Model for Transversely Isotropic Materials Based on Distinct Lattice Spring Model (DLSM)

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***Abstract*—A two-phase DLSM model is proposed to model transverse isotropic materials, in which the layer structure of the anisotropic materials is explicitly represented. Equations are derived to obtain elastic parameters of the two phase model from the elastic modules in parallel to the bedding planes and that in normal direction of the bedding planes. Then, the mechanical properties of the two-phase DLSM model are studied and compared with experimental observations. Results indicate that the mechanical properties of transverse isotropic materials can be well represented by the proposed model. Following this, the model is applied to study the influence of the weakness layers (bedding planes) on the anisotropic deformation and Damaged Zone (DZ) evolution of underground disposal galleries in transverse isotropic medium. Finally, some conclusions are presented.**

***Index Terms*—transverse isotropy, DLSM, third term, a two-phase model, weakness layers**

I. INTRODUCTION

In the various sedimentation processes or the subsequent burial history of rock formations, well defined fabric elements can be found in many kinds of rocks, in the form of bedding, stratification, foliation, fissuring or jointing. In general, these rocks have anisotropic properties in physical, thermal, mechanical, hydraulic those varied with direction [1]. For most practices, anisotropic rocks can be classified as orthotropic or transverse isotropic medium. Orthotropic implies that three orthogonal planes of elastic symmetry exist at each point in the rock and that these planes have the same orientation throughout the rock. Transverse isotropy indicates that at each point in the rock there is an axis of symmetry of rotation, and the rock has isotropic properties in a plane normal to that axis. The plane is called the plane of transverse isotropy [2]. This paper deals exclusively with the modeling of transverse isotropic materials.

The mechanical behavior of anisotropic rock can be modeled through distinct element approach [3], discontinuities can be explicit represented. However, when the numbers of discontinuities are excessive large, the distinct element approach is not accessible due to the limitation of computational power. Here, a continuum description of the anisotropic material is needed to provide an average description of the response of the rock under mechanical unloading. This kind of model is called as implicit joint model [4], in which joints are implicit in the choice of constitutive law for the equivalent continuum. For example, the representative one is the Ubiquitous Joint model in FLAC [5] which can be used to model anisotropic material. However, the implicit joint model also suffer some inherit shortcomings, e.g., recently it is reported that the Ubiquitous Joint model will completely break down when the rock layer undergo bending during loading [4].

The Distinct Lattice Spring Model (DLSM) [6, 7] is a discrete numerical model, in which the material is discretized into particles linked by springs. As a discrete approach, it is more suitable to simulate the problems of failure process than conventional continuum based methods, it have already been used in the study of dynamic cracking propagation of PMMA [8] and dynamic strength of rock materials [9]. Recently, the DLSM is extended to study the anisotropic materials [10]. In this paper, formulations of the two-phase model in DLSM to represent the transverse isotropic materials are introduced. Then, the mechanical properties of the two-phase DLSM model are studied and compared with experimental observations. Following this, verifications and application of the two-phase model are presented. The influence of particle size and orientation of bedding planes on the mechanical behavior of the transverse

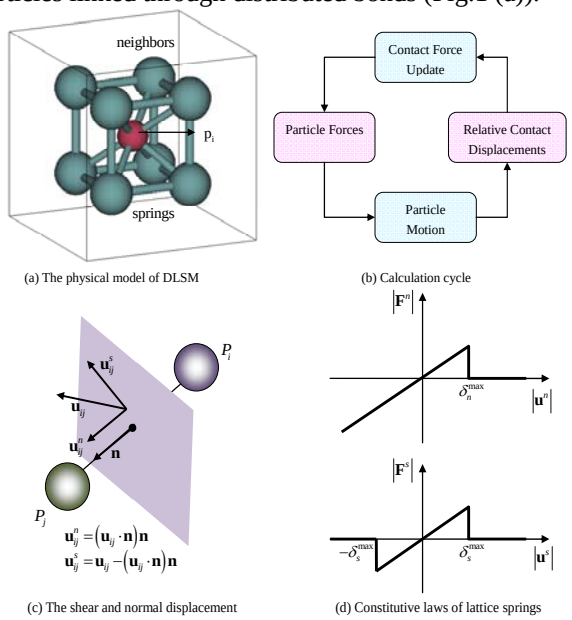
doi:10.4304/jcp.6.6.1139-1144

isotropic rock are studied and compared with experimental observations. Then, the influence of direction of bedding planes on the Damaged Zone (DZ) evolution of underground circular galleries is simulated by the proposed two-phase DLSM model. Finally, some conclusions are concluded and discussed.

II. Model Transverse Isotropic Material based on DLSM

A. Distinct Lattice Spring Mode (DLSM)

The Distinct Lattice Spring Model (DLSM) is a microstructure based numerical model based on the RMIB theory [7, 11] which is an extension of VMIB model [12]. In DLSM, material is discretized into mass particles linked through distributed bonds (Fig.1 (a)).



(a) The physical model of DLSM (b) Calculation cycle

(c) The shear and normal displacement (d) Constitutive laws of lattice springs

Figure 1. The physical model, calculation cycle, particle spring deformation and its constitutive law in DLSM .

Due to the explicit considerations of the microstructure of the material, the proposed model can give more realistic modeling of material failure behavior than continuum based methods. Based on Cauchy-born rules and the hyperelastic theory, the relationship between the micromechanical parameters and the macro material constants, i.e. the Young's modulus and the Poisson ratio is obtained:

$$k_n = \frac{3E}{\alpha^{3D}(1-2v)}, k_s = \frac{3(1-4v)E}{\alpha^{3D}(1+v)(1-2v)} \tag{1}$$

where k_n is the normal stiffness of the spring, k_s shear stiffness, E Young's modulus, v Poisson ratio and α^{3D} is a microstructure geometry coefficient which can be obtained from:

$$\alpha^{3D} = \frac{\sum l_i^2}{V} \tag{2}$$

where l_i is the original length of the ith bond, V is the volume of the geometry model. The details of this model can be found in [9]. DLSM is a particle based numerical method. The particles and springs make a whole system which represents the material. For this system, its motion equation is expressed as

$$[\mathbf{K}]\mathbf{u} + [\mathbf{C}]\dot{\mathbf{u}} + [\mathbf{M}]\ddot{\mathbf{u}} = \mathbf{F}(t) \tag{3}$$

where u represents the vector of displacement, $[\mathbf{M}]$ the diagonal mass matrix, $[\mathbf{C}]$ the damping matrix and $\mathbf{F}(t)$ the vector of external forces on particles. The calculation cycle is illustrated in Fig.1 (b). Given the particle displacements (either prescribed initially or obtained from the previous time step), new contacts and broken bonds are detected. The list of neighboring particles for each particle is updated. Then, contact and spring forces between particles are calculated according to the prescribed force-displacement relations.

The interaction between particles is represented by one normal spring and one shear spring. The shear spring is a multi body spring which is different from the conventional lattice spring methods. The multi-body shear spring is introduced to make the model handle problems which Poisson's ratio is beyond 0.25. The behavior of normal spring is in a conventional way. For example, there existing one bond between particle i and particle j. The unit normal n (nx, ny, nz) points form particle i to particle j. The relative displacement is calculated as

$$\mathbf{u}_{ij} = \mathbf{u}_j - \mathbf{u}_i \tag{4}$$

Then, the vector of normal displacement and interaction force between two particles (Fig 1(c, d)) can be given as

$$\mathbf{u}_{ij}^n = (\mathbf{u}_{ij} \bullet \mathbf{n})\mathbf{n} \text{ and } \mathbf{F}_{ij}^n = k_n \mathbf{u}_{ij}^n \tag{5}$$

where k_n is the stiffness of the normal spring and $\mathbf{n}$ is the normal of the bonds. The multi-body shear spring between two particles is introduced through a spring with a multi-body shear displacement vector which can be obtained from:

$$\hat{\mathbf{u}}_{ij}^s = [\boldsymbol{\varepsilon}]_{bond}\mathbf{n}^{\mathbf{T}} - \left(\left([\boldsymbol{\varepsilon}]_{bond}\mathbf{n}^{\mathbf{T}}\right)\bullet\mathbf{n}\right)\mathbf{n} \tag{6}$$

where $[\boldsymbol{\varepsilon}]_{bond}$ is the strain state of the bond, it is e-valuated by DLSM method. Then, the shear interaction between two particles is given as

$$\mathbf{F}_{ij}^s = k_s \hat{\mathbf{u}}_{ij}^s \tag{7}$$

where k_s is the stiffness of the shear spring. Equations (4) to (6) provide the formulas for force update in DLSM. For the displacement update, the particle velocity is advanced individually as

$$\dot{\mathbf{u}}_i^{(t+\Delta t/2)} = \dot{\mathbf{u}}_i^{(t-\Delta t/2)} + \frac{\sum \mathbf{F}_j^{(t)}}{m_p}\Delta t \tag{8}$$

where $\dot{\mathbf{u}}_i^{(t+\Delta t/2)}$ is the particle velocity at $t+\Delta t/2$, $\dot{\mathbf{u}}_i^{(t-\Delta t/2)}$ is the particle velocity at $t-\Delta t/2$, m_p is the particle mass, $\sum \mathbf{F}_j^{(t)}$ is the sum of forces acting on the particle i including applied external forces and Δt is the time step. Finally, the new displacement of particle is obtained as

$$\mathbf{u}_i^{(t+\Delta t)} = \mathbf{u}_i^{(t)} + \dot{\mathbf{u}}_i^{(t+\Delta t/2)}\Delta t \tag{9}$$

where $\mathbf{u}_i^{(t+\Delta t)}$ is the displacement at $t+\Delta t$, $\mathbf{u}_i^{(t)}$ the displacement at t. These two equations are the main procedure involved in the implementation of DLSM. Details of the implementation of DLSM and verifications of DLSM on modeling elastic problems can be found in [6, 7].

B. Represent Transverse Isotropic Material in DLSM

The transverse isotropic materials are represented by a two phase model which made up from the matrix and the weakness materials (see Fig.2 (a)). Moreover, two macro elastic moduli are considered, i.e., the elastic modulus parallel with the bedding plane E_p^{macro} and that in normal direction of the bedding plane E_n^{macro}. The moduli are obtained as [13]:

$$E_p^{macro} = E_{base}\left(1+\beta(\alpha-1)\right) \tag{10}$$

$$E_n^{macro} = E_{base}\frac{\alpha}{\alpha(1-\beta)+\beta} \tag{11}$$

where E_{base} is the elastic modulus of the matrix material, $\alpha = E_{weak}/E_{base}$ is the ratio of the elastic modulus of weak material to that of the matrix material and β is the volume contain ratio of the weak material. Then, the anisotropic ratio κ is obtained as

$$\kappa = \frac{E_n^{macro}}{E_p^{macro}} = \frac{\alpha}{\left(\alpha(1-\beta)+\beta\right)\left(1+\beta(\alpha-1)\right)} \tag{12}$$

The 3D surface of Equation (12) is given in Fig. 2(b). Then, the volume ratio β and anisotropic ratio κ are given, the α can be determined through the curve of κ versus α. The elastic modules of the matrix and weak material can be obtained as

$$E_{base} = \frac{E_p^{macro}}{\left(1+\beta(\alpha-1)\right)} \text{ and } E_{weak} = \alpha E_{base} \tag{13}$$

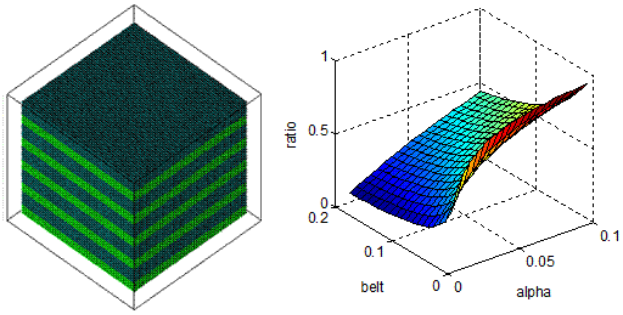
The Poisson's ratio of both the matrix material and the weak material is taken the value of the macro Poisson's ratio in direction parallel with the bedding plane. Relationship between macro and micro failure parameter is given as [7]

$$u^* = \frac{\sigma_t d^*}{E} \tag{14}$$

where u^* is the ultimate deformation for the bond spring in DLSM and d^* is the mean particle size. For the anisotropic DLSM model, there will have two parameters related to matrix and weak materials, respectively. They are suggested to taken as

$$u_{base}^* = \frac{\sigma_t^p d^*}{E_{base}},\ u_{weak}^* = \gamma\frac{\sigma_t^n d^*}{E_{weak}} \tag{15}$$

where u_{base}^* and u_{weak}^* are the ultimate bond deformation of matrix and weak material, respectively, γ is a safety factor for the weak material. The reason is that weakness layers usually can bear larger deformation than that of the matrix material (in this paper $\gamma = 100$).



(a) Two-phase model (b) The anisotropic ratio κ

Figure 2. Representation of transverse isotropic material in DLSM and relationship between the anisotropic ratio κ and ratio of elastic modulus α and ratio of volume β between the matrix and the weak materials.

III. Verifications

In this section, the two-phase DLSM model is verified through numerical experiments. The computational models for uniaxial compressive test are shown Fig.3. Two models with different orientation of bedding plane are presented. The apparent elastic modulus (the elastic modulus in the loading direction) is obtained from the strain stress curves of the two-phase models.

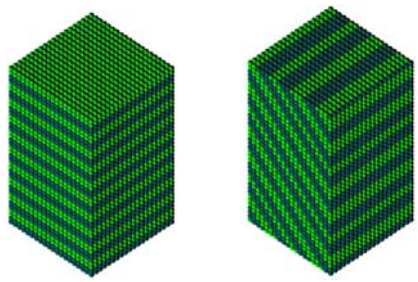
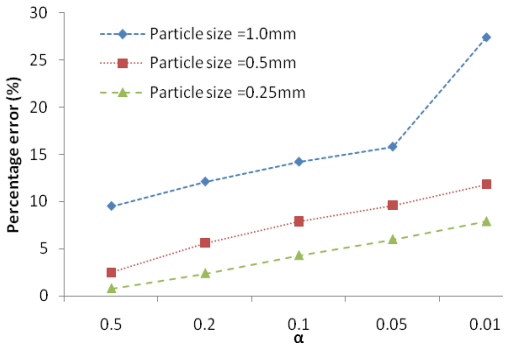


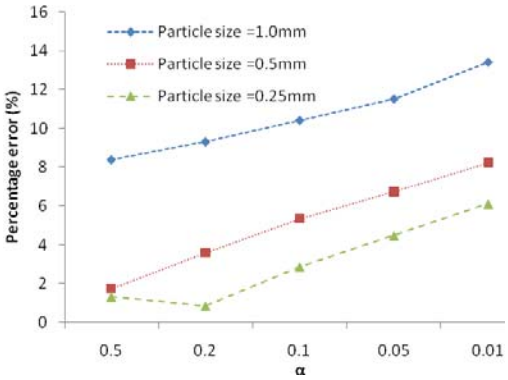
Figure 3. The two phase DLSM model used for the uniaxial compression test.

First, the two-phase models with volume ratio under different particle size are simulated. The orientation of bedding plane is 0 degree and the apparent elastic modulus can be obtained from Equation (11). The percentage errors between the value obtained from the two-phase DLSM model and that from Equation (11) are shown in Fig.4. It indicates that the numerical error decrease with the particle size and increase with the weakness ratio. The requirement on particle size for the two-phase model is high, e.g., only the 0.25mm particle

model can provide an average error less than 5%. It means the two-phase DLSM model with coarse particle size can only provide a quantitative analysis.



(a) β=0.5



(b) β=0.25

Figure 4. The percentage error of the two-phase DLSM models with different particle size.

Then, the numerical experiment on transverse isotropy rock is performed by the two-phase DLSM model. For these simulations, the particle size is taken as 0.5mm and the volume ratio is taken as 0.5. In reference [1], the Young's moduli in the plane of isotropy and in a direction normal to it are 741MPa and 196MPa separately. The elastic parameters of the two-phase DLSM models are obtained from Equation (10) to (13). The modeling results are shown in Fig. 5, where the results between the two-phase model and experimental data are well fitted.

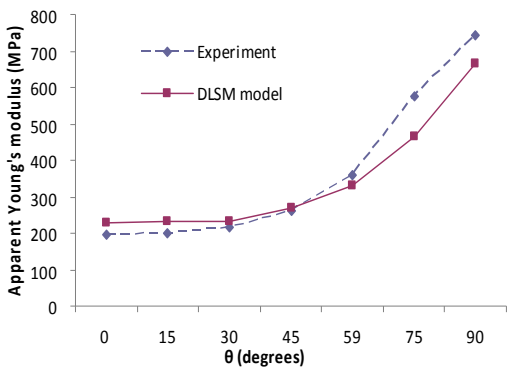
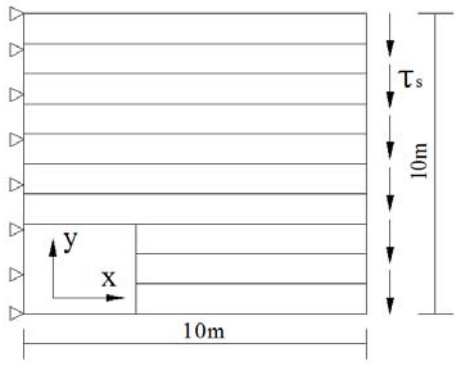


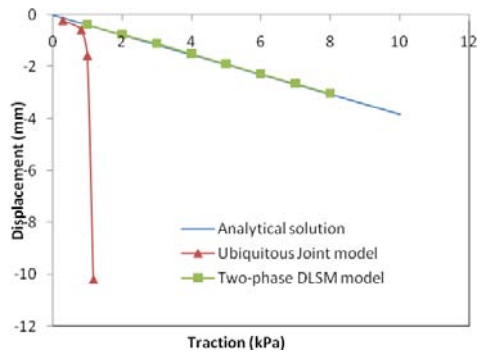
Figure 5. The apparent Young's modulus predicted by two-phase DLSM and the experimental data.

Moreover, a beam bending problem as described in [4] are solved by the two-phase DLSM model. As shown in Fig.6 (a), the beam is made up from layers which clamped on the left side and traction is applied on the right side. The material properties of the rock layer are taken as E=10GPa, v=0.2 and the shear strength of the joint plane is assuming zero. The deformation solution of y direction is given [4]:

$$u_y = \frac{4\tau_s l^3}{Eh^2}\left(1-v^2\right) \tag{16}$$



(a) The schematic of the beam bending problem



(b) Results of the two-phase DLSM model

Figure 6. The two-phase DLSM model for the beam bending problem and modeling results.

The simulation results are shown in Fig.6 (b), the two-phase DLSM model agrees well with the analytical solution, whereas the Ubiquitous Joint model produces rubber like material without bending stiffness. It can be concluded that the proposed two-phase DLSM model can reflect the mechanical properties of transverse isotropy rock and provides a solution for the DLSM to solve anisotropic problems. However, the requirement on particle size is very strict for precision analysis. For this reason, a more quantitative application is performed in following.

IV. Applications

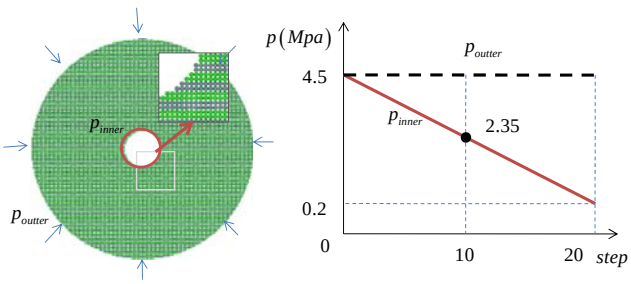
A. Computational Model

The computational model used to simulate a kind of indurated clay (Opalinus Clay), which is the potential host rock for nuclear waste disposal galleries in

Switzerland (Fig. 7(a)). Recently, many researches have awareness that the importance of bedding plane to sound estimate the construction of the galleries of Opalinus Clay [14]. The two-phase model as described in previous section is used to simulate this transverse isotropic rock. The dimension of the model is the same as the testing sample in the laboratory, the diameters of inner and outer hole are 14mm and 86mm, respectively. The mechanical parameters are taken from Mont Terri rock laboratory in Switzerland [15]. Here, the volume ratio β used in DLSM model is given as 0.5. From Equations (10) to (15), the mechanical parameters needed for the two phase model are calculated and listed in TABLE I

TABLE I. . *THE MECHANICAL PARAMETERS USED IN THE TWO PHASE MODEL OF DLSM.*

	E (MPa)	ν	$\rho\left(kg/m^3\right)$	$u^*(mm)$
Matrix material	12850	0.24	2400	7.80e-5
Weak material	1500	0.25	2400	3.20e-1



(a) DLSM model (b) The unloading process

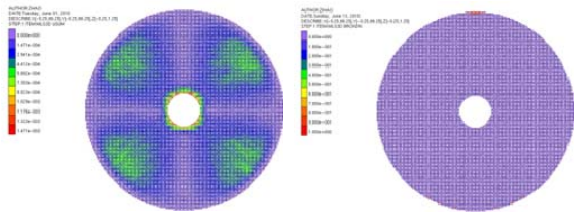
Figure 7. Two-phase computational model for transverse isotropic rock (green layers are matrix, grey layers are weakness materials).

B. Modeling Failure Process of Indurated Clay

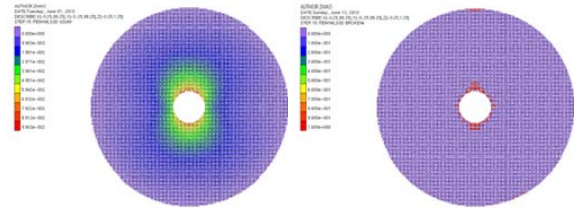
The numerical simulation is aim to describe the failure process and evolution of Damaged Zone (DZ) under mechanical unloading, those are similar to that will be experienced by host rocks around disposal galleries. The mechanical unloading process is simulated by reducing the borehole stress from 4.5 to 0.2MPa step by step, however, the stress around outer boundary keeps constant at 4.5MPa (Fig.7 (b)). The initial strain and stress state is obtained by a static analysis. Then, the explicit dynamic calculation is performed. The DLSM modeling results of mapping displacement contour and fractures extension around the disposal galleries in transverse isotropic medium are shown in Fig. 8. In the process of decrease of borehole pressure, the main cracks develop along the matrix layer by layer and then extend normal to bedding planes. The movement of particles in bedding planes direction is smaller than in the vertical ones, an oval-shape damage zone induced. It indicates bedding planes seriously effect on the mechanical anisotropy of this indurated clay.

V. CONCLUSIONS

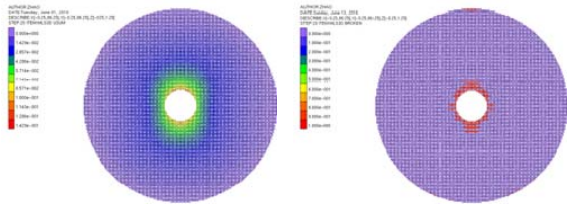
A two phase model is proposed to enhance the usability of DLSM model on transverse isotropic materials. Verifications and applications of the proposed model are presented to show the correctness and applicability of the two-phase DLSM model. The results the two-phase model are well represented the failure process of the transverse isotropic rock mass, the anisotropy deformation can be observed around the galleries. It can be concluded that the proposed model can provide a quantitative analysis tool for transverse isotropic materials although using a coarse particle model. Moreover, as the explicit consideration of the layer structure, the introduced model can overcome some shortcomings of the implicit joint model. Further improvement on the computational power of the DLSM and study on the failure mechanism on the two-phase model are needed.



(a) Inner pressure is 4.5MPa



(b) Inner pressure is 1.5MPa



(c) Inner pressure is 0.2MPa

Figure 8. Evolution of DZ around the nuclear waste disposal galleries decreasing the central pressure.

ACKNOWLEDGMENT

The authors wish to thank the financial supports from the China Scholarship Council, TIMODAZ project which is co-funded by the European Commission and performed as part of the sixth EURATOM Framework Programme for nuclear research and training activities (2002-2006) under contract FI6W-CT-2007-036449, and the Swiss National Science Foundation (200021-116536). Further acknowledgements go to LMR-EPFL who gives the technical support in the experiments.

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