On the Determination of the Damping Coefficient of Non-linear Spring-dashpot System to Model Hertz Contact for Simulation by Discrete Element Method

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Abstract—**The Discrete Element Method (DEM) is widely used in the simulation of a particle system. The viscoelastic contact models are the most common ones in the DEM simulation. However, these models stills have a few unrealistic behaviors, which will influence the reasonableness and accuracy of the simulation results. A general form of damping coefficient is proposed through dimensional analysis, and the condition is found so that the unrealistic behaviors can be solved. Regardless of adhesiveness and plasticity, an improved model based on Hertzian theory is proposed, in which the approximated expression relating the damping constant to the restitution coefficient is given. The impact of a single ball to a wall is simulated by using three different models, of which the results of contact force are presented and discussed.**

Index Terms—**Contact model, contact parameters, damping coefficient, discrete element method.**

I. Introduction

The Discrete Element Method (DEM), proven to be a reliable and promising approach for investigating granular systems, has been widely used in many areas, for example originally in geotechnical mechanics by Cundall and Strack [1], later in pneumatic transport technology by Tsuji *et al.* [2], in fluidized beds by Kawaguchi *et al.* [3], in tumbling ball mills by Mishra and Rajamani [4], and Cleary and Hoyer [5].

The basic assumption in DEM is that during a small time step, the disturbances cannot propagate from any particle to the others except its immediate neighbors. So the actions at any particle at any time are limited to the several neighboring particles. The process of DEM can be divided into two parts: (a) using a contact model to calculate the contact forces acting on any contact point; (b) applying Newton's second law of motion to solve the behavior of particles by any resultant force.

The contact model in DEM plays a critical role in simulating the behavior of elements in the particle assemblies. If it can not well characterize the real contact between particles, the results of simulation will not be reasonable and satisfying. It is important to study the basic contact model and the correlative parameters, such as stiffness, damping, etc. Currently, there are some contact models that have been introduced in many literatures involving in DEM or its applications on analyzing granular assemblies. The linear spring-dashpot model, first proposed by Cundall and Strack [1] to simulate dry granolas, is the most commonly used one in the actual applications of DEM because of its simplicity and efficiency. In that model, a spring is employed to model the elastic part and a dashpot is adopted to describe the dissipative mechanism, as the contact is considered to be viscoelastic in the both normal and tangential directions, regardless of plastic deformation and adhesiveness, etc. Once the macro-slip of the contact surface occurs, the slider works instead to describe the resulting energy dissipation, i.e. Coulomb friction replaces the spring-dashpot in tangential direction.

Some other investigators in the view of contact mechanics, introduced a few more accurate and sophisticated contact relations [6], [7]. For example,

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Hertzian contact theory is used to provide the elastic force in the normal direction, which is called normal force-displacement model based on Hertzian theory. As for the tangential direction, the Mindlin solution or its simplification is applied to solve the tangential impact force, which is the non-linear tangential force-displacement model considering the case of no-slip or micro-slip.

However, there are still some comprehensive problems for those commonly used linear and non-linear models. Zhang and Whiten [8] examined the reasonableness and accuracy of linear and non-linear DEM models, and found that before the particles separate the forces change direction. Thus the force is pulling the two particles towards each other instead of making them apart before they separate, which is inconsistent with the actual contact process of dry particles. So it is necessary to find a better form of the damping term to make the impacting force and the displacement return to zero at the same time in a contact period.

In this paper, a new non-linear damping is proposed to keep the impacting force in the contact model from changing direction. And the approximated expression, relating the damping ratio to restitution coefficient in the model applying a new damping, is derived by curve fitting method.

II. NORMAL CONTACT

The widely used normal contact model is composed of two parts: elastic force and energy dissipation mechanism. In the schematic of the normal contact model, a linear or non-linear spring is adopted to provide the elastic force $\vec{f}_n^e$ and a damping term provides the dissipative contribution to the normal force.

A. Linear Spring and Damping Models

The linear model, given by Cundall and Strack [1], is the most widely used one in the DEM. When particle *i* has a central impact with particle *j*, their normal force acting at the contact point consists of the elastic repulsive force $\vec{f}_n^e$ and the damping force $\vec{f}_n^d$. The former can be given by a linear spring with a constant stiffness k_n, while the latter is provided by a damper dependant on the velocity with a constant damping coefficient c_n. Then the normal contact force acting on particle *i*, is given by the sum of the two component forces:

$$\vec{f}_n = \vec{f}_n^{\,e} + \vec{f}_n^{\,d} = -k_n\delta_n - c_n\dot{\delta}_n \tag{1}$$

where k_n is the spring stiffness or elastic constant, c_n is the damping coefficient, δ_n and $\dot{\delta}_n$ denote the relative displacement and velocity of particle *i* and *j*, respectively.

The damping coefficient c_n can be defined as follows:

$$c_n = \alpha \cdot c_n^c = \alpha \cdot 2\sqrt{m^* k_n} \tag{2}$$

where α is the so-called damping ratio without dimension, c_n^c is the critical damping coefficient, and m^* is the equivalent mass of particle *i* and *j*, given by: $\left(m^*\right)^{-1} = m_i^{-1} + m_j^{-1}$.

That model has been specially analyzed by Zhang and Whiten. Their article provided the corresponding analytic solutions for this model when the dimensionless damping ratio α is taken different values, i.e. for the cases $\alpha < 1$, $\alpha = 1$ and $\alpha > 1$. When $\alpha < 1$, the explicit expression related to the damping ratio α and the restitution coefficient e_n can be derived as follows:

$$\alpha = -\ln\left(e_n\right)\sqrt{\frac{1}{\ln^2\left(e_n\right)+\pi^2}}. \tag{3}$$

As the constant restitution coefficient is defined, the damping factor c_n can be determined via (2) and (3).

Di Maio and Di Renzo [9] gave the equivalent stiffness k_n derived from the actual force-displacement relation, and the following expression for k_n can be obtained by using a common ratio of estimated maximum impact force to displacement:

$$k_n = \left(\frac{320}{81} m^* \dot{\delta}_{n0}^2 E^{*4} R^{*2}\right)^{1/5} \tag{4}$$

where E^* and R^* are the equivalent Young's modulus and radius of particle *i* and *j*, respectively.

B. Non-Linear Spring and Damping Models

The normal elastic constant based on Hertz contact theory is

$$k_n = \frac{4}{3} E^* \sqrt{R^*}. \tag{5}$$

Tsuji *et al.* introduced a non-linear damping term, which is a function of displacement δ_n and velocity $\dot{\delta}_n$. Zhang and Whiten noted that Tsuji's non-linear contact model is more realistic and closer to the experimental results than the linear model. The damping coefficient was found heuristically and defined as:

$$\eta = \alpha_1 \sqrt{m^* k_n}\,\delta_n^{1/4} \tag{6}$$

where α_1 is a constant depending only on the coefficient of restitution e_n.

Then the damping force and the normal contact force in the non-linear model are given by: $\vec{f}_n^d = -\eta\dot{\delta}_n = -\alpha_1\sqrt{m^* k_n}\,\delta_n^{1/4}\dot{\delta}_n$, and $\vec{f}_n == -k_n\delta_n^{3/2} - \eta\dot{\delta}_n = -\frac{4}{3}E^*\sqrt{R^*}\,\delta_n^{3/2} - \alpha_1\sqrt{m^* k_n}\,\delta_n^{1/4}\dot{\delta}_n$.

There are analytic solutions with respect to the damping constant α_1 and the restitution coefficient e_n in the governing equation of the non-linear model, and the explicit expression is:

$$\alpha_1 = -\ln\left(e_n\right)\sqrt{\frac{5}{\ln^2\left(e_n\right)+\pi^2}}.. \tag{7}$$

III. THE NEW NON-LINEAR DAMPING COEFFICIENT

Hunt and Crossley [10] proposed a damping term $\lambda\delta(t)^p \dot{\delta}(t)^q$, where the exponent q equals to 1 commonly, and the coefficient λ and the exponent p are unidentified. Then the damping force is $\vec{f}_n^d = \lambda\delta_n^p \dot{\delta}_n$. The non-linear elastic force-displacement relationship is defined as: $\vec{f}_n^e = k_{nw}\delta_n^w$, where w>0, and the dimension of the so-called elastic constant k_{nw} is N/m^w.

Putting the elastic force and the damping force together, we obtain a new non-linear contact model. The governing equation of motion using this model is as follows:

$$m^*\ddot{\delta}_n(t) + \lambda\delta_n(t)^p \dot{\delta}_n(t) + k_{nw}\delta_n(t)^w = 0. \tag{8}$$

where λ is the non-linear damping coefficient.

By a dimensional analysis, a new term of the damping coefficient λ is proposed as: $\lambda = \alpha_2 m^* \dot{\delta}_{n0}\left(k_{nw}/m^*\dot{\delta}_{n0}^2\right)^{(1+p)/(1+w)}$, where α_2 is a dimensionless damping constant depending only on the restitution coefficient e_n, $\dot{\delta}_{n0}$ represents the estimated initial impact velocity in the normal direction.

To make each term non-dimensional, we replace displacement δ_n and time t in (8) with: $\delta_n = \left(m^*\dot{\delta}_{n0}^2/k_{nw}\right)^{2/5}\hat{\delta}_n$ and $t = \left[\left(m^*\dot{\delta}_{n0}^2/k_{nw}\right)^{2/5}/\dot{\delta}_{n0}\right]\hat{t}$. Equation (8) can be rewritten as:

$$\ddot{\hat{\delta}}_n(\hat{t}) + \alpha_2\hat{\delta}_n(\hat{t})^p \dot{\hat{\delta}}_n(\hat{t}) + \hat{\delta}_n(\hat{t})^w = 0.. \tag{9}$$

The initial conditions for the equation is set to $\hat{\delta}_n(\hat{t})\big|_{\hat{t}=0} = 0$, $\dot{\hat{\delta}}_n(\hat{t})\big|_{\hat{t}=0} = \dot{\delta}_n(t)\big|_{t=0}/\dot{\delta}_{n0} = 1$. Heuristically, a case is found: $p = (w-1)/2$, so that there are analytic solutions for (9).

The relational expression between the restitution coefficient e_n and the damping constant α_2 can be derived by setting the normalized displacement to zero, which is:

$$\alpha_2 = -\ln(e_n)\sqrt{\frac{2(1+w)}{\ln^2(e_n)+\pi^2}}.. \tag{10}$$

It is observed that (3) and (7) are the specific cases of (10), in which w = 1 and w = 3/2, respectively. Though, there is a little difference for (3), due to the damping constant is double of the damping ratio as w selects the value of 1.

The purpose of our study is to find a realistic model in which the contact force does not change its direction until the end of collision. The normalized damping force $\hat{f}_n^d$ and elastic force $\hat{f}_n^e$ constitute the total contact force, and can be respectively written as follows: $\hat{f}_n^d = \alpha\hat{\delta}_n(\hat{t})^p \dot{\hat{\delta}}_n(\hat{t})$ and $\hat{f}_n^e = \hat{\delta}_n(\hat{t})^w$.

In a short time period before the two particles separate, the value of the displacement is very small and tends to zero, meanwhile the sign of the velocity is opposite to the displacement, so that the damping force component is mainly depended on the displacement, and acting as resisting force for separation. Therefore, only if the factors p and w satisfy the case $p \geq w$, the absolute value of damping force will be smaller than that of the elastic force, and thus the total contact force will be still acting as repulsive force during collision.

While $p = (w-1)/2$, it contradicts the condition $p < w$, which means that the model has no such analytic solutions. Instead, we can get their numerical solutions via the computing power.

Hunt and Crossley noted that when the velocity of initial collision is small, the exponent p in the damping coefficient may be selected to a value identical with the non-linear exponent in the elastic force. In the light of Hertzian contact theory, the nonlinear elastic factor w should be selected as 3/2.The exponent of the damping coefficient p is chosen as 3/2, which is equal to the elastic factor w. The corresponding damping coefficient λ becomes

$$\lambda = \alpha_1 k_{nw}/\dot{\delta}_{n0}. \tag{11}$$

Putting the value of p (9), we obtain:

$$\ddot{\hat{\delta}}_n(\hat{t}) + \alpha_2\hat{\delta}_n(\hat{t})^{3/2} \dot{\hat{\delta}}_n(\hat{t}) + \hat{\delta}_n(\hat{t})^{3/2} = 0.. \tag{12}$$

According to the initial conditions, we make numerical differentiation calculations with respect to (12), and get the relationship between the damping constant α_2 and the restitution coefficient e_n, as shown in Fig. 1.

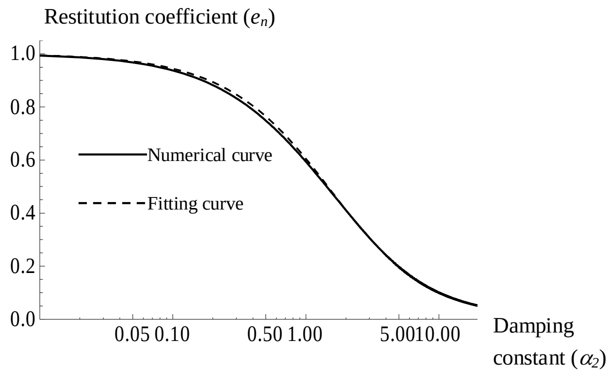

Figure 1. The relationship between the damping constant α_2 and the restitution coefficient e_n

The approximate relation expression between the parameter α_2 and the restitution coefficient e_n by numerical fitting is expressed by:

$$\alpha_2 = -\ln(e_n)\frac{b}{c+\ln(e_n)}. \tag{13}$$

where b and c are constants, and b = 6.66264, c = 3.85238, respectively; and the domain of variables in the above equation are $\alpha_2 \in [0,20]$, or $e_n \in [0.05,1.0]$.

In Fig. 1, the fitting dash curve corresponding to (13) is close to the solid curve corresponding to the numerical solution. However, to make sure the value calculated by (13) is sufficiently precise, the constant α_2 should be selected as small as possible.

When the coefficient of restitution e_n is given, the value of α_2 can be determined from Fig. 1, and then the damping coefficient λ is obtained by (11).

IV. NUMERICAL SIMULATION

The normal impact between a single ball and a plane wall with an initial velocity of 6.331m/s is considered, which will be simulated by using the linear model, the nonlinear model given by Tsuji and our nonlinear model based on Hertzian theory, respectively. According to the above discussion, the basic parameters in the contact models are determined and displayed in Table I, with a uniform estimated overlap $1/f$ = 0.5%.

TABLE I
KEY PARAMETERS IN THE CONTACT MODELS

Parameters	Linear model	Tsuji model	Our model
k_n	3.357×107(N/m)	5.935×109 (N/m$^{3/2}$)	
α	0.034	0.075	0.187
m*	2.094×10^{-3} (kg)		

The governing equations of the ball's motion can be numerically solved by directly using the parameters in Table I. The results of the numerical simulation are presented in Fig. 2, which shows the variations of the normal impact force with displacement. The area of the closed curves in Fig. 2, corresponding to the three contact models respectively, is the dissipative energy in the impact. Unlike the other two cases, for our non-linear models using the new damping, the area consisted of the loading and unloading curves decreases sharply as the displacement δ_n approaches to zero. It means that the energy dissipation is mainly confined in the area with medium and large displacements. The damping term in this model is a function of displacement δ_n and velocity.

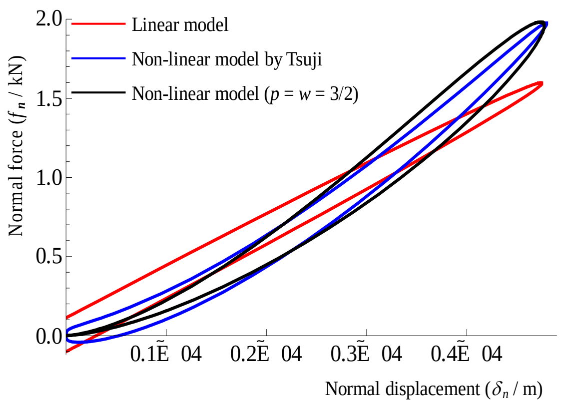

Figure 2. Normal force vs. normal displacement using three different contact models

The exponent 3/2 of δ_n makes the dissipation contributed by the damping emphasis on the displacement δ_n, instead of the velocity as in the linear case.

It is observed that the maximum displacements $\delta_{n,\max}$ in the three curves are equal, because of the uniform preestimated overlap used in the derivation of the equivalent parameters. In the non-linear force-displacement laws based on Hertzian theory, the elastic force is proportion to the 3/2 power of the displacement, instead of the linear proportional relation in the linear case. As a result, the maximum contact force (about 1600 N) of the linear model is smaller than the two maximum forces (both about 2000 N) corresponding to the non-linear ones.

It is noted that in the area near to the displacement δ_n = 0, the direction of the contact force in the linear as well as the non-linear (Tsuji) cases both changes, which is unrealistic. Moreover, for the linear case, the force is not zero at both the start and end points of the collision. Nevertheless, the non-linear model using the damping term introduced in our study has no those problems and looks more realistic, in which the contact force is zero only as the displacement equals zero, without any unexpected negative force occurring closed to the end of the collision.

V. CONCLUSION

In a discrete element simulation, the reasonableness and accuracy of contact models used to solve impact forces affect the computed results. In this paper, a general damping is proposed by means of the dimensional analysis, in order to constitute a new viscoelastic DEM contact model without the unrealistic behavior that an unexpected attractive force, instead of the repulsive force, exists towards the end of collision. A general expression for the damping constant α_2 and the restitution coefficient e_n is derived, which makes the results of the new viscoelastic model are more reasonable.

For the case of p = w = 3/2, the relationship between the damping constant α_2 and the restitution coefficient e_n is given by numerical calculations. An impact simulation of a single ball is conducted by using three approaches. The simulation results of three approaches show that our non-linear approach corresponding to p = w = 3/2 is more realistic than the other two cases, while the non-linear approach by Tsuji looks better than the linear approach.

We have to note that all the analysis above is based on theoretic study. For further research in the accuracy of the damping term in the contact mode in the DEM, more experimental analysis is needed.

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