Dynamic Real-time Optimization of Reservoir Production

Kai Zhang, Jun Yao, Liming Zhang, Yajun Li
College of Petroleum Engineering
China University of Petroleum (East China)
QingDao, Shandong, China 266555
E-mail: reservoirs@163.com

***Abstract*—Water flooding is a common technology of enhanced oil recovery and has been widely used to the world's oil fields. But it is difficult to maintain a uniform displacement of water, which causes the ineffective circulation of injected water. Therefore, the real-time dynamic control strategy is studied to work out development plans scientifically and make better economic profits in this paper. The net present value of crude oil exploitation during a certain period of time is choosen to performance index of the control problem. Hydrodynamic equations of oil, water and gas, which describe the displacement process, are used as governed equations. Meanwhile, the boundary constraints of wells and balance relationship of injection-production are taken into account. Based on the above equations, a control model of oil field development is established. The control vector parameterization method is illustrated in detail to deal with the unconstrained, control constrained and state constrained optimal control problem. Finally, a simulation example is employed to verify the validity of the proposed control algorithm.**

***Index Terms*—optimal control, reservoir simulation, gradient method, smart field**

I. INTRODUCTION

The world's main onshore oil and gas fields in the basin have been developed for a long time. Most of them are reaching a mid or late stage of development. One of the main problems is non-equilibrium of water flooding, which causes the complex distribution of remaining oil and the difficult production adjustment. The space of increasing reserve and enhancing production is very limited.

According to simulation and analysis results, the problem is mainly resulted in by interlayer rhythm, reservoir plane heterogeneity and different well patterns etc. They are the reasons why a large number of inefficient and ineffective circulation wells exit in real oil fields too. For example, Gudong reservoir in China has 1650 production wells, 620 of which are located in the main stream channels. High capacity channel occurs in the center of these main stream channels. So water flooding spread unevenly in the area (shown in Fig.1). Meanwhile, the layer heterogeneity (shown in Fig.2) makes the big difference of vertical flooding and lead to severe interlayer interference.

In order to produce more oil, the traditional method aiming at this problem is to do a lot of tests (orthogonal test design and reservoir simulation) and compare results to decide the optimal development plan. The plan could minimize the impact of heterogeneity as much as possible. However, its shortcoming is very obvious that lots of time and manpower are expended. What's more, the final obtained result is always limited in a small scope of original designs. Dynamic real-time optimization method of reservoir production is proposed for the problems. It combines reservoir numerical simulation and optimization method, and directly solves gradients of control variables to achieve optimal control settings.

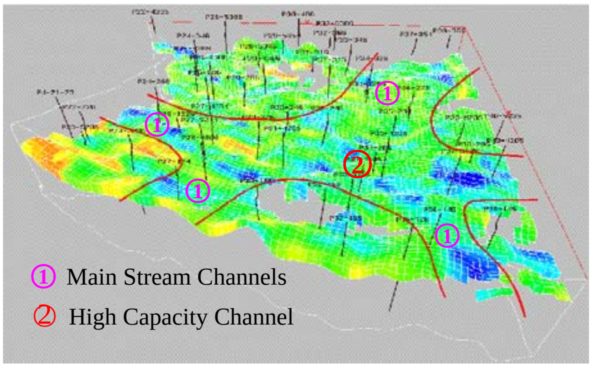

Figure 1. High capacity and main stream channels in Gudong reservoir

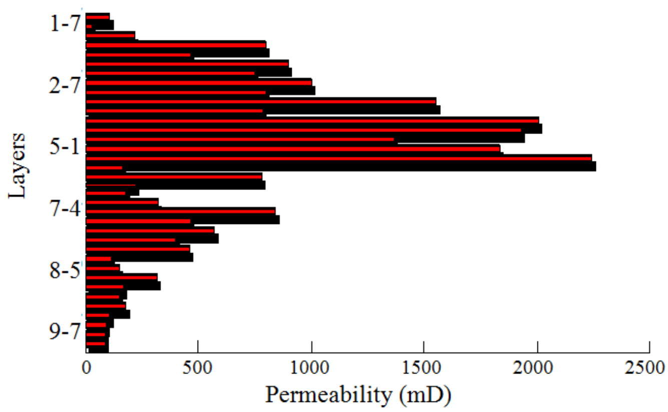

Figure 2. The layer heterogeneity of permeability

This work is supported by "National Nature Science Fund" under Grant 61004095", "China Important National Science & Technology Specific Projects" under Grant 2008ZX05030-05-002 and "the Fundamental Research Funds for the Central Universities" under Grant 09CX05007A

doi:10.4304/jcp.6.3.610-617

There is only a few and not enough relevant research carried out in the early 20th century, which are simple applications of optimization theory (such as the energy equation of allocation[1], multi-objective optimization of oil field development[2], etc). But with the proposal of smart field system, many oil companies and research universities have been studying production optimization since 2005.

In 2006, the method combining empirical formula with optimization theory is proposed by Saputelli[3,4], Elgsaeter[5], and Awasthi[6] and etc. Their goal focused on the short-term optimization of production and gained the optimized development program in time with convenient empirical formula. The advantages of this method are the certain universality in optimization results and high speed. But its disadvantages are also obvious that it is poor targeted, such as it can't consider the heterogenetic characteristic of formation and realize production optimization control of single-well.

Alghareeb[7] calculated the optimal flow control valve settings of intelligent well with genetic algorithm. Chunhong[8] optimized production in small-scale reservoir with SPSA method. Lorentzen[9] and Yan[10] simplified the calculation of production optimization with EnKF and other methods. These studies achieved certain results, which can search the global optimal solution of optimization production problem. But low-speed computation is the biggest drawback of the method in large-scale reservoir model, so this kind of method is difficult to be applied to the oil field.

Brouwer[11] and Chunhong[8] studied production optimization problems with gradient algorithms. But their algorithms still have some limitations. Chunhong used the perturbation gradient algorithm. In her method, each time only one gradient of control variable in one time step could be calculated though two simulation runs, in which the speed basically can't meet the requirements of engineering applications. But such problems always need a large amount of calculations and the running speed is first and foremost premise. Therefore, compared to stochastic and empirical algorithms, the gradient algorithm is more advisable.

II. Mathematical Model Of Optimal Control For Reservoir Production Working System

Reservoir development optimization could maximize oil production by adjusting control parameters of production and injection wells in a reservoir, which belongs to the scope of optimal control problems. This issue starts from an economic point of view and the working system is optimized to improve production capacity and increase oil recovery efficiency[12, 13]. Compared with conventional simulation methods, the method's greatest advantage is to automatically obtain optimal programs at every time under the principle of stabilizing oil production and controlling water cut. Relying on the method, production figures go up not adopting any measures(acid, fracture, EOR etc.) . That is, costs would come down.

A. Performance indicator of optimal control

Reservoir production optimization issues need a cost function aiming at the actual working system as performance indicator of optimal control. Different performance indicator will get different optimization result. The actual situation in oil field must be considered to choose appropriate performance indicator. Generally, oil industry can be regarded as a business of high-input and high-return. Oil exploitation is closely linked with economic factors and its main purpose is to make maximum profits. So the net present value(NPV) is used as the cost function in this paper, which is given by the following equation (see Nomenclature for the definitions of the symbols)

$$H_{p,l}\left(u_p\right)=\frac{\left(1+\beta\right)^{t_{p,l}}}{\Delta t_{p,l}}\left[\sum_{n=1}^{N_P}\left(m_o q_{p,o,n}-m_w q_{p,w,n}\right)-\sum_{j=1}^{N_I} m_{iw} q_{p,iw,n}\right] \tag{1}$$

$$q=\sum_{n=1}^{N_p}\mathrm{WI}_n\left[\frac{\rho_{ps,n}K_{ps,n}X_{ps,n}}{\mu_{ps,n}}\left(P_{ps,n}-P_{cgo,n}-P_{wf,n}\right)\right] \tag{2}$$

where, H is NPV, ￥; p is control step; l is simulation step in each control step; ps is fluid phase(oil, water and gas); q is flow rate, m^3; $P_{ps,n}$ is pressure in grid block, MPa; $P_{cgo,n}$ is capillary pressure, MPa; $P_{wf,n}$ is bottom hole pressure(BHP) of wells, MPa; u_p is control variable(such as p_{wf}, q_o, q_w and q_{wi}); m_o is oil price per cubic meter; m_w is water production cost per cubic meter; m_{iw} is water injection cost per cubic meter. WI_n is well index in the n layer. $\rho_{ps,n}$ is density of ps phase, kg/m^3; $K_{ps,n}$ is permeability of ps phase,μm^2; $X_{ps,n}$ is compositional factor; $\mu_{ps,n}$ is viscosity of ps phase, mPa·s. Δt is time period, year; b is discounting factor. N_{P} is number of production wells; N_{I} is number of injection wells.

B. Reservoir numerical simulation

Optimization calculation is based on flow system of oil and water. Usually, this system is described by a theory of reservoir numerical simulation. The goal of reservoir simulation is to optimize and simulate the development program in the future on the basis of repeating the whole process of oil field development in numerical way. Optimization here refers to better result relatively in a small scope of original designs. Researcher would compare several design plans and get relative optimal plan in general.

This paper combine optimization theory and reservoir simulation using black oil model. This model must satisfy several assumptions in production phase as follows: 1) reservoir exists oil-water-gas three-phase flow; 2) rock and fluid can be compressed; 3) fluid flow obeys Darcy's law in reservoir; 4) rock is anisotropic and heterogeneous; 5) the effects of gravity and capillary pressure are considered in the flow.

Black oil model can be described as follows[14]

$$\psi=\nabla\left[\omega_{ps,g}\rho_g\frac{kk_{rg}}{\mu_g}\nabla\left(p_g-\rho_g gD\right)\right.$$
$$\left.+\omega_{ps,o}\rho_o\frac{kk_{ro}}{\mu_o}\nabla\left(p_o-\rho_o gD\right)+\omega_{ps,w}\rho_w\frac{kk_{rw}}{\mu_w}\nabla\left(p_w-\rho_w gD\right)\right]$$
$$+q_{ps}-\frac{\partial}{\partial t}\left[\varphi\left(\omega_{ps,g}\rho_g S_g+\omega_{ps,o}\rho_o S_o+\omega_{ps,w}\rho_w S_w\right)\right]=0 \quad (3)$$

In order to solve this equation, some assistant equations must be provided:

①Saturation normalized equation

$$S_o+S_w+S_g=1 \quad (4)$$

②Mass fraction normalized equations

$$\sum_{i=1}^{N}\omega_{ps,g}=1;\quad \sum_{i=1}^{N}\omega_{ps,o}=1;\quad \sum_{i=1}^{N}\omega_{ps,w}=1 \quad (5)$$

③Equilibrium constant equation

$$K_{ps,go}=\frac{\omega_{ps,g}}{\omega_{ps,o}};\quad K_{ps,gw}=\frac{\omega_{ps,g}}{\omega_{ps,w}} \quad (6)$$

④Capillary pressure equation

$$p_{cgo}=p_g-p_o;\quad p_{cow}=p_o-p_w \quad (7)$$

Initial condions are

$$p_o\big|_{t=0}=p_{oi};\quad S_w\big|_{t=0}=S_{wi} \quad (8)$$

Closed outer boundary conditions is

$$\left.\frac{\partial P}{\partial n_r}\right|_{\Gamma=0}=0 \quad (9)$$

Outer boundary conditions of specified pressure is

$$P\big|_{\Gamma}=0 \quad (10)$$

where, ψ is flow equation. k_r is relative permeability, μm^2; D is height difference, m;

Flow is simulated by Eqn.(3) to (10). State variables like pressure and saturation can be solved for optimal control model. This forward model uses differential solution.

C. Mathematical Model of Optimal Control

According to the general form of optimal control model, the model for reservoir production working system is expressed as follows

$$\max \quad C=\sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}H_{p,l}\left(x_{p,l+1},u_p\right) \quad (11)$$

subject to

$$\psi_{p,l}\left(x_{p,l+1},x_{p,l},u_p\right)=0 \quad (12)$$

$$u_{p,low}\le u_p\le u_{p,up} \quad (13)$$

where, C is objective function of optimization and summation of performance indicator H in every period. x is state variable(pressure and saturation in each grid block).

The problem can be described as follows: under the constraints of reservoir flow equation $\psi_{p,l}=0$ and single-well production limit ($u_{p,low}\le u_p\le u_{p,up}$), mathematical model is solved to achieve optimal control variables $x*$ and corresponding optimal states $u*(t)$.

III. GRADIENTS CALCULATION

It was stated earlier that the gradients of the cost function with respect to the controls could be calculated very efficiently using maximum principle. The gradient equations can be obtained from the necessary conditions of optimality of the optimization problem defined by Eq.(11)~(13). These necessary conditions of optimality are derived from the classical theory of calculus of variations. For a relatively simple treatment of this subject, refer to Stengel[15]. A more detailed and rigorous analysis of the problem and generalization to infinite dimensional problems in arbitrary vector spaces is given by reference [16]. The essence of the theory is that the cost function of (11) along with all the constraints can be written equivalently in the form of an augmented cost function given by Eq.(14).

$$C_A=\sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}H_{p,l}\left(x_{p,l+1},u_p\right)+\sum_{m=0}^{N-1}\sum_{n=0}^{N_m-1}\left(\lambda_{p,l+1}\right)^T\psi_{p,l}\left(x_{p,l+1},x_{p,l},u_p\right) \quad (14)$$

The vector λ is known as Lagrange multiplier, which can be thought of as elements of the dual space of the vector space to which u belongs. One Lagrange multiplier is required for each constraint with which the cost function is augmented. That is, the total number of Lagrange multipliers is equal to the product of the number of dynamic states and control steps.

For optimality of the augmented cost function, the first variation of the function must equals to zero, which is given by:

$$\delta C_A=\sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}\left(\frac{\partial C_{A,p,l}}{\partial x_{p,l}}\right)\delta x_{p,l}+\sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}\left(\frac{\partial C_{A,p,l}}{\partial x_{p,l+1}}\right)\delta x_{p,l+1}+\sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}\left(\frac{\partial C_{A,p,l}}{\partial u_p}\right)\delta u_p \quad (15)$$

Eq.(15) is extended as follows

$$\delta C_A=\sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}\left(\frac{\partial C_{A,p,l-1}}{\partial x_{p,l}}+\frac{\partial C_{A,p,l}}{\partial x_{p,l}}\right)\delta x_{p,l}$$
$$+\sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}\left(\frac{\partial C_{A,p,l}}{\partial u_p}\right)\delta u_p+\sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}\left(\frac{\partial C_{A,N-1,N_m-1}}{\partial x_{N-1,N_m-1}}\right)\delta x_{N-1,N_m-1} \quad (16)$$

Because these items in Eq.(16) are independent of each other, according to the necessary conditions getting extremum is $\delta C_A=0$, (16) are changed as follows

$$\sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}\left(\frac{\partial C_{A,p,l-1}}{\partial x_{p,l}}+\frac{\partial C_{A,p,l}}{\partial x_{p,l}}\right)=0$$
$$\sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}\left(\frac{\partial C_{A,N-1,N_m-1}}{\partial x_{N-1,N_m-1}}\right)=0,\quad \sum_{p=0}^{N-1}\sum_{l=0}^{N_m-1}\left(\frac{\partial C_{A,p,l}}{\partial u_p}\right)=0 \quad (17)$$

Each item are extended, (16) is expressed in the following form

$$\begin{cases}\dfrac{\partial H_{p,l-1}}{\partial x_{p,l}}+\left(\lambda_{p,l+1}\right)^T\dfrac{\partial \psi_{p,l}}{\partial x_{p,l}}+\left(\lambda_{p,l}\right)^T\dfrac{\partial \psi_{p,l-1}}{\partial x_{p,l}}=0\\ \dfrac{\partial H_{N-1,N_m-1}}{\partial x_{N-1,N_m}}+\left(\lambda_{N-1,N_m}\right)^T\dfrac{\partial \psi_{N-1,N_m-1}}{\partial x_{N-1,N_m}}=0\\ \dfrac{\partial C_{A,p,l}}{\partial u_p}=\dfrac{\partial H_{p,l}}{\partial u_p}+\left(\lambda^{m,n+1}\right)^T\dfrac{\partial \psi_{p,l}}{\partial u_p}\end{cases} \quad (18)$$

λ can be calculated at each time step and grid block by the first two equations(state equations) in (16). And then λ are using in the third equation(gradient equations) to obtain gradients $\partial C_{A,p,l}/\partial u_p$ of the various control variables. When gradients tends to zero through iterative calculation, the optimal solution is found. That is, the optimal control plan for reservoir production working system is ascertained.

IV. CONSTRAINED OPTIMIZATION

Oilfield development is based on the real reservoir. According to the geological conditions and the constraint of the development facilities and environment, the oil/water wells always produce under certain constraint condition. For example, there is the maximum and minimum value for the production of single well and bottom hole pressure. According to the reservoir pressure, the single well can not produce infinitely and the bottom hole pressure can not reduce to zero. So there is always a production limit for the single well. But for the entire block, because the reservoir controls optimization, on one hand is to compare the optimization effect, on the other hand is to maintain balanced injection and production rate combined with the real reservoir development, the general constraint (such as the total injection rate of block)is needed for injection/production condition of the whole reservoir. The much more convenient method of constraint control is discussed combined with the above problems.

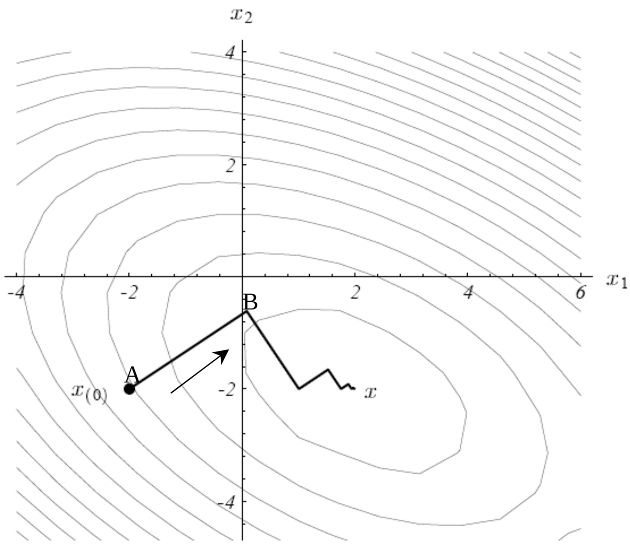

Figure 3. Searching schematic diagram of the steepest descent algorithm

The reservoir simulation calculation pertains to the area of large-scale mathematical computation. In the computation process, if considering the constraint conditions, such as the production limit of single well and the total injection rate of block, then the optimization pertains to large-scale constrained optimization problems. The gradient computational method was used in this article, but the gradient is only scalar field gradient. In the scalar field, the gradient of control variables points to the fastest increasing direction of the scalar field. The length of gradient is the largest rate of change. As shown in Fig. 3, the gradient direction is from point A to B and the distance between A and B is the search step size in the calculated iteration. Constrained optimization method is used to restrict the gradients.

When optimal control would be executed, the control variables' gradients can be obtained through the adjoint model. The sign of gradients may be positive or negative. For bottom hole pressure controlled by the single well, if the gradient is positive, it indicates that the bottom hole pressure should increase towards the direction of control variables rising (or increasing) in well optimization program; If the gradient is negative, it should be regulated towards the corresponding descent direction of the bottom hole pressure. For the block optimization, if the injection and production rate can not be balanced, free optimization is inconsistent with the real situation, and if the production is based on regulation scheme after optimizing in the forecast period, the injection flow rate of the entire block increases or decreases, and the same fluctuation to the corresponding crude oil and water production. The schemes obtained before and after optimization are not comparable and the real results of optimization can not be reflected because of the different total injection rate. Therefore, the injection balance of the optimized reservoir block must be considered in the entire calculation progress, and the injection well is controlled by injection rate. Combined with the maximum and minimum values of single well-controlled constraints, the optimal control model becomes

$$\max \quad J=\sum_{m=0}^{N-1}\sum_{n=0}^{N_m-1} Y^{m,n}\left(x^{m,n+1},u^m\right) \tag{19}$$

Constraint condition:

$$L^{m,n}\left(x^{m,n+1},x^{m,n},u^m\right)=0,$$
$$\forall n\in\left(0,\cdots,N_m-1\right),m\in\left(0,\cdots,N-1\right) \tag{20}$$

$$\sum_{i=1}^{N_{inj}} Q_{wi,i}^m = Q_t,\forall i\in\left(1,2,\cdots,N_{inj}\right) \tag{21}$$

$$LB\le Q_{wi,i}^m \le UB,\forall i\in\left(1,2,\cdots,N_{inj}\right) \tag{22}$$

$$LB\le P_{wf,k}^m \le UB,\ \forall k\in\left(1,2,\cdots,N_{oil}\right) \tag{23}$$

In the formula, $Q_{wi,i}^m$ is the injection rate of the single well i in m control step, m^3/day; $P_{wf,k}^m$ is the bottom hole pressure of the single production well k in m control step, m^3/day; Q_t is the constraint total injection rate, m^3/day, the total injection rate is constant all the times; N_{inj} is the total number of injection wells; N_{pro} is the total number of production wells.

The adjoint gradient $\frac{\partial J_A}{\partial u}$ obtained by previous solution is solved by Eq.(20) and (21). Considering the linear constraint of Eq.(22), a simple average gradient calculation method is used to restrict. First, solve the average gradient of all the water wells in adjoint model

$$Ga=\frac{1}{N_{inj}}\sum_{i=1}^{N_{inj}}\frac{\partial J^m}{\partial Q_{wi,i}^m} \tag{24}$$

In the formula, Ga is the accumulated gradient item.

Generally, the fluid in the reservoir should flow following the direction of oil/water wells in optimized process which is more favorable for the production. So based on this norm, if the single well calculated control gradient $\frac{\partial J^m}{\partial Q_{wi,i}^m}$ is greater than the average gradient value Ga in one time step, then the gradient is considered favorable; if $\frac{\partial J^m}{\partial Q_{wi,i}^m}$ is less than the average gradient value Ga, then the gradient is considered unfavorable. For the control of injection wells, if all of the control gradient subtract the average gradient in one time step, then the symbols of favorable gradient are all positive at this time; while the symbols of the unfavorable gradient are all negative. The advantages of this approach retain gradient amplitude fluctuations, and the gradient is an improved gradient, which the sum of the gradient is zero. The specific expression is

$$\left(\frac{\partial J^m}{\partial Q_{wi,i}^m}\right)_{\text{mod}}=\frac{\partial J^m}{\partial Q_{wi,i}^m}-Ga$$
$$=\frac{\partial J^m}{\partial Q_{wi,i}^m}-\frac{1}{N_{inj}}\sum_{i=1}^{N_{inj}}\frac{\partial J^m}{\partial Q_{wi,i}^m} \tag{25}$$

After the improved gradient obtained in the calculation, the injection gradient of each injection well divide by the absolute value of the maximum gradient, So that all the gradient values drift between -1 and +1. Here only referring to the constrained optimization of the total injection rate, if the total fluid production rate is constrained by the same method, it can be controlled and optimized under balanced production and injection situation in the calculation. The expression is

$$\left(\frac{\partial J^m}{\partial Q_{pro,i}^m}\right)_{\text{mod}}=\frac{\partial J^m}{\partial Q_{pro,i}^m}-Ga$$
$$=\frac{\partial J^m}{\partial Q_{pro,i}^m}-\frac{1}{N_{pro}}\sum_{i=1}^{N_{pro}}\frac{\partial J^m}{\partial Q_{pro,i}^m} \tag{26}$$

where, $Q_{pro,i}^m$ is the fluid production rate of the single well i in m control step, m^3/day.

The linear constraint of the Eq .(3) is considered in the process above. But for the Eq. (4) and (5), two constrained optimization conditions have been transformed into unconstrained optimization by the logarithmic transformation method [12] and the clipping and approaching constraint has been achieved[12]. Using the Eq.(9), the control variables will be calculated by transforming into the variables within the upper and lower constraint bound.

$$s^m=\ln\left(\frac{u^m-u_{low}^m}{u_{up}^m-u^m}\right) \tag{27}$$

In the formula, u_{low}^m is the lower boundary of control variable; u_{up}^m is the upper boundary of control variables; s^m is the control variable after logarithm transform.

V. Numerical Solution

A. Solution of state and gradient equations

State and gradient equations of this optimal control problem are a set of partial differential equations, in which the state equations are nonlinear and the initial conditions are known so as to require clockwise solving. However, as for gradient equation, its terminal condition equation are known. They need to solve in the counterclockwise direction. Gradient equations are a set of linear partial differential equations and the coefficients can be calculated by a known state variables.

Both state equations and gradient equations require discrete solution, therefore, finite difference method is used in this paper. The flat area is divided into $n_x \times n_y$ grid blocks which are arranged in a certain order. The serial number of the grid blocks is i, $i \in 1,2,...,n_x \times n_y$. The coordinates of the i th grid block is written as (xi, yi) .Assume $Te(x, y, t)$ are variables defined in the region $\Omega \subset R^2$ and difference form is used to replace the calculation of derivatives. A example to calculate the discrete variable $Pe_{xi,yi}$, in which differential direction is x, is expressed as follows

$$\frac{\partial Te}{\partial t} = \frac{Te^{p+1} - Te^{p}}{\Delta t} \tag{28}$$

$$\frac{\partial Te}{\partial x} = \frac{Te_{xi+1,yi} - Te_{xi-1,yi}}{2\Delta x_{xi,yi}} \tag{29}$$

$$\frac{\partial}{\partial x}(k\frac{\partial Pe}{\partial x}) = \frac{1}{\Delta x_{xi,yi}}\left[\frac{k_{xi+\frac{1}{2},yi}}{\Delta x_{xi+\frac{1}{2},yi}}(Pe_{xi+1,yi} - Pe_{xi,yi}) + \frac{k_{xi-\frac{1}{2},yi}}{\Delta x_{xi-\frac{1}{2},yi}}(Pe_{xi-1,yi} - Pe_{xi,yi})\right] \tag{30}$$

where, p and $p+1$ are discrete time step, $p \in 0,1,\cdots N$. Some coefficients need to be calculated at the location $\frac{1}{2}$ by the principle of "the upper power" in Eq.(30). [14] According to the above definitions, the state and gradient equations can be discretized. A full implicit discrete method [14] is used in this paper. Because discretized state equations are nonlinear and gradient equations are linear, the two equation groups are solved by Newton-Raphson method and deepest descent algorithm respectively. In the calculation process, the technology of sparse matrix storage is used. The object of Newton-Raphson solution is a Jacobian matrix consisting of derivatives of flow equation with respect to state variables(pressure and saturation). And these derivatives also will be used in equation group (18) for gradients. Therefore, these matrixes are saved and would be directly extracted for gradient solution. This combined method will simplify calculation process and improve running efficiency.

The calculation of Eqn.(18) is mainly depending on some large matrixes. For example, if the dimension of grid block in a reservoir is $N_m \times N_m \times 1$ (N_m in the x and y direction). For the first equaton in Eqn.(18), the dimension of matrix λ and $\frac{\partial H}{\partial x}$ is N_m but $\frac{\partial \psi}{\partial x}$ is $N_m \times N_m$. So this equation can be written as

$$\left[\frac{\partial H_{p,l-1}}{\partial x_{p,l}}\right]_{N_m} + \left[\left(\lambda_{p,l+1}\right)^T\right]_{N_m}\left[\frac{\partial \psi_{p,l}}{\partial x_{p,l}}\right]_{N_m \times N_m} + \left[\left(\lambda_{p,l}\right)^T\right]_{N_m}\left[\frac{\partial \psi_{p,l-1}}{\partial x_{p,l}}\right]_{N_m \times N_m} = 0 \tag{31}$$

The second equaton is similar as the first in Eqn.(18). For the gradient equation, it will be written as

$$\left[\frac{\partial C_{A,p,l}}{\partial u_p}\right]_{1\times N_u} = \left[\frac{\partial H_{p,l}}{\partial u_p}\right]_{1\times N_u} + \left[\left(\lambda^{m,n+1}\right)^T\right]_{1\times N_m}\left[\frac{\partial \psi_{p,l}}{\partial u_p}\right]_{N_m \times N_u} \tag{32}$$

B. Solution steps

The main steps required for gradient-based method are summarized as follows:

a) Solve the forward model equations for all time steps with given initial condition and initial control strategy. Get the dynamic states(pressure and saturation) at each time step.

b) Store the Jacobian matrixes of the simulation equations and the well derivatives at each time step.

c) Solve the adjoint model equations using the stored Jacobian matrixes and derivatives to calculate the Lagrange multipliers with the first two equations in (18).

d) Use the Lagrange multipliers to calculate gradients using the third equation in (18) for all control steps.

e) According to specified boundary conditions of control variables, gradients could be constrainted by the method of logarithmic transformation;

f) Calculate new gradients $\left(\frac{\partial J^m}{\partial Q^m_{wi,i}}\right)_{\mathrm{mod}}$ *or* $\left(\frac{\partial J^m}{\partial Q^m_{wi,i}}\right)_{\mathrm{mod}}$ *to constrain total injection and production rates using the average gradient method discussed above,*

g) Use these gradients with the deepest algorithm to choose new search direction and working system.

h) Repeat process until optimum is achieved, that is, all gradients are close enough to zero.

VI. CASE STUDY – DYNAMIC WATER FLOODING

This case is a simple example of two-dimensional three-phase reservoir model with 4 production wells and 1 injection well, which covers an area of 1220×1220 m^2 and has a thickness of 15 m and is modeled by a 20×20×1 horizontal 2D grid. The heterogeneous permeability and porosity fields of the reservoir and well configuration are shown in Fig.4 and Fig.5. The initial reservoir pressure is 41.37 MPa. Bottom hole pressure of each oil well is 34.48 MPa. The total injection rate is 2023.25 m^3/d. The price of crude oil is 4000 ￥/t , the cost of production water is 1500 ￥/m^3 and the cost of injection water is 500 ￥/m^3. The discount factor is zero. The model is produced about 950 days. This time period is divided into eight control steps of 0, 120, 240, 360, 480, 600, 720 and 840 days.

Two optimization modes are used: the first starts from the initial moment and the second starts at a specified time in the development. In this case, the specified time is fixed on 480 days after production.

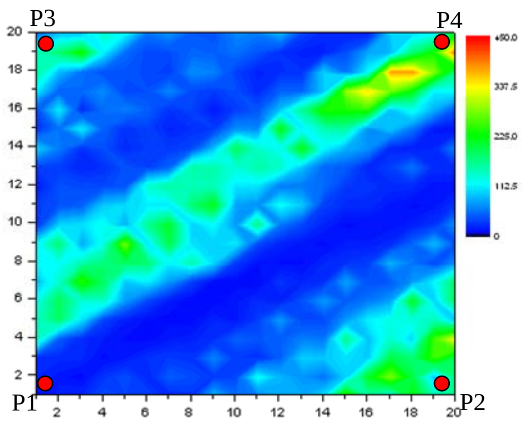

Figure 4. Permeability field of the reservoir

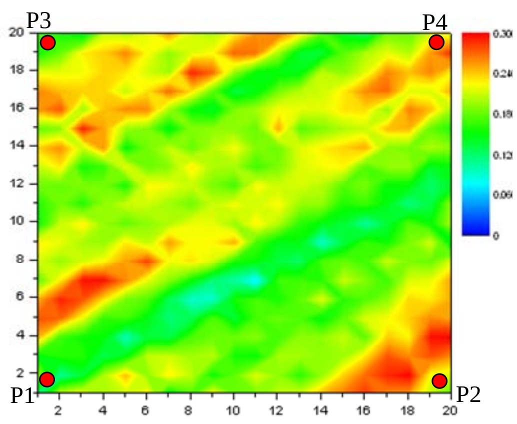

Figure 5. Porosity field of the reservoir

Fig.6 describes the comparison of accumulative oil rate and accumulative produced water rate before and after optimization. It is easy to see that the effect of optimization is very obvious, which fingering phenomenon of water flooding is inhibited and final recovery of oil is remarkably improved. After several iterations as shown in Fig.7, the effect of optimization from 0 day is better than from 480 day.

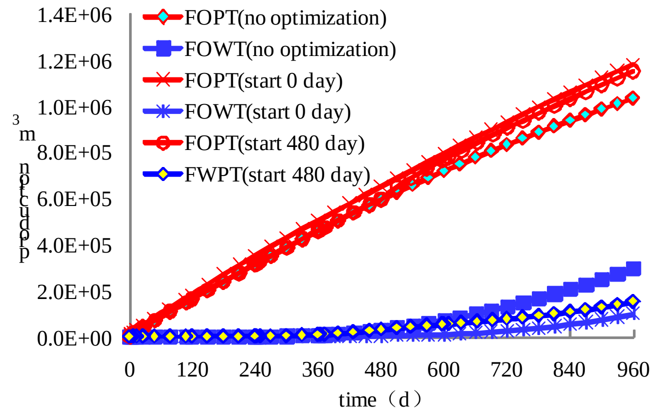

Figure 6. Comparison of accumulative oil rate and produced water rate before and after optimization (starts from 0day and 480 day respectively)

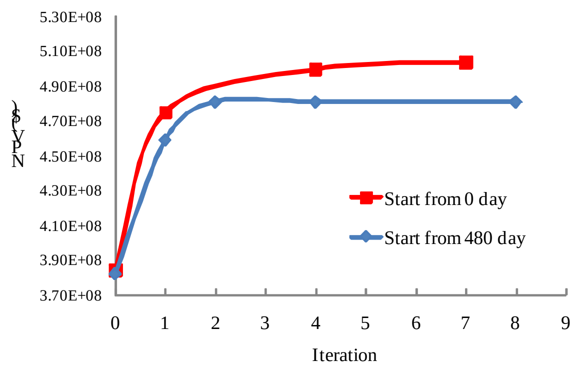

Figure 7. Comparison of NPV

Fig.8 show the saturation distributions before and after optimization. It is clear that injected water is inhibited to flow along high permeability channel.

The optimal control plan of reservoir production working system is as shown in Fig.9 and 10. Bottom hole pressure goes up and the relative production will go down. Prod 4 is located in the edge of high permeability channel. So BHP of this well is closed to the upper limit after optimization and BHP of others goes down to increase oil production.

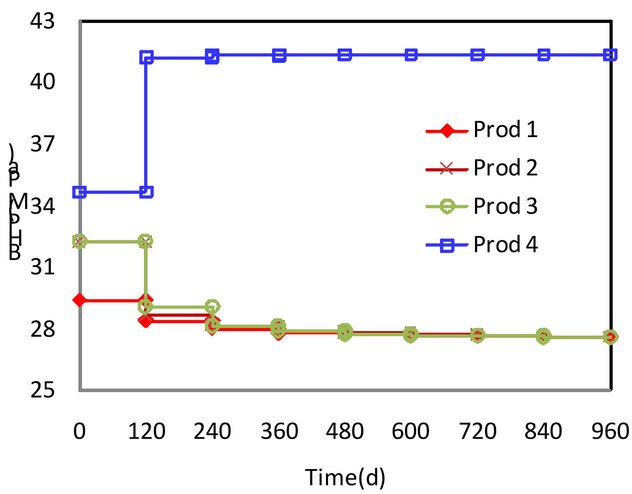

Figure 9. BHP controls after optimization from 0 day

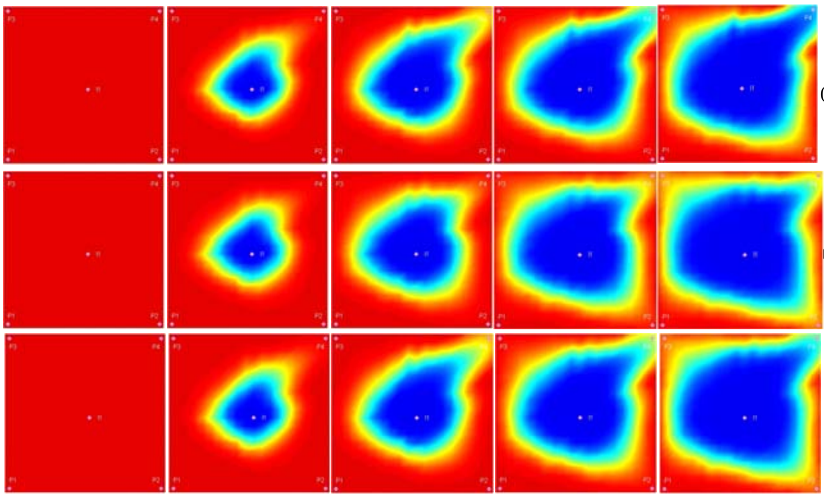

Figure 8. Oil saturation distribution before and after optimization

(a:before optimization, b:optimize from 0 day, c:optimize from 480 day)

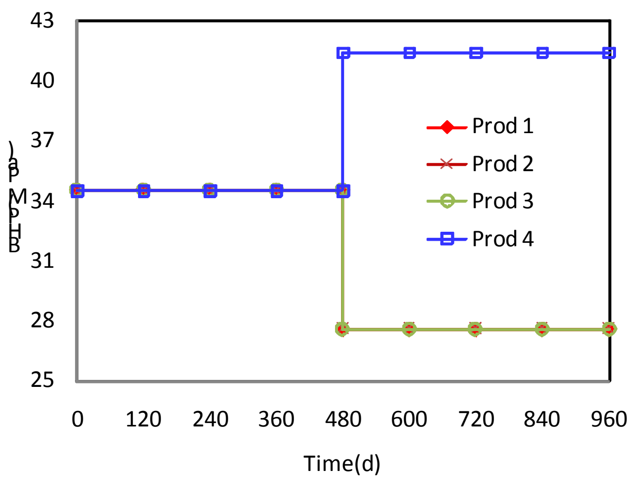

Figure 10. BHP controls after optimization from 480 day

VII. CONCLUSIONS

This work explores a new algorithm for optimization of reservoir production working system using optimal control theory. Maximum principle is used as the main solution method. Combined full-implicit reservoir simulation, optimal control model is solved to obtain accurate gradients of control variables to optimize reservoir production.

The methodology is applied to a synthetic example, enabling comparison with traditional production strategies. Significant improvement in NPV, acceleration of oil production, cumulative oil recovery, and reduction of water production were realized. Results were better than those obtained with water flood optimization based on comparison with many results of simulation.

REFERENCES

[1] G.F. Wang, Y.L. Hu, Z.P. Li, et al, “Oil/Gas Reservoir Energy Equation and A New Theroy of Optimal Production Allocation,” Journal of Southwest Petroleum University, vol. 29, pp. 57-59, October 2007 (in Chinese).

[2] J.L. Zhang, X.F. Ding, X.Y. Sa, et al, “Study of the Multiple Objective Optimization of Oilfield Development and Integrative Judgment Matrix,” Journal of Southwest Petroleum University, Vol. 26, pp. 42-45, December 2004 (in Chinese).

[3] S. Mochizuki; L.A. Saputelli, and C.S. Kabir, et al, “Real-Time Optimization:Classification and Assessment,” SPE 90213, 2006.

[4] M. Nikolaou, A.S. Cullick, and L. Saputelli, “Production Optimization - A Moving-Horizon Approach,” SPE 99358, 2006.

[5] Elgsaeter, M. Steinar, Slupphaug, et al, “Production optimization: System identification and uncertainty estimation,” SPE 112186, 2008.

[6] Awasthi, Ankur, Sankaran, et al. “Meeting the challenges of real-time production optimization - A parametric model-based approach,” SPE 111853, 2008.

[7] Z.M. Alghareeb RNH, B.B. Yuen, S. H. Shenawi, “Proactive Optimization of Oil Recovery in Multilateral Wells Using Real Time Production Data,” SPE 124999, 2009.

[8] CH Wang, GM Li and Reynolds AC, “Production Optimization in Closed-Loop Reservoir Management,” SPE 109805, 2007.

[9] R.J. Lorentzen, M.A. Berg, and G. Naevdal, “A New Approach for Dynamic Optimization of Water Flooding Problems,” SPE 99690, 2006.

[10] Y. Chen, G. Oliver, and D.X. Zhang. “Efficient ensemble-based closed-loop production optimization,” SPE 112873, 2008

[11] D.R. Brouwer and J.D. Jansen, “Dynamic optimization of waterflooding with smart wells using optimal control theory,” SPE 78278, 2004.

[12] M. Lien, D.R. Brouwer, T. Manseth, J.D. Jansen, ” Multiscale regularization of flooding optimization for smart field management,” SPE 99728, 2006.

[13] M.M.J.J. Naus, N. Dolle, J.D. Jansen, “Optimization of Commingled Production Using Infinitely Variable Inflow Control Valves,” SPE 90959, 2006.

[14] H.Q. Liu, “Topic of reservoir numerical simulation methods,” Dongying: Petroleum University Press, pp. 90-102, 2001(in Chinese).

[15] R.F. Stengel, “Optimal Control and Estimation,” Dover Books on Advanced Mathematics, New York, 1985.

[16] X.S. Jie, “Optimal Control Theory and Applications,” Tsinghua University Press, pp. 57, 1986(in Chinese)