Performance Analysis of Some New CFAR Detectors under Clutter

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Abstract—In order to improve the radar’s detection performance under Pearson distributed background clutter, the problem of designing CFAR detectors is studied in this paper. Three kinds of distributed fuzzy CA scheme and two kinds of OS scheme are proposed. The numerical simulations prove that CA CFAR detector based on algebraic sum fusion rule provides more robust detection performance in homogenous background clutter, CASO based on algebraic-sum rule shows detection improvement in multi-target environment, CAGO based on MIN rule provides better false-alarm-control capacity under clutter-edge condition, and OSSO CFAR offers better detection performance than OS CFAR in multiple interrupt-target situation.

Index Terms—radar network, detection, CFAR, fuzzy, CAGO, CASO, OS

I. INTRODUCTION

Constant false alarm rate (CFAR) technique plays an important role in radar network signal processing. CA CFAR detector is a classical scheme, in which the decision threshold is computed adaptively on the assumption that the background noise amplitude samples are independent identically distributed random variables. CA CFAR is optimum in homogeneous noise or clutter\(^1\), unfortunately, it suffers performance degradation in multiple-target or non-homogeneous environment. In order to alleviate this problem, Rohling\(^2\) modified the common CA CFAR technique by replacing the arithmetic averaging estimator of clutter power with a new data fusion rule based on order statistics method, which is called OS CFAR detector. Similar information fusion works included CAGO\(^3\), CASO\(^4\), CMLD\(^5\), TM\(^6\), etc.

CFAR detectors should consider the clutter model when deciding decision threshold and expressing false alarm rate. Conventional CFAR detector treats the background clutter as Gaussian model. Nowadays, heavy-tailed distribution especially Pearson distribution has been proved to be more appropriate to model actual data such as impulsive signal\(^7\), sea clutter\(^8\) and active sonar returns\(^9\). It has been proved that CA, CAGO/CASO, OS detectors still maintain constant false alarm rate under homogeneous Pearson distributed clutter\(^10-11\).

Traditional statistical detection technique treats observations as random variables and determines the most probable choice between ‘signal presence’ and ‘signal absence’. However, these quantized functions result in incomplete information. Fuzzy logic detection is a new choice, where the decision is not restricted to the presence or absence of a signal. Some work dealing with fuzzy CFAR detection has been reported in literature\(^12-14\).

In this paper, we propose three new detection network systems adopting fuzzy CA, CAGO, and CASO schemes, and develop two kinds of new information fusion technique called OSGO and OSSO based on OS under Pearson distributed background clutter. The work is organized as follows. In section II, we introduce Pearson distribution and some classical detectors under such clutter, in section III, we derive the mathematic model of fuzzy CA/CAGO/CASO-CFAR detectors, in section IV, four kinds of fusion rules used in radar network system are introduced, in section V, we introduce OSGO and OSSO schemes, some simulation results are presented in Section VI. Section VII is the conclusion.

II. PEARSON DISTRIBUTION AND DETECTORS

A. Pearson Distribution

Pearson distribution is a kind of symmetrical alpha-stable distribution when the character parameter \(\alpha\) equals to 1/2. If a variable follows Pearson distribution, the probability density function is \(^15, p.10\):

\[
p_X(x) = (\gamma / \sqrt{2\pi}) (1 / x^{3/2}) e^{-\gamma^2 x}
\]

(1)

The cumulative density function is:

\[
P_X(x) = P_r\{X \leq x\} = 2 \left(1 - \phi(X / \sqrt{x})\right), \quad x \geq 0
\]

(2)

here \(\gamma\) is the scale parameter which reflects the clutter power level, \(\phi(x)\) denotes the cumulative density function of the standard Gaussian distribution \(N(0,1)\):

\[
\phi(x) = 1 / \sqrt{2\pi} \int_{-\infty}^{x} \exp(-y^2 / 2) dy
\]

(3)
so Pearson distribution belongs to single parameter distribution, it means the scale parameter could be estimated by common methods such as CA, OS, CAGO/CASO.

B. CA/CAGO/CASO CFAR Detectors

The structure of CA-CFAR detector is depicted in Fig. 1, where \( q_{\text{CUT}} \) is the output of the cell under test (CUT), others are output of reference sliding cells, \( N \) is the length of reference window, \( T \) is the threshold parameter to achieve a certain \( P_f \) for a given window of size \( N \). \( z \) is the estimation of the power level of background clutter, which must be adaptive to the change of clutter power level. Various CFAR systems are distinguished by the way this threshold is obtained. The adaptive decision rule is

\[
H_1: \text{CUT} \geq Tz, \quad H_0: \text{target absence}
\]

In our study, \( z \) is the mean of \( N \) variables following Pearson distribution, so \( z \) also follow the Pearson distribution with the scale parameter equaling to \( N \). The probability density function of \( z \) is

\[
f_{z}(z) = \frac{\sqrt{N}}{2\pi} e^{-Ny^2/2z}
\]

then \( P_{fa} \) of the CA-CFAR is:

\[
P_{fa}^{CA} = \Pr(q_{\text{CUT}} > Tz | H_0) = \int_{Tz}^{\infty} f_{z}(z)dz
\]

\[
= \int_{Tz}^{\infty} 2\Phi \left( \frac{y}{\sqrt{Tz-1}} \right) \sqrt{\frac{N}{2\pi}} \frac{1}{z^{1/2}} e^{-Ny^2/2z} dz
\]

\[
= \int_{0}^{\infty} \left[ 2\Phi \left( \frac{y}{\sqrt{Tz-1}} \right) \right] \sqrt{\frac{N}{2\pi}} \frac{1}{z^{1/2}} e^{-Ny^2/2z} dz
\]

let \( y = \frac{y}{\sqrt{N}} \), \( \text{Erf} (x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt \), \( P_{fa} \) can be expressed as

\[
P_{fa}^{CA} = \int_{0}^{\infty} \left[ 2\Phi \left( \frac{y}{\sqrt{Tz-1}} \right) \right] \sqrt{\frac{N}{2\pi}} \frac{1}{z^{1/2}} e^{-Ny^2/2z} dy
\]

\[
= \frac{2\Phi \left( \frac{y}{\sqrt{Tz-1}} \right)}{\sqrt{\pi}}
\]

by (6), we can see that CA is also a CFAR method for Pearson background. We have

\[
\int_{0}^{\infty} \text{Erf}(bx) e^{-bx^2} dx = \frac{1}{\sqrt{\pi}} \text{arctan} \left( \frac{c}{\sqrt{p}} \right)
\]

by using (7) in (6), we could get the exact elementary expression of \( P_{fa} \):

\[
P_{fa}^{CA} = \frac{2}{\pi} \text{arctan} \left( \frac{1}{\sqrt{TN}} \right)
\]

C. OS CFAR Detectors

The procedure that takes place in an OS CFAR system is shown in Fig. 3.
\[ p_{o_s}^{os} = \sqrt{\frac{2}{\pi}} \left(1/B(k, N-k+1)\right) \int_0^\infty \text{erf}(y/\sqrt{2T}) \]

\[ \{\text{erf}(y/\sqrt{2})\}^{y-x} \times \{\text{erf}(y/\sqrt{2})\}^{y-x} \times e^{-y^2/2} dy \]  

(13)

The detection probability is

\[ p_o^{os} = \sqrt{\frac{2}{\pi}} \frac{1}{B(k, N-k+1)} \int_0^\infty \exp(-y^2/\sigma^2)(y/2\sqrt{T}) dy \]

\[ \{\text{erf}(y/\sqrt{2})\}^{y-x} \times \{\text{erf}(y/\sqrt{2})\}^{y-x} \times e^{-y^2/2} dy \]  

(14)

where \( \text{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2} dt \).

III. FUZZY DETECTOR

In the fuzzy detector proposed in [13], the membership function \( w \) is defined so that it maps the observation space to a value between 0 and 1 indicating the degree to which the test is indicative to the hypothesis 'no signal' and 'signal'. The membership function corresponding to the false alarm space was defined as

\[ w(y) = \Pr(Z > y|Z \in N(0,\sigma^2)) \]  

(15)

here \( y \) is the observation. The membership function monotonically decreases ensuring that stronger observations are assigned smaller membership to the 'no target' hypothesis. If \( w(y) < p_o \), the detector declares a target present. In our case, the clutter power level is unknown, so the threshold should be adaptive. We define the membership function as

\[ w(x) = \Pr(X > x|H_0) = 1 - F_X(x) \]  

(16)

where \( x = q_{cct}/z \), \( q_{cct} \) is the output of cell under test, \( z \) is the estimate of background clutter power level, \( X \) is a variable defined as \( X = Q_{cct}/Z \), here \( Q_{cct} \) and \( Z \) denote the variables corresponding to \( q_{cct} \) and \( z \), and \( F_X(x) \) is the cumulative density function of \( X \), so \( w(x) \) monotonically decreases with \( x \). If \( w(x) \) is smaller than the decision threshold, we declares a target present. It is worth noting that the random variable formed by applying the cumulative distribution function to any continuous random variable is uniformly distributed on [0,1], therefore the distribution of the membership function \( w(x) \) is uniformly distributed on [0,1]. Comparing (10) to (5), we could get the membership function of fuzzy CFAR by changing \( T \) in (8) to \( x \). As to CA-CFAR under Pearson distributed background clutter, we have

\[ w(x) = \frac{2}{\pi} \text{arctan} \left( \frac{1}{\sqrt{xN}} \right) \]  

(17)

The structure of the fuzzy CA-CFAR under Pearson distributed background clutter is depicted in Fig. 4.

IV. DISTRIBUTED FUZZY DETECTOR

In this paper, we consider a distributed system consisting of two antennas and a fusion center as shown in Fig. 6. Each antenna computes the value of membership function \( \mu_{01} \) and \( \mu_{02} \) by the received vector \( Q \), the values are sent to fusion center to produce the global value. Detector compares the global value to a decision threshold to decide whether a target is present. In this paper, we consider four kinds of fusion rules: MAX, MIN, algebraic-product, algebraic-sum.

A. MAX

\[ \mu = \max(\mu_{01}, \mu_{02}) \]  

(20)

the pdf of \( \mu_{pc} \) is given as

\[ f_{\mu}(m) = f_{\mu_{01}}(m) \cdot f_{\mu_{02}}(m) + f_{\mu_{02}}(m) \cdot f_{\mu_{01}}(m) \]  

(21)

since \( \mu_{01}, \mu_{02} \) is uniformly distributed on [0,1], so we have

\[ f_{\mu_{01}}(m) = \frac{1}{m} \]  

\[ f_{\mu_{02}}(m) = \frac{1}{m} \]  

The structure of the fuzzy CA-CFAR under Pearson distributed background clutter is depicted in Fig. 4.

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Figure 6. Two-element distributed fuzzy CFAR detector

\[ f_a = 2 \cdot m, \ 0 \leq m \leq 1 \]  

(22)

The relationship between \( P_\mu \) and \( T \) is given by

\[ P_\mu = \int_0^1 2 \cdot m \cdot dm = T^2 \Rightarrow T = \sqrt{P_\mu} \]  

(23)

If we utilize the binary AND in this CFAR system, the relationship between the global false alarm rate \( P_\mu \) and local false alarm rate \( (P_\mu)_l \) is

\[ P_\mu = (P_\mu)_l \Rightarrow (P_\mu)_l = \sqrt{P_\mu} \]  

(24)

If a target present was declared when using MAX rule, we have

\[ \max(\mu_{\mu_1}, \mu_{\mu_2}) < T \Leftrightarrow \max(\mu_{\mu_1}, \mu_{\mu_2}) < \sqrt{P_\mu} \]

\[ \Leftrightarrow \mu_{\mu_1} < \sqrt{P_\mu} \ \text{and} \ \mu_{\mu_2} < \sqrt{P_\mu} \]  

\[ \Leftrightarrow \mu_{\mu_1} < (P_\mu)_l \ \text{and} \ \mu_{\mu_2} < (P_\mu)_l. \]

So the binary AND fusion rule also declares that there is a target. It indicates that the MAX fusion rule is equivalent to the binary AND.

B. MIN

\[ \mu = \min(\mu_{\mu_1}, \mu_{\mu_2}) \]  

(26)

\[ f_{\mu_{\mu_1}}(m) \cdot (1 - F_{\mu_{\mu_1}}(m)) + f_{\mu_{\mu_2}}(m) \cdot (1 - F_{\mu_{\mu_2}}(m)) = 2(1 - m) \]  

(27)

\[ P_\mu = \int_0^1 2(1 - m) \cdot dm = 1 - (1 - T)^2 \]  

\[ \Rightarrow T = 1 - \sqrt{1 - P_\mu} \]  

(28)

like the MAX fusion rule, we show that the MIN fusion rule is equivalent to the binary OR. In this case, the relationship between the global false alarm probability \( P_\mu \) and the local false alarm probability \( (P_\mu)_l \) is derived as

\[ P_\mu = 2(P_\mu)_l - (P_\mu)_l^2 \Rightarrow (P_\mu)_l = 1 - \sqrt{1 - P_\mu} \]  

(29)

If a target present was declared when using MIN rule, we have

\[ \min(\mu_{\mu_1}, \mu_{\mu_2}) < T \]

\[ \Leftrightarrow \min(\mu_{\mu_1}, \mu_{\mu_2}) < 1 - \sqrt{1 - P_\mu} \]

\[ \Leftrightarrow \mu_{\mu_1} < 1 - \sqrt{1 - P_\mu} \ \text{or} \ \mu_{\mu_2} < 1 - \sqrt{1 - P_\mu} \]  

\[ \Leftrightarrow \mu_{\mu_1} < (P_\mu)_l \ \text{or} \ \mu_{\mu_2} < (P_\mu)_l.\]

So the binary OR fusion rule declares that there is a target. And we see that the MIN fusion rule is equivalent to the binary OR logic.

C. Algebraic product

\[ \mu = \mu_{\mu_1} \cdot \mu_{\mu_2} \]  

(31)

\[ f_\mu(m) = \int_0^1 \frac{du}{u} = -\ln(m) \]  

(32)

The relationship between \( P_\mu \) and \( T \) is given by

\[ P_\mu = \int_0^T -\ln(u)du = T(1 - \ln(T)) \]  

(33)

D. Algebraic sum

\[ \mu = \mu_{\mu_1} + \mu_{\mu_2} \]  

(34)

\[ f_\mu(m) = m \]  

(35)

The relationship between \( P_\mu \) and \( T \) is given by

\[ P_\mu = \int_0^T m dm = \frac{1}{2} T^2 \]  

(36)

V. OSGO/OSSO CFAR DETECTOR

The OSGO and OSSO CFAR detectors’ structure is depicted in Fig. 7, the precious and following sets of cells are independently ordered. Respective outputs are \( Z_1 \) and \( Z_2 \), which are the \( k_1 \)th largest sample of the precious reference window, and the \( k_2 \)th of the following window. The probability density function of \( Z_1 \) and \( Z_2 \) is given by

\[ \frac{1}{2(\gamma/\sqrt{x})^{k_1-1/2} \Gamma(k_1/2)} \exp\left(-\frac{\gamma}{\sqrt{x}}\right) \frac{1}{\sqrt{2\pi}} e^{-\gamma^2/2x} \]  

(37)

\[ \frac{1}{2(\gamma/\sqrt{x})^{k_2-1/2} \Gamma(k_2/2)} \exp\left(-\frac{\gamma}{\sqrt{x}}\right) \frac{1}{\sqrt{2\pi}} e^{-\gamma^2/2x} \]  

(38)

In the data fusion center, OSGO CFAR utilizes the maximum criterion to \( Z_1 \) and \( Z_2 \) to get the clutter level estimation \( Z \), and OSSO CFAR utilizes the minimum criterion. The respective scale factor \( T \) of the two CFAR schemes are obtained according to the given desired false alarm probability, which will be discussed later.
A. OSGO Detector

In OSGO CFAR, as discussed above, the clutter level is estimated by selecting the greater one of \( Z_1 \) and \( Z_2 \), therefore the statistic \( Z_{\text{OSGO}} \) is given by

\[
Z_{\text{OSGO}} = \max(Z_1, Z_2)
\]  

(39)

the probability density function of \( Z_{\text{OSGO}} \) is

\[
P_{\text{OSGO}}(z) = p_{z_1}(z)p_{z_2}(z) + p_{z_2}(z)p_{z_1}(z)
\]  

(40)

here \( p_{z_1}(z) \) and \( p_{z_2}(z) \) is the cumulative density function of \( Z_1 \) and \( Z_2 \). According to this, the probability of false alarm rate for OSGO scheme can be calculated for a fixed scaling factor \( T \) and it does not depend on the dispersion \( \sigma \) of the Pearson-distributed parent population.

The probability of detection of OSGO CFAR is expressed as

\[
P_d^{\text{OSGO}} = \Pr\{Y \geq TZ\} = \int_0^\infty \Pr\{Y \geq Tz\}p_{\text{OSGO}}(z)dz
\]  

(41)

using (37), (38) and (40) in (41), it could be expressed as

\[
P_d^{\text{OSGO}} = \frac{1}{\pi} \frac{2^{1+k+1-1}}{B(k_1, N/2 - k_1 + 1)B(k_2, N/2 - k_2 + 1)} \int_0^\infty \int_0^\infty [2\phi(y) - 1][2\phi(y) - 1]^{1/2-11}/\sqrt{2\pi}e^{-y^2/2}dzdy
\]  

(42)

like OS CFAR, the false alarm rate is controlled by the scaling factor \( T \) and it does not depend on the dispersion \( \gamma \) of the Pearson-distributed parent population.

VI. SIMULATION AND DISCUSSION

We utilize Monte-Carlo simulation method to illustrate the performance of fuzzy CA/CAGO/CASO CFAR detectors using four kinds of fusion rules. The length \( N \) of reference window is 32, and target fluctuates according to Swerling II model, the desired false alarm rate \( P_f = 10^{-4} \). We define the general signal-to-noise ratio GSNR as
\[ GSNR = 20 \log \frac{\sigma}{\gamma} \]  

(51)

Here \( \sigma \) is the fluctuating parameter.

In Fig. 8, we compare the detection performance of identical detection systems using MAX, MIN fusion rules with binary AND, OR logic rules. We take CA-CFAR for example, it is shown that MAX fusion rule performs exactly as binary AND, and MIN fusion rule performs exactly like binary OR, which is constant with the theory analysis.

In Fig. 9, we plot the detection probability of fuzzy CA CFAR detector against different general GSNR under homogenous clutter. We could observe that the detection performance of detector system using algebraic sum and MAX fusion rules achieves at \( P_d \geq 0.5 \) when \( GSNR = 70dB \). When \( 70dB \leq GSNR \leq 95dB \), the detector using algebraic sum fusion rule offers best detection performance. When \( GSNR > 95dB \), the performance of the algebraic sum fusion rule is very close to algebraic product fusion rule, and better than MAX and MIN fusion rule. We could conclude from the simulation results that the algebraic sum fusion rule is the most robust scheme compared to other fuzzy fusion rules.

We analyze the detection performance of CASO detector based on fusion rules under multi-target environment. Fig. 10 is the case that there are two interrupt targets in left reference window, Fig. 11 is the case when there are two in the right window and one in the left window with equal generalized interference signal-to-noise which is 20dB. As we can see, in two cases, CASO detector based on algebraic-sum fusion rules is the most robust to the interrupt targets.

In Table I, we compare the false-alarm-control capacity of fuzzy CAGO CFAR detector under non-homogeneous condition such as clutter-edge environment, which is very common due to snow, rain and wind in practical condition. The numerical results in table I show that the false alarm rate control capacity of MIN rule is better than those of other rules.

We compare the detection performance of the OSGO, OSSO and OS CFAR detectors in the presence of multiple targets with equal generalized interference signal-to-noise ratio (GINR), GINR=20dB. In Fig. 12, there are two interrupt targets in the reference window, OSSO is the best detector. When detection probability is

<table>
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<th>Table I. False Alarm Rate Control of Four Fuzzy Rule</th>
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<tr>
<td>ratio</td>
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<tr>
<td>20dB</td>
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50%, OSSO exhibits 4dB superiority in GINR than OS, OSGO shows the worst detection probability.

In Fig. 13, when four interrupt targets exist in left reference window or right reference window, the performance of OS and OSGO CFAR detectors began to degrade, and suffer more serious detection loss, while OSSO still keep robust, 6dB GINR improvement is gained. OSGO shows comparable performance with OS when interrupt target increases. When the interrupt targets number is beyond 4, as shown in Fig. 14, the OS and OSGO CFAR detection schemes appear totally invalidated, but the OSSO CFAR is still not very affected.

It is worth to point out the OSGO CFAR detector has all the advantages of OS CFAR with a negligible CFAR loss when the interrupt target number is not too large, OS complexity is $O(N^2)$, as to OSGO and OSSO, the complexity is $(1/2)O(N^2)$, so it needs only one half of processing time compared to OS CFAR detector.

VII. CONCLUSION

In this paper, we have proposed several kinds of new CFAR detector, analyzed the performance in multi-target and clutter-edge environment, when the background clutter follows Pearson distribution. Fuzzy CA/CAGO/CASO detectors offer detection superiority and better false-alarm-control capacity, OSSO CFAR is more robust to interrupt targets than OS, OSGO CFAR owns all the advantages of OS CFAR. Performance analysis of multiple-sensor network system is the next step of research.

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