

# Research on Multipath-Caused MLS Angle-Measurement Error Using Main Lobe Gaussian Replacement Technology

Huaxin Wu, Xiubin Zhao and Chuanjin Dai

Telecommunication Engineering Institute, Air Force Engineering University, Xi'an, China

Email: wnewhua@163.com

**Abstract**—Electromagnetic scattering caused by the site environment of airports will cause multipath effect. In the presence of multipath, the measured angle position, which is derived through the measuring of the time interval between “From” and “To” scanning pulse pairs, will deviate from the actual position, leading lower system guidance accuracy and higher risk. An error research method based on the equivalent replacement of the scanning beaming main lobe is presented. Through the equivalent Gaussian replacement of the beam main-lobe of “From” and “To” pulse, the error formula in the presence of several multipath components is derived. The method is used to calculate the angle-measuring performance error of the receiver for the special multipath environment and a practical angle-measurement error model is established. The results derived from the replacement model are compared with that of the classical model. The new model can overcome the limitation and deficiency of classical model in exact error calculation. Moreover, the accuracy is more improved than the classical model. The displacement error of envelope peak position and displacement error of envelope peak level caused by multipath effect is further analyzed based on the replacement method.

**Index Terms**— Microwave Landing System, electromagnetic scattering, multipath effect, scanning beam error, angle error, simulation and modeling

## I. INTRODUCTION

Microwave landing system (MLS), which has been recommended for the new type aircraft by International Civil Aviation Organization (ICAO), is intended to gradually replace the existing other landing systems [1][2]. The key advantages of microwave landing system are its large capability, more channels and high accuracy. In the next period, it will be widely installed and used. However, in the work process, it is always disturbed by electromagnetic scattering caused by the site environment of airports, such as hangers, large buildings and humped runway etc, causing multipath problem for the system. The measured angle position in receiver deviates from the actual position, leading lower system guidance accuracy and higher risk. On the other hand, the expanded coverage and increased accuracy requirements of MLS make it more susceptible to the features of the site in which it is located. What's more, the MLS is designed to

provide multiple approaching corridors to aircraft, there is thus less margin for error in MLS than other landing system in which error become catastrophic only when they are so large as to affect obstruction clearance [3][4]. The sensitivity of MLS receiver is further enhanced because of the reason.

The experimental method of the MLS site environment evaluation has certain important drawbacks [5][6]. Setting up the equipment and performing the experiment takes a long time and is very expensive. Another, and perhaps more serious, limitation of the experimental method is that it provides information on the MLS performance only for an existing site. If the site is found unsuitable, the method cannot provide guidance regarding nature and extent of site development necessary to make it acceptable. There is, thus, a strong motivation for research into analytical methods for the site environment multipath angle error of MLS Receiver, preferably avoiding experimental procedures. In 1990s, an error model developed by Mathias is based on the essence equation of MLS angle measurement. In the condition of multipath, the leading edge error of -3dB dwell gate is ignored and only the trailing edge error is computed when the separation angle equate to half beamwidth in the model. So, in the worst conditions, the model is necessary to fast determine the angle error [7][8]. But, if the leading error is considered, the model is not appropriate.

In order to more accurately analyze the multipath angle error of the MLS receiver, an error research method based on the equivalent replacement of the scanning beaming main lobe is presented. Through the equivalent Gaussian replacement of the beam main-lobe of “From” and “To” pulse, the error model formula in the presence of several multipath components is derived. The method is used to calculate the angle-measurement performance error of the receiver for the special multipath environment and a practical angle-measurement error model is established. The results of replacement model computation are compared with the computed results of Mathias model.

## II. THE ANGLE MEASUREMENT PRINCIPLE OF MLS

The Time Reference Scanning Beam (TRSB) MLS is based on the principle of converting the angular position

of the aircraft into a time difference between two received pulses. It uses two narrow beams which are scanned in an oscillatory manner in the azimuth and elevation sectors. At every position within the scan sector, an aircraft will receive two pulses from each beam corresponding to the “To” and “From” scans. Through the measuring of the time interval between “From” scanning pulse pairs and “To” scanning pulse pairs, the MLS receiver can acquire the angle position of aircraft in space. In the proportional guidance sector, the relationship of the guidance angle and pulse interval can be expressed by:

$$\theta = (T_0 - t) \cdot \nu / 2 \tag{1}$$

where  $\theta$  is the guidance angular position in degrees,  $T_0$  is the time separation between “To” and “From” beam centers referenced to 0 degrees measured in microseconds.  $t$  is time separation between “To” and “From” beam centers measured in microseconds.  $\nu$  is beam scan velocity in degree per microseconds.

As an air-derived position system, MLS receivers use a dwell gate technique to decode the aircraft’s angular position. The angle processor envelope detects the TRSB “To” and “From” beam pulse and forms a dwell gate about the  $-3dB$  threshold crossings of each pulse. Fig. 1 shows how the angle processor forms the dwell gate. When an aircraft located in the angular position  $\theta$  was scanned into receiver, the beam envelope peak is detected. Then take the level,  $-3dB$  lower than the peak level, as the threshold crossing level. Only the signal higher than the threshold crossing level can be transited through the dwell gate into the angle processor. The angle processor calculates the dwell gate midpoints, with the resulting time separation between midpoints corresponding to  $t$  in the angle coding (1). It is pointed that the dwell gate initiates at the  $-3dB$  threshold on the leading edge of beam and cease at the  $-3dB$  threshold on the trailing edge of beam[9][10]. This is done for each of the “To” and “From” scan signals, and then the angular position is calculated by taking the time interval between dwell gate midpoints. The dwell gate midpoint also represents the beam centroid for undistorted beam. So, the scanning beam pulse centroid is defined as:

$$t_{To/From} = (T_l + T_t) / 2 \tag{2}$$

Where  $T_l$  is the time corresponding to the leading edge of  $-3dB$  threshold crossing;  $T_t$  is the time corresponding to the trailing edge of  $-3dB$  threshold crossing.

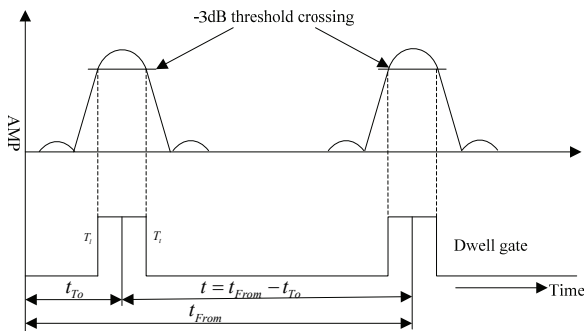


Figure 1. Interval time measurement principle

### III. MULTIPATH EFFECT ON MLS

The presence of obstructions, such as hangars, building, vehicle, aircraft, and imperfect terrain etc., within the vicinity of the MLS ground equipment will result in electromagnetic scattering of MLS signals as shown in Fig. 2. The electromagnetic scattering is commonly referred to as multipath, which may result in detectable distortion of the direct MLS beam signals. When present, this distortion degrades the accuracy of the time measurements in angle processor, thus the accuracy of the MLS guidance information.

#### A. Separation Angle

In order to effectively analyze the error caused by the electromagnetic scattering effect of site environment, separation angle is defined.

Let  $\vec{R}_i = \{R_{ix}, R_{iy}, R_{iz}\}$  denote the transmitting path from phase middle of the transmitting antenna to the scattering point of the obstacle in the environment and  $\vec{R}_0 = \{R_{0x}, R_{0y}, R_{0z}\}$  denote the direct path from the antenna to the receiver. Separation angle is defined as the angle of  $\vec{R}_i$ 's and  $\vec{R}_0$ 's projection on the scanning beam plane. For the horizontal plane of azimuth scanning beam, separation angle is defined as

$$SEP|_{AZ} = \cos^{-1} \left( \frac{R_{ix}R_{0x} + R_{iy}R_{0y}}{\sqrt{R_{ix}^2 + R_{iy}^2} \sqrt{R_{0x}^2 + R_{0y}^2}} \right) \tag{3}$$

For the vertical plane of elevation scanning beam, separation angle is defined as

$$SEP|_{EL} = \cos^{-1} \left( \frac{R_{iy}R_{0y} + R_{iz}R_{0z}}{\sqrt{R_{iy}^2 + R_{iz}^2} \sqrt{R_{0y}^2 + R_{0z}^2}} \right) \tag{4}$$

MLS adopt the phased array antenna technique to scan in the operational region. The multipath error caused by the obstructions can be divided into two key regions: in-beam (if the separation angle  $\theta_s > 1.7\theta_{BW}$ ) and out-beam (if the separation angle  $\theta_s \leq 1.7\theta_{BW}$ ), as depicted in Fig. 3 and Fig. 4. The out-beam error can be eliminated by the dwell gate tracing techniques. The in-beam error, as the key factor, is the biggest interference of MLS operational performance. Thus, the in-beam multipath error is the main problem studied in this paper.

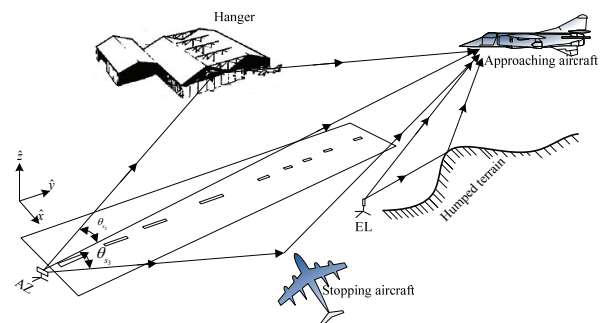


Figure 2. Sketch of multipath effect

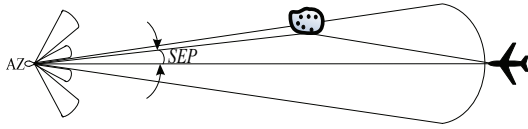


Figure 3. In-beam scattering

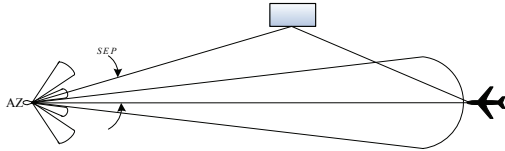


Figure 4. Out-beam scattering

**B. Multipath Angle Error**

If the scanning beams are not perturbed by multipath, the dwell gate midpoint and scanning beam peak will coincide, resulting in accurate angular guidance. However, if the guidance beams are distorted by in-beam multipath, the dwell gate midpoint and the desired beam will differ. At the same time, the leading and trailing edge of threshold crossing pulses will be displaced. From (1) we can get MLS guidance angle error

$$\Delta\theta = v \cdot \Delta t / 2 \tag{5}$$

$\Delta t$  represents measurement error of time interval between “From” and “To” dwell gate midpoints and depends on the specific form of distortion in scanning pulse. If the leading error is  $\varepsilon_L$ , and the trailing error is  $\varepsilon_r$ , the time displaced difference  $\varepsilon$  between the dwell gate midpoint and desired beam peak is the arithmetic mean of  $\varepsilon_L$  and  $\varepsilon_r$ . So, the error is expressed by:

$$\varepsilon = (\varepsilon_L + \varepsilon_r) / 2 \tag{6}$$

As we known, the “From” and “To” scanning beam are symmetrical, and the displacement error are symmetrical too. So the time interval  $\Delta t$  between “From” and “To” dwell gate midpoints is  $2\varepsilon$

**C. Multipath Beam Envelope Modeling**

MLS ground transmitted antennas are phased array antennas[11], so the receiver located at the point  $R(\theta_0, \varphi_0)$  can receive the scanning beam pulse, which is proportional to  $F_a(\sin vt - \sin \theta_0)F_b(\theta_0, \varphi_0)$ . Here:  $F_a$  is the phased array antenna factor[12][13];  $F_b$  is the antenna element pattern. For the sake of convenience, the initial time is removed to the time when the scanning across the runway center line. And then, at the initial time antenna begins to scan as the uniform speed  $v$  from the left of the runway. The time when the scanning beam across the runway center line is marked as  $T_0$ , and the time when the scanning beam across any point in space is marked as  $t'$ . The time removal displacement can be expressed by  $t = t' - T_0$ . Thus, if the Doppler frequency shift is ignored, the “From” and “To” scanning signal in multipath condition can be express as:

$$e(t) = F_a(\sin vt - \sin \theta_0)F_b(\theta_0, \varphi_0) + \sum_{i=1}^M \rho_i F_a[\sin v(t - \tau_i) - \sin \theta_i]F_b(\theta_i, \varphi_i)e^{j\delta_i} \tag{7}$$

Where:  $M$  is the number of propagated paths; the direct signals is  $F_a(\sin vt - \sin \theta_0)F_b(\theta_0, \varphi_0)$ ; if  $i \geq 1$ , (7) is the multipath signal.  $\rho_i$ ,  $\delta_i$ ,  $\tau_i$  are the amplitude, phase and time delay of the ratio of the  $i^{\text{th}}$  multipath signal relative to the direct signal. The DOA  $(\theta_i, \varphi_i)$  of the multipath is the separation angle of obstruction. The modulus of (7) is the “From”/“To” scanning beam envelope.

In MLS far field, the ground antenna elements can be regarded as non-directional nod-source. Antennas start to scan from runway center line, so at the instant  $t$ , the scanning angle is  $\theta = vt$ , and the separation angle is  $\theta_i = v\tau_i$ . Thus the received single scanning beam envelope pulse also can be expressed as:

$$e(\theta) = F_a(\theta) + \sum_{i=1}^M \rho_i F_a(\theta - \theta_i)e^{j\delta_i} \tag{8}$$

Equation (8) shows that the envelope of scanning signal is the antenna pattern embodied in time axis. Also, this is the beam envelope of TRSB MLS considering the multipath effect.

**IV. GAUSSIAN REPLACEMENT BASED ANGLE ERROR MODEL**

**A. Gaussian Replacement of Main-lobe Beaming**

As we all known, when the approaching airplane was scanned, the signal higher than the threshold crossing level (-3dB) can be transited through the dwell gate into the angle processor. The signals are also the beam envelope, a part of antenna main-lobe beam. The main-lobe beam interference caused by the multipath effect will lead to angle-measurement error. Hence, only the transform of the antenna main-lobe signal need to be analyzed.

It is supposed that the antenna beamwidth of MLS is equated to  $BW$ . The main-lobe of the antenna pattern can model based on the Gaussian pattern replacement. Through the equivalent Gaussian replacement of the beam main-lobe, then:

$$A(\theta) = \exp[-k(\theta / BW)^2]; k = 2 \ln 2 \tag{9}$$

Where  $A(\theta)$  is the replacement pattern of the MLS antenna pattern when the ignoring the side lobe. From (9), we can see:

$$A(\theta)|_{\theta=\pm 0.5BW} = \sqrt{2} / 2(-3dB) \tag{10}$$

Therefore, the Gaussian replacement of the main-lobe beam can quickly and effectively analyze the displacement of the leading edge and trailing edge of -3dB dwell gate. It is the different that the Gaussian replacement pattern has no side-lobe. However, the key of the MLS interval measurement is the beamwidth of the 3dB the threshold crossing. As a result, the method of equivalent replacement is available. Hence, the received

single scanning beam-envelope pulse (8) can be transformed as:

$$e(\theta) = A(\theta) + \sum_{i=1}^M \rho_i A(\theta - \theta_i) e^{j\delta_i} \quad (11)$$

*B. Angle-measurement error model*

As we known from (11), with the  $i^{th}$  of  $M$  multipath components,  $1 \leq i \leq M$ , associate relative amplitude  $\rho_i$ , separation angle  $\theta_i$  (BW) and phase angle  $\delta_i$ . The assumed antenna pattern is Gaussian replacement beam. With all the multipath components present, the received pulse squared envelope (as a function of scan angle) is:

$$|e(\theta)|^2 = e(\theta) \overline{e(\theta)} = \left| A(\theta) + \sum_{i=1}^M \rho_i A(\theta - \theta_i) e^{j\delta_i} \right|^2 \quad (12)$$

If it notes  $x = \theta / BW$ ;  $x_i = \theta_i / BW$ , (12) can be expressed:

$$e^2(x) = A^2(x) \left[ 1 + 2 \sum_{i=1}^M \eta_i e^{2kx_i} \cos \delta_i + O(\eta_i^2) \right] \quad (13)$$

Where  $\eta_i = \rho_i e^{-kx_i^2}$  is the “effective” multipath amplitude. The criteria used for the threshold crossing assumes a threshold with respect to the unperturbed beam maximum. The maximum level of squared compounded signals is noted  $[e^2(x)]_{max}$ , and then if a real time thresholding system is used, the one seeks solution of

$$e^2(x) \Big|_{-3dB} = e^{-2k\lambda^2} * [e^2(x)]_{max} \quad (14)$$

$$e^{-k\lambda^2} = 0.707$$

Here  $\lambda = 0.5$  being the nominal threshold crossing location in beamwidths (in the absence of multipath). Taking logarithms in (13), and it is noted as  $f(x)$ , then

$$f(x) = -2kx^2 + \ln \left[ 1 + 2 \sum_{i=1}^M \eta_i e^{2kx_i} \cos \delta_i + O(\eta_i^2) \right] \quad (15)$$

The first order terms of the Taylor series expansion of  $f(x)$  in power of  $\eta_i$  at the point zero:

$$f(x) = -2kx^2 + 2 * \sum_{i=1}^M \eta_i e^{2kx_i} \cos \delta_i \quad (16)$$

The objective is to find the leading and trailing edge threshold crossing  $x_-, x_+$ . The squared threshold level is assumed to be  $e^{-2k\lambda^2} e^{f(x_0)}$ , where  $x_0$  is the location of the envelope peak. Moreover, for all  $x$ ,  $f(x_0) \geq f(x)$  is always satisfied. Furthermore, it is known that  $x_0$  is approximate to zero and the equation  $e^{2kx_i} - 1 = 2kx_i$  exists. So,  $f(x)$  is a quadratic function of  $x$ , and  $f(x)$  is maximum at  $x_0 = \sum_{i=1}^M \eta_i x_i \cos \delta_i$ . Resulting in

$$f(x_0) = f(0) + 2k \cdot O(\eta_i^2) \quad (17)$$

Ignoring the terms of higher order than  $\eta_i^2$  In (17), then  $f(x_0) \approx f(0)$ . Considering (14), we get:

$$f(x_{\pm}) = -2k\lambda^2 + f(x_0) \doteq -2k\lambda^2 + f(0) \quad (18)$$

Then substitute into (17), we can get:

$$kx_{\pm}^2 = k\lambda^2 + 2 \sum_{i=1}^M \eta_i e^{2kx_i} \cos \delta_i \quad (19)$$

For the leading edge crossing, replace  $x_-$  by  $x_- = -\lambda + \varepsilon_l$ , and for the trailing edge crossing, replace  $x_+$  by  $x_+ = \lambda + \varepsilon_t$ . Where  $\varepsilon_l$  is the leading edge error,  $\varepsilon_t$  is the trailing edge error. For (19), in the exponential term,  $x$  can be replaced by the nominal value  $(-\lambda)$  alone. Maintaining only the linear term in  $\varepsilon_l$  and  $\varepsilon_t$ , then yield the solutions

$$\varepsilon_l = -\frac{1}{2k\lambda} \sum_{i=1}^M \eta_i e^{-2k\lambda x_i} \cos \delta_i \quad (20)$$

$$\varepsilon_t = \frac{1}{2k\lambda} \sum_{i=1}^M \eta_i e^{2k\lambda x_i} \cos \delta_i$$

The average of  $\varepsilon_l$  and  $\varepsilon_t$  is the error made by a dwell gate processor:

$$\varepsilon = \frac{1}{2}(\varepsilon_l + \varepsilon_t) = \sum_{i=1}^M \rho_i x_i e^{-kx_i^2} \frac{\sinh 2k\lambda x_i}{2k\lambda x_i} \cos \delta_i \quad (21)$$

Where the measurement units of  $\varepsilon$  is BW. So, the angle measuring error  $\Delta\theta$  of “From” and “To” scanning signals equated to  $2\varepsilon$ . It is expressed as  $\Delta\theta = 2\varepsilon$ .

Hence, it is seen to relate the error caused by multipath signals to 1)the multipath/direct signal ratio  $\rho_i$ ; 2)the elements related to separation angle  $x_i e^{-kx_i^2}$ ; 3)the elements related to the threshold crossing  $\sinh(2k\lambda x_i) / 2k\lambda x_i$ ; 4)RF phase difference  $\delta_i$ .

*C. Simulation and Discussion*

It is supposed that the three parameters  $x_i, \rho_i, \delta_i$  can be acquired in the presence of a single multipath signal component. The angle measuring error in the multipath environment can be achieved. Assumed  $\rho = 0.7$  and  $BW = 1.3^\circ$ , the simulated result is shown in Fig. 5. In the condition of separation angle  $\theta_m \in (-3BW \sim 3BW)$ , it shows the relationship of the separation angle and the measure error in receiver. In  $(-1.7BW \sim 1.7BW)$ , the angle measuring error responded to separation angle is near to the cosine curve. The error is maximized at  $\theta_m = 0.5BW$ . But, the error will be disappeared except the separation angle located in  $(-1.7BW \sim 1.7BW)$ . And, the results are identical to the real experiment.

However, responded to the RF phase difference, the angle error is cosine movement. The simulated results given in Fig.6 can express the relative relationship between the angle error and RF phase difference. Comparing the curve Fig.5 with Fig.6, we can get that the angle error caused by the separation angle is more serious than the error caused by the RF phase difference.

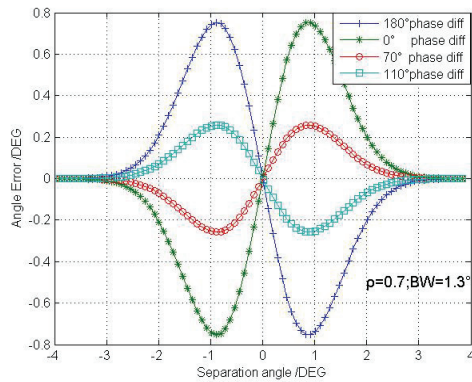


Figure 5. Measure error and separation angle in Receiver Dwell Gate processor

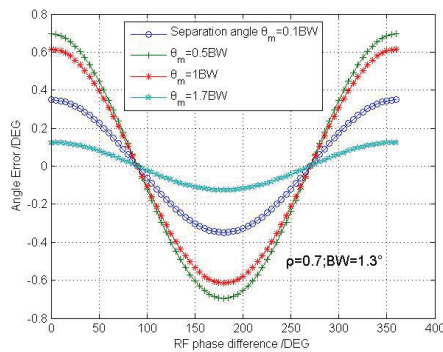


Figure 6. Measure error and RF phase difference in Receiver Dwell Gate processor

**D. Verification and Validation**

References[1][2][7] demonstrate that the Mathias Model recommended by ICAO can quickly compute the angle measuring error of receiver in multipath condition. The Mathias Model can be simply expressed as:

$$\Delta\theta = \sum_{i=1}^M \rho_i \cos \delta_i \quad (22)$$

Where  $M$  is the number of propagated paths,  $\rho_i$ ,  $\delta_i$  are the amplitude and phase of the ratio of the  $i^{th}$  multipath signal relative to the direct signal, the measurement units of  $\Delta\theta$  is  $BW$ . In Mathias model, the angle error is only related to two parameters,  $\rho_i$  and  $\delta_i$ . But, there is no relationship with the parameter - separation angle  $\theta_m$ . Obviously, it is restricted within the specific used-limits. If Mathias model is available, the following two conditions should be satisfied.

1) Assumed the separation angle  $\theta_m \approx 0.5BW$ . If  $\theta_m$  is near to the  $0.5BW$ , the error is maximum and the receiver performance ability is most seriously disturbed. Hence, in order to save time, only  $\theta_m \approx 0.5BW$  is considered in Mathias model.

2) Only the trailing edge error  $\varepsilon_t$  is considered, and ignoring the leading edge error. Then, the angle measuring error, the beam peak location error, is expressed as  $\varepsilon_t / 2$ .

Then, resulting in:

$$\frac{\Delta e_R}{\varepsilon_t} = \frac{de_\Sigma(\theta)}{d\theta}; \quad \left. \frac{de_\Sigma(\theta)}{d\theta} \right|_{-3dB} \approx \frac{e_D}{BW}; \quad (23)$$

where  $e_D$  and  $e_R$  represent peak direct signal amplitude and peak indirect signal amplitude;  $e_\Sigma(\theta)$  represents the compounded beam envelope.  $\Delta e_R$  represents the small disturbing signals. (23) is combined with (5), thus the Mathias Model, (22) is achieved.

If the above condition can be satisfied, the angle error can be correctly predicted by the Mathias model. The model is always used in the MLS site evaluation and the determination of MMLS (Mobile Microwave landing system) station location. So, the Mathias model can be used to validate the feasibility of the Gaussian replacement error model in the specified conditions. It is supposed to satisfy the assumed conditions, and  $\rho = 0.7; BW = 1.3^\circ$ . The simulated results based on the Gaussian replacement method is shown in Fig.7, moreover, the Mathias model results is given in the figure at the same time as comparison. The error curves given in Fig.7 can prove the availability of the suggested methods.

However, the Mathias model has certain application limits. If the separation angle located in  $\theta_m \in (-1.7BW \sim 1.7BW)$ , the model cannot compute the multipath angle error. But, the Gaussian replacement method presented in this paper can well breaks the limitation and overcome its drawbacks.

**V. BEAM ERROR ANALYSIS**

The multipath effect can cause received scanning beam error in two ways, the displacement error of scanning beam envelope peak position and displacement error of the envelope peak level. Both of the beam errors caused by multipath can affect the performance of MLS. The Gaussian replacement model, which has been validated, can be used effectively to analyze these two kind of scanning beam errors.

**A. Displacement Error of Envelope Peak Position**

As we known from (11), when there is a single multipath signal, the received “To” and “From” scanning beam envelope is

$$e(\theta) = A(\theta) + \rho A(\theta - \theta_m) e^{j\delta} \quad (24)$$

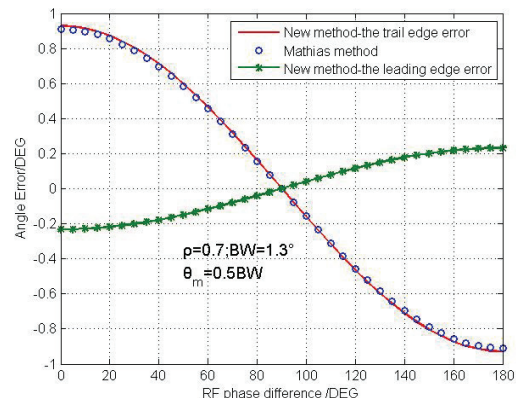


Figure 7. Verification and results



Where  $\theta_m$  is the separation angle, and the envelope peak for “To” and “From” scanning beam is

$$|e(\theta)| = \sqrt{e(\theta)e^*(\theta)}$$

$$= \sqrt{A^2(\theta) + \rho^2 A^2(\theta - \theta_m) + 2\rho A(\theta)A(\theta - \theta_m)\cos\delta}$$
(25)

Let  $G = |e(\theta)|^2$ , and in the position of “To” and “From” scanning beam envelope peak,  $\nabla G = 0$  is satisfied. We can get

$$\frac{\theta}{\theta_m} = \frac{\rho^2 [A(\theta - \theta_m)/A(\theta)]^2 + \rho \cos\delta [A(\theta - \theta_m)/A(\theta)]}{1 + \rho^2 [A(\theta - \theta_m)/A(\theta)]^2 + 2\rho \cos\delta [A(\theta - \theta_m)/A(\theta)]}$$
(26)

From (9) and (26), we can get

$$\frac{\theta}{\theta_m} = \frac{\rho^2 Y^2 + Y \cos\delta}{1 + \rho^2 Y^2 + 2Y \cos\delta}; \quad Y = e^{\left[ -k \left( \frac{\theta_m}{BW} \right)^2 \left( 1 - 2 \frac{\theta}{\theta_m} \right) \right]}$$
(27)

By solving (27), we can get envelope peak position. Because (27) is nonlinear equation, linearization for the equation is needed and the iterative way is adopted to get the solution.

The equivalent equation of equation (27) is  $F(\theta) = 0$ , and  $F(\theta)$  can be denoted as:

$$F(\theta) = \theta(1 + \rho^2 Y^2 + 2Y \cos\delta) - \theta_m(\rho^2 Y^2 + Y \cos\delta)$$
(28)

Suppose the approximate root for  $F(\theta) = 0$  is  $\theta_k$ , spread function  $F(\theta)$  at  $\theta_k$ , we get

$$F(\theta) \approx F(\theta_k) + F'(\theta_k)(\theta - \theta_k)$$
(29)

And equation (29) can approximately denoted as

$$F(\theta_k) + F'(\theta_k)(\theta - \theta_k) = 0$$
(30)

Equation (30) is linear equation. Assume  $\theta_{k+1}$  be its root, then

$$\theta_{k+1} = \theta_k - F(\theta_k) / F'(\theta_k)$$
(31)

Equation (31) is then the linearized iterative equation.

The algorithm to calculate is as follows:

The first step is initialization. The initial approximate value  $\theta_0 = \alpha$  is selected as the estimate direction of the scanning beam. Calculate  $F_0 = F(\theta_0)$ ;  $F'_0 = F'(\theta_0)$ .

The second step is iteration. With the equation  $\theta_1 = \theta_0 - F_0 / F'_0$ , one iteration can get a new approximate value  $\theta_1$ . Calculate  $F_1$  and  $F'_1$ .

The third step is to control. If  $|\theta_{k+1} - \theta_k| \leq \sigma$  is satisfied, stop iteration and  $\theta_k$  is taken as the root of the equation, otherwise, go to step four.

The fourth step is to modify. If the iteration times arrive a pre-set value N or  $F'_k = 0$ , the method is invalid, otherwise, iteration continues with equation (31).

When  $\theta_m, \rho, \delta$  are known, it is needed to seek the optimized solution for  $F(\theta) = 0$  in all positions of airspace. Because in the instant  $t=0$ , the antenna start “From” and “To” scans from the middle of the runway, the displacement error of envelope peak position is the

root of the equation. Simulation results are shown in Fig. 8 and 9.

Fig. 8 depicts the relationship between envelope peak position and the phase difference of multipath signal and direct signal. As we can see from the figure, when the phase difference is  $0^\circ$  and  $180^\circ$ , the maximum displacement error of envelope peak position appear. With the variation of phase difference, error of envelope peak position will undulate similar to that of a cosine function, but the margin for the undulation is slim. Fig. 9 shows the variation of displacement error of envelope peak position when separation angle are in interval  $(-1.7BW \sim 1.7BW)$ . The displacement error of envelope peak position reaches its maximum when separation angle  $\theta_m = \pm 0.5BW$ . Compared with phase difference, separation angle has the much more influence on the displacement of envelope peak position.

### B. Displacement Error of Envelope Peak level

It is known from the principle of angular measurement that envelope peak level of the scanning beam pulse is key for MLS receiver angle measurement using the dwell gate. Each envelope peak level corresponds to a time instant and a certain angular position. Research can be

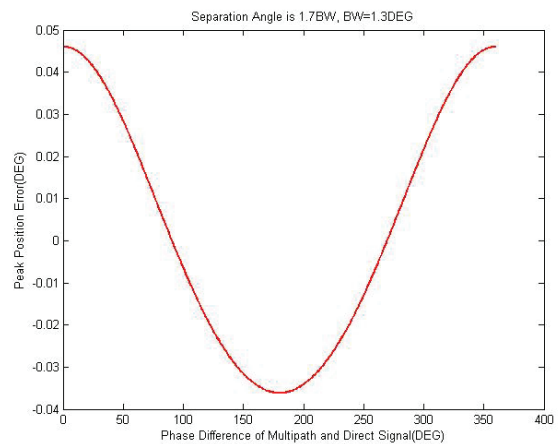


Figure 8. Envelope peak position displacement variation with phase difference of multipath signal and direct signal

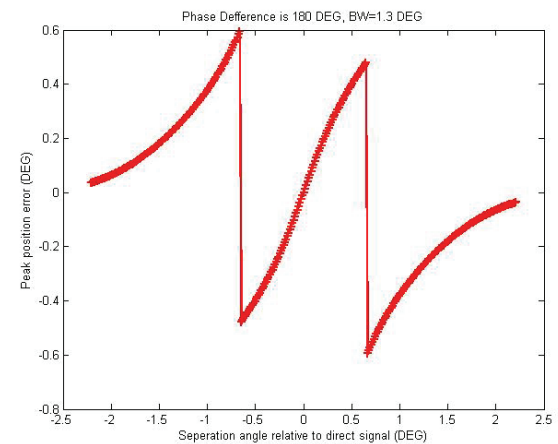


Figure 9. Envelope peak position displacement variation with separation angle of multipath signal relative to direct signal

performed on the displacement of envelope peak level with the Gaussian replacement model. Further analysis can be given on displacement of envelope peak level based on the analysis of the displacement of scanning beam envelope peak position.

By applying the Gaussian replacement of main-lobe beaming, the model for scanning beam envelope peak level of the MLS receiver considering scattering effect can be expressed as:

$$F(\theta) = \left| e^{-k\theta^2} + \rho e^{j\phi} e^{-k(\theta-\theta_m)^2} \right|; k = 2 \ln 2 \quad (32)$$

Equation (32) denotes the sum of the two Gaussian substituted equivalent beams separated by separation angle. Fig. 8 shows the variation of the peak level of the scanning beam envelope at  $\theta = 0^\circ, \theta = \theta_m$  and the position of real peak, when separation angle are in the interval 0-2BW,  $phase = 0^\circ$  and the level ratio of scattering wave and direct wave  $\rho = -3dB$ . Envelope peak level are 0.6dB higher compared with the original envelope peak level  $F(0)$  when  $\theta_m = 0.68BW$ . Fig. 9 shows the summed Gaussian envelope values at the peak, direct and multipath locations when separation angle are in the interval 0-2BW,  $phase = 180^\circ$  and the level ratio of scattering wave and direct wave  $\rho = -3dB$ . Envelope peak level are 1.7dB higher compared with the original envelope peak level  $F(0)$  when  $\theta_m = 0.45BW$ , which is the largest displacement error.

With the diminishment of  $\rho$ , the displacement error for envelope peak level are decreased, leading to higher measurement accuracy.

### VI. CONCLUSION

In order to effectively analyze the multipath angle error of the MLS receiver, an error research method based on the equivalent replacement of the scanning beaming main lobe is presented. The practical error model is established. Through the equivalent Gauss replacement of the beam main-lobe of "From" and "To" pulse, the error formula in the presence of several multipath components is derived. The method is used to calculate the angle-measuring performance error of the receiver for the specified multipath environment. The results of replacement method computation are compared with the computed results of the classical Mathias model. Further analysis on beam error caused by multipath effect is given based on the replacement method.

Thus, the principle results can be attained as follows:

1) Angle error is related to the four parameters: a) multipath/direct signal ratio; b) The elements related to separation angle; c) The elements related to the threshold crossing; d) RF phase difference; So, the error can be reduced through their interference.

2) The new method presented in this paper can give the determinative relationship between the angle error and the four parameters. Furthermore, in the specified condition, the simulated results can be coincident to the Mathias model.

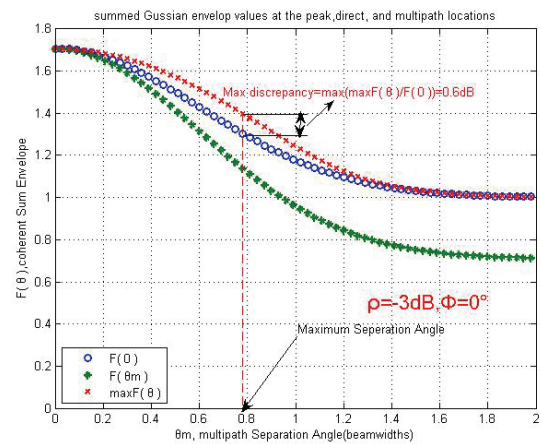


Figure 10. Envelope peak level variations when  $\rho = -3dB$   $phase = 0^\circ$

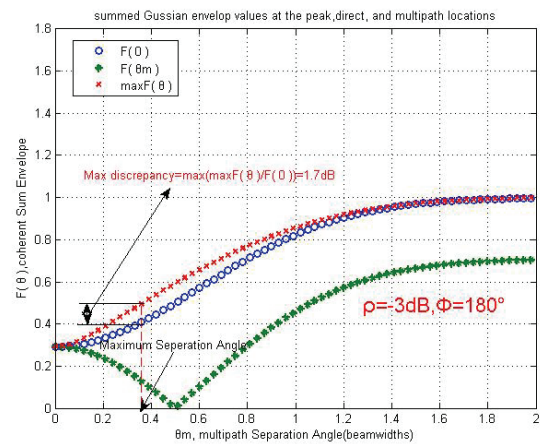


Figure 11. Envelope peak level variations when  $\rho = -3dB$   $phase = 180^\circ$

3) The new method can overcome the limitation and deficiency of classical model in the exact error calculation. It can be applied to the environment that separation angle located in.

4) Analysis based on the new method show that separation angle has the much more influence on the displacement of envelope peak position, compared with phase difference and that with the diminishment of  $\rho$ , the displacement error for envelope peak level are decreased, leading to higher measurement accuracy in MLS angle measurement.

More practical value of error model can be discovered with the equivalent replacement of the main lobe in the precision evaluation of angle measuring performance ability of the MLS receiver under multipath condition.

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**Huaxin Wu** was born in Hunan province, China in 1982. He received his Master degree in Circuits and Systems from the Air Force Engineering University of (AFEU), Xi'an , China in 2008. Currently he is working on doctoral degree on communication and information system at AFEU.

His main research interests focus on the multipath effect on electronic navigation systems.

**Xiubin Zhao** was born in Hubei province, China in 1965. He received his Master degree in Electromagnetics from Nanjing University of Aeronautics and Astronautics, Nanjin, China in 1991. He received his PhD in Electric Science and Technology from Northwestern Polytechnical University, Xi'an, China in 2006.

He is currently a Professor and Doctoral Advisor at AFEU, in Xi'an, China. His main research interests focus on the navigation systems. He is the author of more than 20 peer-reviewed international journal and conference papers. He has served as a reviewer for many international journal and conferences.

Prof.Zhao is currently a senior member of Chinese Institute of Electronics.

**Chuanjin Dai** was born in Anhui province, China in 1982. He received his M.Sc. Engineering in Navigation from the Air Force Engineering University of (AFEU), Xi'an , China in 2007. Currently He is a PhD student in communication and information system at AFEU.

His main research interests focus on the modeling of site environment for electronic navigation systems.