Acquisition Performance Analysis of BOC Signal Considering the Code Search Step Size

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Abstract—The Binary Offset Carrier (BOC) proposed for next generation of Global Navigation Satellite Systems (GNSS) will enhance navigation performance and spectrum compatibility. However, the acquisition process is made more complex, due to the ambiguities in the autocorrelation function. This paper analyzes the performance of two methods allowing to acquire a BOC signal unambiguously: the Binary Phase Shift Keying like (BPSK-like) technique and the Sub Carrier Phase Cancellation (SCPC) technique. In order to measure the acquisition performance, statistical analysis model of these methods using the non-coherent combining scheme is developed. The code search step size effects for these techniques are analyzed and compared with the direct acquisition technique. The effects of the code search step size on acquisition performance for different cases are assessed in terms of probability of detection and probability of false alarm.

Index Terms—acquisition performance; BPSK-like; SCPC; code search step size

I. INTRODUCTION

The Binary Offset Carrier (BOC) signal has been selected for next generation of Global Navigation Satellite Systems (GNSS). A Sin/CosBOC(\(m, n\)) signal is created by a sine/cosine square subcarrier modulation, where the signal is multiplied by a sine/cosine rectangular subcarrier at subcarrier frequency. The parameters are equal to \(m = f_{c} / f_{rc}\), and \(n = f_{c} / f_{sc}\), where \(f_{c}\) is subcarrier frequency, \(f_{c}\) is the Pseudorandom Noise (PRN) code rate and \(f_{ref} = 1.023\)MHz is the reference frequency [1]. The autocorrelation function (ACF) of BOC signal has multiple positive and negative peaks, which induces a complexity in the acquisition phase and a risk of biased measures while tracking [2].

Several techniques can be used to eliminate the ambiguity of the ACF envelop. This paper assesses the performance of two methods allowing to acquire a BOC signal unambiguously: the Binary Phase Shift Keying like (BPSK-like) technique [3], [4], [5], [6] and the Sub Carrier Phase Cancellation (SCPC) technique [6], [7]. The BPSK-like technique only consists in considering the received BOC(\(m, n\)) signal as the sum of two BPSK(\(n\)) signals. The BPSK-like methods approached in the literature can be divided into two categories: Fishman & al. method and Martin & al. method [8]. The main lobes of the signal and of reference BOC-modulated PRN code are selected and then correlated in the former method. However, the latter is less complex, where both the main lobes and secondary lobes between the main lobes of the signal are kept and the reference code is based on the BPSK-modulated PRN code. Therefore, only one filter is needed in Martin & al. method. Moreover, the use of two filters in the Fishman & al. method induces unacceptable error due to filtering effect on the signal, as explained in [6]. In this paper, the BPSK-like method just stands for the Martin & al. method.

The Sub Carrier Phase Cancellation (SCPC) technique also can be used to get rid of the ambiguous ACF envelop, in which the local PRN codes modulated by in-phases subcarrier and quadrature subcarrier correlate with the received signal respectively. The acquisition performance of the SCPC method will be discussed and compared with the BPSK-like method.

The traditional analysis of BOC signal acquisition performance focuses on the Receiver Operating Characteristic (ROC) curves, without considering the effect of the code search step size. However, the code search step size has direct influence on signal acquisition process. The objective of this paper is to analyze the effect of the code search step size on autocorrelation value and acquisition performance.

This paper is organized as follows. First the received Intermediate Frequency (IF) signal model will be introduced. Second theoretical analysis of the BPSK-like method and the SCPC method is presented here. This process is based on a two hypothesis test statistic:

- \(H_{0}\) : absence of input signal
- \(H_{1}\) : presence of input signal

The statistical characteristics of the acquisition output variable using non-coherent combining scheme will be
obtained. Next the effect of code search step size on the autocorrelation value is analyzed for BOC signals. The effects of the code search step size on acquisition performance for different cases are assessed in terms of probability of detection ($P_d$) and probability of false alarm ($P_{fa}$).

II. STATISTICAL ANALYSIS OF ACQUISITION PERFORMANCE

A. Signal Model

As depicted in the introduction, the BOC baseband signal can be written as

$$s(n) = c(n) s_{\text{sc}}(n)$$

(1)

where $c(n)$ is the PRN code; $s_{\text{sc}}(n)$ is subcarrier of the BOC signal. BOC waveforms have autocorrelation functions (ACFs) containing multiple peaks, as presented in Fig. 1.

The down-converted radio signal after Analog to Digital Converter (ADC) can be represented as [9]

$$r_{\text{IF}}(n) = s_{\text{IF}}(n) + \delta_{\text{IF}}(n)$$

$$= A_{\text{in}} d(n) c(n - \tau) s_{\text{sc}}(n - \tau)$$

$$\times \cos(2\pi(f_{\text{IF}} + f_s)nT_c + \phi_0) + \delta_{\text{IF}}(n)$$

(2)

where $A_{\text{in}}$ is the amplitude of the signal; $d(n)$ is navigation data; $f_{\text{IF}}$ and $f_s$ denote the Intermediate Frequency (IF) and the Doppler shift respectively; $\tau$ is the time delay; $T_c = 1/f_s$ is the sampling period; $\phi_0$ is the initial phase of the received signal and $\delta_{\text{IF}}(n)$ is additive white Gaussian noise with zero mean and the variance $\sigma_{\text{IF}}^2$. A convenient choice is to sample the IF signal with a sampling frequency $f_s = 2B_{\text{IF}}$, where $B_{\text{IF}}$ is the front-end bandwidth. In this case, it is easily shown that the noise variance becomes [10]

$$\sigma_{\text{IF}}^2 = E[\delta_{\text{IF}}^2(n)] = N_0 f_s/2 = N_0 B_{\text{IF}}$$

(3)

where $N_0/2$ is the power spectral density of $\delta_{\text{IF}}(t)$.

B. The BPSK-like Method

As mentioned above, the BPSK-like method uses only one centered filter with a bandwidth including the two principal lobes of the spectrum and the secondary lobes between the two principal lobes. Two correlation channels are generated: right channel where the filtered signal situated at $f_{\text{IF}} + f_s$ and correlated with a BPSK signal and left channel where the filtered signal situated at $f_{\text{IF}} - f_s$ and correlated with a BPSK signal. The principle of this method is illustrated in Fig. 2. The correlation outputs of the two channels are combined [6].

At the output of the correlator, we get the correlation value between the incoming signal and the local generated one, which can be represented as [6], [10]

$$R_{\text{IF}} = \frac{1}{N} \sum_{n=0}^{N-1} r_n(s(n) \cos(2\pi(f_{\text{IF}} + f_s)nT_c + \phi_0))$$

$$\times \cos(n - \tau) \exp(j2\pi f_s nT_c)$$

(4)

After calculation, it can be written as

$$R_{\text{IF}} = y_{\text{IF}}(n) + \delta_{\text{IF}}(n)$$

(5)

$$y_{\text{IF}}(n) = \frac{A_{\text{in}}}{2} d(n) R_{\text{BLR}}(\Delta \tau) \cos(\Delta \phi)$$

(6)

where $T_c$ is the coherent integration time; $N = T_c / T_f$ represents the number of samples in coherent integration; $\Delta \tau = \tau - \tau'$, $\Delta f_s = f_{\text{IF}} - f_s$, and $\Delta \phi = \phi_0 - \phi_0'$. $R_{\text{BLR}}(\Delta \tau)$ represents the correlation function between the received BOC signal and the local PRN code modulated by $\exp(j2\pi f_{\text{IF}} t)$. Similarly, other correlation results can be expressed as [6], [10]

$$R_{\text{IF}} = y_{\text{IF}}(n) + \delta_{\text{IF}}(n)$$

(7)

$$R_{\text{IF}} = y_{\text{IF}}(n) + \delta_{\text{IF}}(n)$$

(8)

$$R_{\text{IF}} = y_{\text{IF}}(n) + \delta_{\text{IF}}(n)$$

(9)

with

$$y_{\text{IF}}(n) = \frac{A_{\text{in}}}{2} d(n) R_{\text{BLR}}(\Delta \tau) \cos(\Delta \phi)$$

(10)

$$y_{\text{IF}}(n) = \frac{A_{\text{in}}}{2} d(n) R_{\text{BLR}}(\Delta \tau) \cos(\Delta \phi)$$

(11)

$$y_{\text{IF}}(n) = \frac{A_{\text{in}}}{2} d(n) R_{\text{BLR}}(\Delta \tau) \cos(\Delta \phi)$$

(12)

where $R_{\text{BLR}}(\Delta \tau)$ represents the correlation function between the received BOC signal and the local PRN code modulated by $\exp(j2\pi f_{\text{IF}} t)$.

It can be proved that $\delta_{\text{IF}}(n), \delta_{\text{IF}}(n), \delta_{\text{IF}}(n)$ and $\delta_{\text{IF}}(n)$
are independent Gaussian noises with this variance [10]
\[
\sigma^2 = \text{Var}\left(\frac{1}{N} \sum_{n=0}^{N-1} \left[\delta_{\phi}(n) \cos(2\pi f_c n T_s + \phi_n) \right]
\times c(n - \tau) \exp(j2\pi f_c n T_s)\right) \approx \frac{1}{N} N_c B_s \frac{\sigma_{\phi}^2}{2N}.
\] (13)

that is
\[
\sigma_{\delta_\phi}^2 = \sigma_{\delta_q}^2 = \sigma_{\delta_i}^2 = \sigma_{\delta_{\bar{q}}}^2 = \sigma^2.
\] (14)

C. The SCPC Method

The Sub Carrier Phase Cancellation (SCPC) technique also can be used to make a non-ambiguous acquisition of BOC signal [6], [7]. This method combines correlations of the incoming signal with both sine and cosine subcarrier replicas to create an approximation of a single Phase Shift Keying like (PSK-like) peak. The principle of the SCPC method is presented in Fig. 3.

With this method, two correlation channels are generated. Similarly to the BPSK-like method, four correlation values can be expressed here [6], [10]
\[
R_{S1} = y_{S1}(n) + \delta_{\phi}(n) \tag{15}
\]
\[
R_{S2} = y_{S2}(n) + \delta_{\phi}(n) \tag{16}
\]
\[
R_{S3} = y_{S3}(n) + \delta_{\phi}(n) \tag{17}
\]
\[
R_{S4} = y_{S4}(n) + \delta_{\phi}(n) \tag{18}
\]
with
\[
y_{S1}(n) = \frac{A_n}{2} d(n) R_{S1}(\Delta \tau) \sin c(\pi\Delta f_s T_s) \cos(\Delta \phi) \tag{19}
\]
\[
y_{S2}(n) = \frac{A_n}{2} d(n) R_{S2}(\Delta \tau) \sin c(\pi\Delta f_s T_s) \sin(\Delta \phi)
\]
\[
y_{S3}(n) = \frac{A_n}{2} d(n) R_{S3}(\Delta \tau) \sin c(\pi\Delta f_s T_s) \cos(\Delta \phi)
\]
\[
y_{S4}(n) = \frac{A_n}{2} d(n) R_{S4}(\Delta \tau) \sin c(\pi\Delta f_s T_s) \sin(\Delta \phi)
\]
where \(R_{S1}(\Delta \tau)\) and \(R_{S2}(\Delta \tau)\) represent respectively the cross-correlation function of the received BOC signal with the local in-phase BOC code, and the cross-correlation function of the received BOC signal with the local quadrature BOC code [6]. Then \(\delta_{\phi}(n)\), \(\delta_{\phi}(n)\), \(\delta_{\phi}(n)\) and \(\delta_{\phi}(n)\) are independent Gaussian noises with variances
\[
\sigma_{\delta_{\phi}}^2 = \sigma_{\delta_q}^2 = \sigma_{\delta_i}^2 = \sigma_{\delta_{\bar{q}}}^2 = \sigma^2.
\] (20)

D. Non-coherent Combining

Non-coherent channel combining for increasing the acquisition performance consists in simply summing instances of the output of the correlation channels [10]. The reconstructed ACFs of BOC signals using non-coherent combining scheme are shown in Fig. 4 and Fig. 5, using the ideal filter of bandwidth: \(2(f_c + f_s)\). Noted that the maximum correlation values are all smaller than 1. This is due to the limited bandwidth and the cross-correlation loss introduced by these techniques.

The final decision variable is obtained as [9]
\[
S_{\text{SCPC}} = \sum_{i=1}^{K} \left[R_{S1,i}^2 + R_{S2,i}^2 + R_{S3,i}^2 + R_{S4,i}^2\right] \tag{21}
\]
where \(R_{Sj}\) represents \(R_{Sj,i}\) or \(R_{Sj,i}(i=1,2,3,4)\).

In the \(H_0\) state, where the signal is absent, \(R_{Sj,i}\) is a Gaussian random variable with zero mean \((\mu_{R_{Sj,i}} = 0)\) and variance [6], [10]
\[
\sigma_{R_{Sj,i}}^2 = \frac{\sigma^2}{2N}.
\] (22)

Therefore, \(S_{\text{SCPC}}\) is a \(\chi^2\) distribution with \(4K\) degrees of freedom.

Figure 3. Principle of the SCPC method

Figure 4. Reconstructed ACFs using the BPSK-like method

Figure 5. Reconstructed ACFs using the SCPC method
freedom. The probability of false alarm is then defined
\[ P_{fa} = \int_{\nu} f_{\kappa | \nu} (s | H_i)ds = \exp \left( -\frac{V_i}{2\sigma^2} \right) \sum_{k=1}^{2K} \frac{1}{k!} \left( \frac{V_i}{2\sigma^2} \right)^k. \] 

Under \( H_i \) hypothesis, where the desired signal is present, \( R_i(i=1,2,3,4) \) is a Gaussian random variable with properties [10]
\[ \sigma_{R_i|H_i}^2 = \sigma^2 \] 
\[ \mu_{R_i|H_i} = \frac{A}{\Delta} R_i(\Delta) \sin c(\pi f_c T_c) \cos(\Delta \phi), i = 1, 3 \] 
\[ \mu_{R_i|H_i} = \frac{A}{\Delta} R_i(\Delta) \sin c(\pi f_c T_c) \sin(\Delta \phi), i = 2, 4 \] 
where \( R_i(\Delta)(i=1,2,3,4) \) represents \( R_{i,1}(\Delta) \) or \( R_{i,0}(\Delta) \) with the SCPC method, and represents \( R_{i,1}(\Delta) \) or \( R_{i,0}(\Delta) \) with the BPSK-like method. Then, \( S_{\kappa|H_i} \) is a non-central \( \chi^2 \) distribution with \( 4K \) degrees of freedom. The non-centrality parameter \( \lambda \) of the distribution is defined as
\[ \lambda = K \sum_{i=1}^{4} \mu_{R_i|H_i}^2 = \frac{KA_i^2}{4} \left[ R_{i,1}(\Delta) + R_{i,0}(\Delta) \right] \sin c^2(\pi f_c T_c), BPSK - like \] 
\[ \frac{KA_i^2}{4} \left[ R_{i,1}(\Delta) + R_{i,0}(\Delta) \right] \sin c^2(\pi f_c T_c), SCPC \] 

The probability of detection is defined as [10]
\[ P_d = \int_{\nu} f_{\kappa | \nu} (s | H_i)ds = Q_{2K} \left( \frac{\sqrt{\lambda}}{\sigma}, \sqrt{\frac{V_i}{\sigma^2}} \right) \] 

where \( Q_\lambda (\cdot) \) is a Marcum Q-function.

### III. EFFECT OF CODE SEARCH STEP SIZE

#### A. Effect of Code Search Step Size on Autocorrelation Values

The traditional analysis of BOC signal acquisition performance focuses on the Receiver Operating Characteristic (ROC) curves, without considering the effect of the code search step size. However, the code search step size has direct influence on signal acquisition process, which can be assessed by the autocorrelation value firstly. The effect on autocorrelation value for different scenarios will be analyzed in this paper. The best case value is the highest maximum correlation that can be obtained for any given code delay. This value always corresponds to the peak of the autocorrelation function. The worst case value is the lowest maximum correlation obtained by stepping through the autocorrelation function with steps of a given size. The best case and the worst case are chosen to obtain an insight into the sharpness of the main peak and the effect of the side peaks [11].

In Fig. 6, the effect of the code search step size on autocorrelation values for the direct acquisition method is depicted, using infinite bandwidth. The best case values for SinBOC(1,1), SinBOC(10,5) and CosBOC(15,2.5) are all 1 but the worst case values are different. For the CosBOC(15,2.5) case, not only is the degradation more steep, but also there are local minimum points produced by the regularly spaced autocorrelation nulls between side peaks, as shown in Fig. 1. A typical code search step size of 0.5 experiences a loss of up to 27 dB compared to the best case and up to 12 dB loss compared to SinBOC(1,1) correlation waveform with the same search step.

The effect of the code search step size for the BPSK-like method is reported in Fig. 7 and for the SCPC method in Fig. 8. The worst case correlation values for both methods follow an approximate linear degradation with increasing search step size, as expected with PSK-like correlation functions, as shown in Fig. 4 and Fig. 5. Compared with the direct acquisition method, the worst case correlation values for the BPSK-like
method and the SCPC method degrade slowly and smoothly with increasing search step size. As a result of the limited bandwidth and the cross-correlation loss, the best case correlation values for both methods are smaller than 1.

B. Probability of Detection for the Best Case

In this section, the probability of detection for the best case is evaluated for the acquisition methods described so far. The traditional analysis of BOC signal acquisition performance mainly focuses on the probability of detection for the best case. In all simulations, we used a constant $P_0$ of $10^{-3}$. A PRN code length 1023 and $K$ non-coherent integrations have been used, and the coherent integration time is equal to 1ms.

The Fig. 9 represents the best case probability of detection ($P_d$) for SinBOC(1,1). It is necessary to point out that the acquisition performance of the BPSK-like method seems to be much higher than the performance of the SCPC method. A difference of 0.8dB-Hz on the Carrier Power to Noise Density Ratio ($C/N_0$) can be observed at equal value of $P_d$. The performance obtained with Monte Carlo simulations (indicated with “MC”) and the theoretical ones (indicated with “Theory”) coincides, thus validating the theoretical analysis. The detection probability of SinBOC(10,5) and CosBOC(15,2.5) with the BPSK-like method and the SCPC method has been shown respectively in Fig. 10 and Fig. 11. It should be noticed that these techniques for CosBOC(15,2.5) have similar performance shown in Fig. 11. The different performance with BOC($m_n$) signals can be attributed to the same limited filter bandwidth ($2(f_c + f_s)$) for both techniques to a great extent. As regard to the reconstructed correlation functions (presented in Fig. 4 and Fig. 5), this result was expected.

For the same probability of detection, the direct acquisition method results in an improvement in $C/N_0$ of about 2dB compared with unambiguous acquisition methods in the best case scenario. As regard to the ACFs (presented in Fig. 1) and the reconstructed correlation functions (presented in Fig. 4 and Fig. 5), this result was expected. Moreover, it is worth mentioning that among all these BOC signals the direct acquisition method has the same best case probability of detection. This is due to the same best case correlation value for these signals.

C. Probability of Detection for the Worst Case

The probability of detection for the worst case will be assessed for the acquisition methods discussed so far. The probability of detection for the worst case has relation to the worst case correlation value. In our analysis and simulations, we considered a constant $K$ of 20 hereafter.

The worst case probability of detection for SinBOC(10,5) using different acquisition techniques is shown in Fig. 12, Fig. 13 and Fig. 14. In order to highlight the effect of the code search step size, it is illustrated with three-dimensional top view figures. It should be noticed that the probability of detection for SinBOC(10,5) using direct acquisition technique declines sharply while the search step size is larger than 0.5 chips. However, there
is a low probability of detection region for the code search step size around 0.25 chips. The probability of detection using the BPSK-like method and the SCPC method, in general, is higher than the probability of detection using the direct acquisition method. Moreover, both techniques allow larger code search step size (up to 1 chip) than the direct acquisition technique. The worst case probability of detection for CosBOC(15,2.5) is presented in Fig.15, Fig. 16 and Fig. 17. For the direct acquisition method, the worst case probability of detection for CosBOC(15,2.5) frequently fluctuates and gradually diminishes with increasing code search step size. Compared with the direct acquisition approach, the BPSK-like approach and the SCPC method both result in improvement in the probability of detection for CosBOC(15,2.5).

D. Probability of Detection for the Average Case

As discussed above, the best case and the worst case correspond to the maximum and minimum correlation values of a given search step size. In fact, we will not always encounter these cases. The truth value will have a uniform distribution in \([R_{\text{worst}}, R_{\text{best}}]\), where \(R_{\text{worst}}\) and \(R_{\text{best}}\) denote the worst case correlation value and the best case correlation value of a given search step size respectively. By using the theorem of the total probability of continuous random variables [12], the average probability of detection (\(\bar{P}_d\)) can be expressed as

\[
\bar{P}_d = \frac{1}{R_{\text{best}} - R_{\text{worst}}} \int_{R_{\text{worst}}}^{R_{\text{best}}} P_d(x)dx
\]  

(29)
where $P_d$ is the probability of detection of a given correlation value, as in (28).

Without loss of generality, just the average probability of detection for the CosBOC(15,2.5) is analyzed in this section, as shown in Fig. 18, Fig. 19, and Fig. 20. Compared with the best case and the worst case, the average probability of detection is in between. Fig. 21 and Fig. 22 illustrate the ratio of the average probability of detection for unambiguous acquisition techniques (BPSK-like and SCPC) and for the direct acquisition technique in decibel (dB). As clearly shown in Fig. 21 and Fig. 22, unambiguous acquisition techniques result in an improvement in the average probability of detection for CosBOC(15,2.5) at moderate and high C/N0 values, when the code search step size is larger than 0.3 chips. For a typical code search step size of 0.5 chips, the average probability of detection for unambiguous acquisition methods outperforms the direct acquisition method by about 4dB. Finally it is worth mentioning that the BPSK-like method and the SCPC method seem to have similar average probability of detection.

IV. CONCLUSIONS

In this paper, the acquisition performance analysis of BOC signal considering the code search step size has been shown. The performance of the direct acquisition approach and unambiguous acquisition approaches for different scenarios are analyzed by means of the probability of detection. It can be concluded that the code search step size plays an important role in determining the probability of detection and that the BPSK-like technique and the SCPC technique are good candidates for the acquisition process in BOC signal receivers.
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