}_2 \) and \( \bar{V}_2 \), which are the spacecraft’s position and velocity in the equator reference frame centered into the Sun at the time \( t_\text{m\infty} \) MJD2000.

Finally, we get the heliocentric transfer orbit by solving the Lambert’s problem according to \( \bar{R}_1 \), \( \bar{R}_2 \) and \( t_\text{m\infty} - t_\text{e\infty} \). There are two another velocities got from the solution of Lambert’s problem, initial velocity \( \bar{V}_{1j} \) and end velocity \( \bar{V}_{2j} \) of transfer orbit. If \( \left| \bar{V}_{1j} - \bar{V}_1 \right| < \varepsilon_1 \)

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and \[|\vec{V}_1 - \vec{V}_2'| < \varepsilon_2\], then we consider that these three orbits can be patched, otherwise not.

So, the total fuel is a function of these variables mentioned above and can be expressed as follows (defined by our):

\[
\Delta v = f(r_{ep}, v_{ep}, t_e, \Omega_e, \epsilon_e, t_{mp}, v_{mp}, t_m, \Omega_m, \epsilon_m) \tag{6}
\]

In fact, some variables are constant in certain practical engineering. And, the general problem is also to minimize \(\Delta v\), namely the function \(f\):

\[
\min \Delta v = \min f \tag{7}
\]

subjects to the conditions:

\[
\begin{align*}
    t_e & \in [t_{e1}, t_{e2}] \text{ MJD2000} \\
    t_m & \in [t_{m1}, t_{m2}] \text{ MJD2000} \\
    v_{ep} & > \sqrt{2\mu_e / (R_e + r_{ep})} \\
    v_{mp} & > \sqrt{2\mu_m / (R_m + r_{mp})} \\
    |\vec{V}_1 - \vec{V}_2'| & < \varepsilon_1 \\
    |\vec{V}_2 - \vec{V}_2'| & < \varepsilon_2 \\
\end{align*}
\]

Where \(\mu_e\) and \(\mu_m\) are the gravitational constants of Earth and Mars respectively. \(R_e\) and \(R_m\) are the mean radius of Earth and Mars respectively. And, “...” implies that we can add any other constraints into this model according to the requirements arbitrarily. In this paper, for example, we add argument of perigee of escape orbit as a constraint.

A lot of global optimization techniques can be applied to this model. But (4) is a multi-dimensional function and does not exist analytical expressions, EAs are very suitable for solving it.

An EA is a population-based meta heuristic optimization algorithm which borrows its mechanisms from biological evolution: reproduction, mutation, recombination, natural selection and survival of the fittest. Candidate solutions to the optimization problem play the role of individuals in a population, and a cost or merit function determines how the individual is suited for the environment. Evolution of the population takes place with the repeated application of prescribed operators.

The present paper deals with impulsive space trajectories. Each trajectory is defined by the real values of a fixed number of variables, which completely determine the trajectory (e.g., relevant dates and corresponding positions, magnitude and orientation of each impulse). The variables can assume values in user-specified ranges. The mission is the function to be minimized.

Differential Evolution (DE) [9], [10] is a method of mathematical optimization of multidimensional multimodal (i.e. exhibiting more than one minimum) functions and belongs to the class of Evolution Strategy optimizers. If the resulting individual exhibits a higher fitness than a predetermined population member, in the next generation the new individual replaces the one it was compared to; otherwise, the old individual is retained. This basic principle can be varied, and in fact there are several practical variants of DE, according to Storn [10], six variants, namely DE/best/1, DE/rand/1, DE/rbt/1, DE/best/2 DE/rand/2, and DE/rbt/2, can be used in the developed algorithm.

We use differential evolution (DE) with orthogonal initialization to do experiments in this paper [9], [11], [12]. In the present paper, DE/rand/1 is used. The scaling factor \(F\) is chosen 0.8. The cross-over probability \(CR\) is fixed at 0.9. The flow is summarized as follow:

1. Set values for NP, CR and \(F\)
2. Set \(k = 1\)
3. Initialize \(x_{i,k}\) for all \(i \in \{1, \ldots, \text{NP}\}\) using orthogonal test, \(x = (r_{ep}, v_{ep}, i_{e}, \epsilon_{e}, \Omega_{e}, t_{e}, r_{mp}, v_{mp}, i_{m}, \epsilon_{m}, \Omega_{m}, t_{m})\)
4. Create the vector of random values \(r \in U[0, 1]\) and the mask \(e = r < CR\)
5. For all \(i \in \{1, \ldots, \text{NP}\}\) do
6. Select three individuals \(x_{i,k}, x_{j,k}, x_{i,k+1}\)
7. Create the vector \(v_{i,k+1} = e [x_{3,k} - x_{1,k}] + F (x_{2,k} - x_{1,k})\)
8. If \(f(x_{i,k} + v_{i,k+1}) < f(x_{i,k})\) and (5) is not meet \(x_{i,k+1} = x_{i,k} + v_{i,k+1}\)
9. If \(f(x_{i,k} + v_{i,k+1}) \geq f(x_{i,k})\) or (5) is not meet \(x_{i,k+1} = x_{i,k}\)
10. End for
11. \(k = k + 1\)
12. Termination unless \(k \geq t_{\text{max}}\), goto Step 4

**IV. NUMERICAL RESULTS**

Take bi-impulsive Earth-Mars transfer for example, we expect to find the optimal launch windows in 2018 and 2020, and the time of flight is between 100 and 400. The matrix of departure and arrival dates presented in Figure 2 and 3 comprises the "mission space" for each departure opportunity [13], [14]. The results were got by calculating (1).
Contour map method is used to analyze the energy distribution. X-axis represents for launch date (or departure date) and Y-axis represents for arrival date. The red line refers to initial fuel and blue line refers to braking fuel. We can see that there is only one minimum which can be easily located by almost all of optimizations method from the Fig. 2 and Fig. 3. However, our aim is not only to find the minimum, but also to make it suitable for constraints. It is a challenging work to solve.

We performed two numerical experiments: one trajectory without constraints and another with constraints, $\omega_e \in [170^\circ, 190^\circ]$.

The parameters for DE:

\[ NP = 80, \quad F = 0.8, \quad CR = 0.9, \quad t_{\text{max}} = 500. \]

The planet ephemerides use JPL DE405 ephemerides in the present paper.

Supposed $i_e = 28.5^\circ$, $i_m = 92.6^\circ$, $e_1 = e_2 = 0.1$, the velocity of Earth parking orbit is 10.801445km/s, and the velocity of orbit round Mars is 4.577651km/s.

The results of the experiments are summarized in Table 1-4. Comparison of the results is very close, as proves the evolutionary patched model is effective, but fuels of evolutionary patched model are smaller than that of patched conic approximation. In addition, although the number of variables of evolutionary patched model is greater than that of patched conic approximation, more repetitive runs need to be done in patched conic approximation to find an orbit which meets the constraints for constrained case.
orbits, patched-conic approximations offer a greatly simplified way of analyzing missions. However, they are limited in their ability to fully represent a particular orbit, such as engineering orbit with some constraints.

EAs were widely applied to solve space trajectory optimization problem. EAs are also suitable to solve multi-dimensional function optimization problems. So, evolutionary patched model was proposed to plan a space trajectory with engineering constraints. Earth-Mars transfer was tested using ODE. The results show that evolutionary patched model improves the efficiency and decrease computational cost and is suitable to solve space trajectory optimization problem with engineering constraints. In addition, this model can be used to solve bi-impulsive Earth-NEAs Transfer, and this methodology can also be extended to interplanetary translation.

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