Research on Optimization of Relief Supplies Distribution Aimed to Minimize Disaster Losses

Yong Gu
School of Logistics Engineering, Wuhan University of Technology, Wuhan, P. R. China
Email: guyong@whut.edu.cn

Abstract—The optimization problem of relief supplies distribution for large-scale emergencies is discussed in this paper. Under the situation that the needs for relief supplies are more numerous and urgent, the optimization goal should be minimum disaster loss instead of minimum delivery cost or shortest travel distance. By setting objective function about minimum disaster loss, scheduling model of relief supplies distribution under determined conditions and fuzzy conditions is proposed. Scheduling model of relief supplies distribution divide into two cases both for single species and for many varieties. We deal with the uncertainty of travel time in relief supplies distribution by applying the approach of selecting the maximum satisfaction path in fuzzy network. For each model, a experimental calculation is conducted to verify its solution approach.

Index Terms—relief supplies, scheduling, distribution, disaster losses, fuzzy set

I. INTRODUCTION

Since entering the 21st century, various catastrophic emergencies occur frequently, which caused huge casualties and property losses, such as 9•11, SARS, Bird flu, Indian Ocean tsunami, New Orleans Hurricane, Wenchuan (China) Earthquake and so on. The effective rescue and aid measures must be taken immediately to minimize the loss caused by emergent events as far as possible. In the course of the emergency preparedness and response, a large amount of emergency supplies were required for casualties' rescue, sanitary and anti-epidemic, restoration and reconstruction, and so on. The demands for disaster relief supplies include mainly supplies of rescuing casualties and livelihood security, medicines and medical equipment. It is necessary that relief supplies are shipped to emergency demand points timely by accurate and reasonable manner in disaster relief operations. The main influence factor of relief supplies distribution are the needs of emergency demand points, travel time from service points to demand points and capacity of transportation vehicles. Therefore, the scheduling decisions of relief supplies include three parts: selecting the emergency service points, the amount of relief supplies that selected emergency service points provide and the route of the transportation vehicles from selected service points to demand points.

Haghani, Ali and Oh, Sei-Chang present a formulation and two solution methods for a logistical problem which is a large-scale multi-commodity, multi-modal network flow problem with time windows in disaster relief management [1]. F. Fiedrich, F. Gehbauer and U. Rickers introduced a dynamic optimization model to find the best assignment of available resources to operational areas after strong earthquakes [2]. Linet Ozdama and Ediz Ekinei developed a planning model of dispatching commodities to distribution centers, which was integrated into a natural disaster logistics decision support system [3]. Sheu J.B. proposed a hybrid fuzzy clustering-optimization approach to the operation of emergency logistics co-distribution responding to the urgent relief demands after natural disasters [4].

When a major disaster event occurs, the demand for relief supplies of emergency demand points will exceed the supplies amount of reserve points which can meet their requirements in limited time. Thus, emergency service points which can not reach demand points in limited time should also provide materials to those points. Therefore, the scheduling strategy proposed in this paper is that all emergency demand points should be provided some relief supplies, if can not fully meet the requirement, by their nearest service points in limited time in order to mitigate the consequences of disasters and carry out relief operations as soon as possible. The insufficient part of each demand points would be met after limited time. This strategy takes into account the needs of all emergency points, which not only reflect the fairness but also prevent further loss of disaster events.

The objective function of emergency scheduling model revolves time, distance and cost in many other study literatures. Actually, in initial period of disaster relief operation, saving life and minimizing losses rather than dispatch expense are the key point of consideration. For each emergency needs point, part of relief supplies should arrived in limited time. Under this premise, the goal is minimize the loss caused by delayed arrival. The factors affecting the loss are the period of delayed time and the number of unmet demand for relief supplies in limited time.

II. ASSUMPTIONS AND DEFINITIONS

A. notations

\[ S = \{S_i | i=1,2,\ldots,n\} : \text{the set of relief supplies service points in the region}; \]
\[ F = \{ F_i \mid j = 1,2, \ldots, m \} \] is the set of relief supplies demand points in the region; 
\( t_i \) : the travel time from point \( i \) to point \( j \); 
\( t \) : the limited time of demand for relief supplies, which is usually determined by the feature of relief supplies; 
\( z_i \) : the backup amount of relief supplies in point \( i \); 
\( r_j \) : the demand amount for relief supplies in point \( j \); 
\( a_y \) : the amount of relief supplies that point \( i \) provide to point \( j \), which is the decision variable; 
\( \lambda_y \) : the 0-1 variable values according to whether the travel time from point \( i \) to point \( j \) exceed \( t \).

\[ \lambda_y = \begin{cases} 1 & t_y > t \\ 0 & t_y \leq t \end{cases} \] 

\( \beta \) : the penalty coefficient of disaster losses caused by per unit relief supplies when delayed per unit time to reach demand point. Obviously, when \( t_y \leq t \), \( \beta = 0 \).

\section*{B. Assumptions}

(i) It was assumed that the backup of all relief supplies service points can meet the need of all relief supplies demand points in the region, that is \( \sum_{j=1}^{n} z_i \geq \sum_{j=1}^{m} r_j \).

If the total amount of all points \( i \) can not meet the needs of all points \( j \), that is \( \sum_{i=1}^{n} z_i < \sum_{j=1}^{m} r_j \). In this case, the relief supplies from outside is needed. Before their arrival, the internal available relief supplies must be used to emergency relief activities. We make variable \( r'_j \) to replace variable \( r_j \).

\[ r'_j = \frac{\sum_{i=1}^{n} z_i}{\sum_{i=1}^{n} r_j} \times r_j \] 

\[ \sum_{j=1}^{m} z_i = \sum_{j=1}^{m} r'_j \] 

Another method is the conversion factor given by the field experts according to the feature of relief supplies. When delay time of relief supplies close to human physiological extreme limit, \( \beta \) increase rapidly as Fig. 1.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{penalty_factor.pdf}
\caption{Example of penalty factor of loss \( \beta \)}
\end{figure}

(iv) We think that emergency transportation vehicles available resources are abundant, by requisitioning social vehicles to ensure the implementation of scheduling decision. So the number restrictions and capacity restrictions of transportation vehicles did not take into account as constraints.

\section*{III. SCHEDULING OF RELIEF SUPPLIES DISTRIBUTION UNDER DETERMINED CONDITIONS}

\subsection*{A. Scheduling Model of Relief Supplies Distribution for Single Species}

The objective function of relief supplies scheduling model should be minimizing the disasters losses of all demand points due to delay, under the premise of ensure that each demand point has its own relief supplies in limited time. The factors affecting the loss are delay time and unmet demand of relief supplies in limited period. The objective function can express as follow.

\begin{equation}
\text{Model I} \quad \min \sum_{i=1}^{n} \sum_{j=1}^{m} \beta \lambda_y \cdot (t_y - t) \cdot a_y 
\end{equation}

Subject to:

(i) The total amount of all emergency service points can meet the needs of all emergency demand points.

\[ \sum_{i=1}^{n} z_i \geq \sum_{j=1}^{m} r_j \] 

(ii) The needs of all emergency demand points can be met.

\[ \sum_{j=1}^{m} a_y = r_j, \quad j = 1,2, \ldots, m \] 

(iii) The supplies number of each emergency service points can not exceed its reserve amount.

\[ \sum_{j=1}^{m} a_y \leq z_i, \quad i = 1,2, \ldots, n \] 

(iv) Each demand point has its own relief supplies in limited time.

\[ \sum_{j=1}^{m} (1-\lambda_y) \cdot a_y > 0, \quad j = 1,2, \ldots, m \]
(v) \( z_i \geq 0; r_j \geq 0; a_{ij} \geq 0; \lambda_j = \{0,1\}; \beta \geq 0 \).

If delay time for each demand point is zero, the model has infinite solutions and objective function should be changed to minimize total transportation time or cost.

B. Experimental Results of Model I

We assume that there are 5 emergency demand points and 10 emergency service points, and \( t = 10 \). The value of \( t_y, r_j \) and \( z_i \) is showed in Table I. The value of \( \beta \) is different according to the value section of \( t_y - t \).

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TABLE II. INITIAL CONDITIONS OF EXPERIMENTAL OF MODEL I

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C. Scheduling Mode of Relief Supplies Distribution for Many Varieties

There are many types of relief supplies, such as drinking water, foods, medicine and so on. When scheduling model of varieties relief supplies distribution is considered, \( k \) is defined as the species of relief supplies, \( k = 1, \ldots, l \).

\( z_i^k \): the backup amount of \( k \) kind of relief supplies in point \( i \);  
\( r_j^k \): the demand amount for \( k \) kind of relief supplies in point \( j \);  
\( a_{ij}^k \): the amount of \( k \) kind of relief supplies that point \( i \) provide to point \( j \), which is the decision variable;  
\( t^k \): the limited time of demand for \( k \) kind of relief supplies. For the same scheduling batch of relief supplies their restriction time is similar, and it is assumed that \( t^k = t^2 = \cdots = t^1 = t \);  
\( \beta^k \): the penalty coefficient of disaster losses caused by per unit \( k \) kind of relief supplies when delayed per unit time to reach demand point;  
Other variables defined as before. The objective function of scheduling model of many varieties relief supplies is also minimizing the disasters losses of all demand points due to delay, and can express as follow.

Model II  \[ \min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{l} \beta^k \cdot (t_j - t) \cdot a_{ij}^k \]  
Subject to:  
(i) The total amount of all emergency service points can meet the needs of all emergency demand points.  
\[ \sum_{i=1}^{n} z_i^k \geq \sum_{j=1}^{n} r_j^k, \quad k = 1, 2, \ldots, l \]  
(ii) The needs of every emergency demand points can be met.  
\[ \sum_{i=1}^{n} a_{ij}^k \geq r_j^k, \quad j = 1, 2, \ldots, m, \quad k = 1, 2, \ldots, l \]  
(iii) The supplies number of each emergency service points can not exceed its reserve amount.  
\[ \sum_{j=1}^{m} a_{ij}^k \leq z_i^k, \quad i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, l \]  
(iv) Each demand point has its own relief supplies reached in limited time.  
\[ \sum_{j=1}^{m} ([1 - \lambda_j] \cdot a_{ij}^k) > 0, \quad j = 1, 2, \ldots, m, k = 1, 2, \ldots, l \]  
(v) \( z_i^k \geq 0; \quad r_j^k \geq 0; \quad a_{ij}^k \geq 0; \quad \lambda_j = \{0,1\}; \quad \beta^k \geq 0 \).
D. Experimental Results of Model II

The solution of Model II is similar with the solution of Model I. For every $k$ from 1 to $l$, $a_{ij}^k$ can be solved. Then the result summarized is the solution of the model. We assume that there are 5 emergency demand points and 10 emergency service points, and $t=10$, $k=3$. The value of $t_{ij}$ is showed in Table III and the value of $r^k$ is showed in Table IV. It is assumed that the penalty coefficient of each kind of relief supplies is the same, so that $\beta^2 = \beta^3 = \cdots = \beta^l = \beta$. The value of $\beta$ is showed in (11).

The results solved by using LINGO are shown in Table V, and the objective function values are 225 ($k=1$), 205 ($k=2$) and 205 ($k=3$).

### TABLE III. INITIAL CONDITIONS(1) OF EXPERIMENTAL OF MODEL II

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### TABLE V. EXPERIMENTAL RESULTS OF MODEL II

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IV. OPTIMAL PATH IN FUZZY NETWORKS

Natural disaster events may cause road damage, so that the travel time between the emergency service points and emergency demand points is uncertain. If there is more than one path from emergency service points to emergency demand points, the travel time must be identified by choosing the optimal path in fuzzy network firstly. Symmetry triangular fuzzy number (STFN) was used to indicate the uncertainty in this paper.

A. Definition

In a given graphic network \( G = \{V, E\} \), \( V \) is the set of points in \( G \), and \( E \) is the set of arc connecting points in \( G \). \( \forall e \in E \), \( w(e) \) is weight of \( e \). \( R \) is the set of all links between point \( i \) and point \( j \). If \( P \) is a route link any two point in \( G \), \( w(P) = \sum_{e \in P} w(e) \).

The weight of arc can be considered as travel time. The shortest route between point \( i \) and point \( j \) is route \( P_0 \) which weight is minimum. \( w(P_0) = \min P w(P) \).

\( t \) is limited time. \( Q = \{P \mid w(P) \leq t, P \in R\} \) is the set of route between point \( i \) and point \( j \) which completely satisfied \( t \). If \( w(e) \) is STFN, \( w(P) \) is also STFN.

The satisfaction function \( F(P,t) \) was defined as the degree of satisfaction that travel time through \( P \) is limited, \( |F(P,t)| = \sum_{e \in P} w(e) \).

Seeking optimal path is finding \( P' \), \( P' \in S \). \( \chi \) is the set of STFN.

\( T(M \leq t) \) means possibility of \( M \leq t \).

\[
T(M \leq t) = \begin{cases} 
0 & a > t \\
\frac{1}{2} \left( \frac{1}{a-b} \right)^2 & a \leq t < b \\
1 - \frac{1}{2} \left( \frac{1}{c-t} \right)^2 & b \leq t < c \\
1 & c \leq t 
\end{cases} \tag{17}
\]

If \( M = [a, b, c] \in \chi \) and \( N = [d, e, f] \in \chi \), \( b = (a+c)/2 \), \( M + N = [a+d, b+e, c+f] \in \chi \).

If \( a \leq t < c \) and \( d \leq t < f \), the sufficient and necessary condition of \( T(M \leq t) \leq T(N \leq t) \) is \( t - a \leq t - d \leq c - a \leq f - d \).

Due to \( w(e) \in \chi \), \( M = [w_1(e), w_0(e), w_2(e)] \) and \( w_0(e) = (w_1(e) + w_2(e))/2 \).

Obviously,

\[
w(P) = \sum_{e \in P} w(e) = [w_1(P), w_0(P), w_2(P)] = [\sum_{e \in P} w_1(e), \sum_{e \in P} w_0(e), \sum_{e \in P} w_2(e)] \tag{18}
\]

B. Transformation of Maximum Satisfaction Factor Path

Let \( F(P,t) = T(\{|w(P)\leq t\}) = T(M \leq t) \leq (M \leq t), 0 \leq F(P,t) \leq 1 \).

If \( w_1(P_0) = \min_{P \in R} w_1(P) = \min_{P \in R} (\sum_{e \in P} w_1(e)) \leq t \), then \( F(P_0,t) = 1 \), \( P_0 \in S \).

If \( w_1(P_0) = \min_{P \in R} w_1(P) = \min_{P \in R} (\sum_{e \in P} w_1(e)) > t \), then \( F(P_0,t) = 0 \).

When \( (19) \) and \( (20) \) is tenable, maximum satisfaction factor path can be obtained with shortest path algorithm.

If \( \min_{P \in R} (\sum_{e \in P} w_1(e)) \leq t < \min_{P \in R} (\sum_{e \in P} w_1(e)) \), let \( P^* \) is solution of

\[
\min_{P \in R} \frac{w_1(P) - t}{w_1(P) - w_1(P)} = \frac{w_1(P') - t}{w_1(P') - w_1(P')} = x^* \tag{21}
\]

Then \( F(P',t) = \max_{P \in R} F(P,t) \) \[5\].

Equivalent form of \( (21) \) is

\[
\max -\frac{t - w_1(P)}{w_1(P) - w_1(P)} = \frac{t - w_1(P')}{w_1(P') - w_1(P')} = x^* \tag{22}
\]

Where \( x^* \in [-1, 0] \), and

\[
F(P',t) = \left\{ \begin{array}{ll} 
1 - 2x^2 & x^* \geq -0.5 \\
2(1 + x^2) & x^* < -0.5 
\end{array} \right.
\]

For \( x \in [-1, 0] \), let \( P(x) \) is corresponding path of optimum solution of the follow problem. \( P(x) \) can be obtained with shortest path algorithm.

\[
\min_{P \in R} \sum_{e \in P} ((w_1(e) - w_1(e)) \cdot x + w_2(e)) = \sum_{e \in P} ((w_1(e) - w_1(e)) \cdot x + w_2(e))
\]

Let \( N(x) = \sum_{e \in P} ((w_1(e) - w_1(e)) \cdot x + w_2(e)) \).

\( N(x) \) is a continuous increasing function. The sufficient and necessary condition of \( N(x) > t \), \( N(x) = t \) and \( N(x) < t \) are \( x < x^* \), \( x = x^* \) and \( x > x^* \).

When \( N(x) = t \), \( P(x) \) is optimal path that satisfy \( (22) \), \( x = x^* \), and vice versa.

C. Solving Step and Example

When \( (19) \) and \( (20) \) does not hold, then \( N(0) > t \) and \( N(\leq t) \). Dichotomy can be used to solving the problem.

Steps 1: let \( x = 0 \), if \( N(0) \leq t \), then \( P(0) \) is optimal path, \( P(0) \in S \), otherwise turn to steps 2.

Steps 2: let \( x = -1 \), if \( N(-1) > t \), then \( P(-1) \) is optimal path, \( P(-1) \in S \), otherwise turn to steps 3.

Steps 3: let \( x^* = -1 \) and \( x^* = 0 \), turn to steps 4.

Steps 4: let \( x = [x^* + x^*]/2 \), if \( N(x) - t < e \), then \( P(x) \) is optimal path, \( P(x) \in S \), calculation stopped. Otherwise turn to steps 5.

Steps 5: if \( N(x) > t \), then let \( x^* = x \), otherwise \( x^* = x \). Turn to steps 2.

We assume that the network between point \( S \) and point \( F \) is shown as Fig. 2 and \( t = 20 \). The travel time between points is STFN. By using the above algorithm, we get the results: maximum satisfaction factor path is \( S \rightarrow A2 \rightarrow A5 \).
Based on Fuzzy Travel Time

The fuzzy travel time from emergency service point \( i \) to emergency demand point \( j \) exceed \( t \); \( t_{ij} = \left( t_{ij}^0, t_{ij}^0, t_{ij}^2 \right) \): the fuzzy travel time from emergency service point \( i \) to emergency demand point \( j \), \( t_{ij}^0 = (t_{ij} + t_{ij}^2)/2 \);

\[
F(t_{ij}, t) = \begin{cases} 
0 & t_{ij}^2 > t \\
 \frac{1}{2} \left( \frac{t - t_{ij}^2}{t_{ij}^2 - t_{ij}^0} \right)^2 & t_{ij}^1 \leq t < t_{ij}^0 \\
 \frac{1}{2} \left( 1 - \frac{t_{ij}^2 - t}{t_{ij}^2 - t_{ij}^0} \right)^2 & t_{ij}^0 \leq t < t_{ij}^2 \\
1 & t_{ij}^2 \leq t 
\end{cases}
\] (23)

\( \lambda_{ij} \): whether the travel time from emergency service point \( i \) to emergency demand point \( j \) exceed \( t \);

\[
\lambda_{ij} = \begin{cases} 
0 & F(t_{ij}, t) = 1 \\
1 & 0 < F(t_{ij}, t) < 1 
\end{cases}
\] (24)

Other variables defined as before. The objective function of relief supplies scheduling model should be minimizing the disasters losses of all demand points due to delay, under the premise of ensure that each demand point has its own relief supplies in limited time. The factors affecting the loss are delay time and unmet demand of relief supplies in limited period. The objective function can express as follow:

\[
\text{Model III} \quad \min \sum_{i=1}^{n} \sum_{j=1}^{m} \beta [1 - F(t_{ij}, t)](t_{ij}^2 - t)a_{ij} 
\] (25)

Subject to:

1. The total amount of all emergency service points can meet the needs of all emergency demand points.

\[
\sum_{j=1}^{m} z_j \geq \sum_{i=1}^{n} r_i 
\]

2. The needs of all emergency demand points can be met.

\[
\sum_{j=1}^{m} a_{ij} = r_j, \quad j = 1, 2, \ldots, m 
\]

3. The supplies number of each emergency service points can not exceed its reserve amount.

\[
\sum_{i=1}^{n} a_{ij} \leq z_i, \quad i = 1, 2, \ldots, n 
\]

4. Each demand point has its own relief supplies in limited time.

\[
\sum_{j=1}^{m} (1 - \lambda_{ij}) \cdot a_{ij} > 0, \quad j = 1, 2, \ldots, m 
\]

5. \( z_i \geq 0; \quad r_j \geq 0; \quad a_{ij} \geq 0; \quad \lambda_{ij} = 0.1; \quad \beta \geq 0 \).

B. Experimental Results

We assume that there are 5 emergency demand points and 10 emergency service points, and \( n = 10 \). The value of \( t_{ij}^1, r_j \) and \( z_i \) is showed in Table VI. The value of \( \beta \) is different according to the value section of \( t_{ij}^2 - t \).

Finally, according to (23) \( F(t_{ij}, t) \) can be calculated.

Then the model is solved by using linear programming tool LINGO. The results are shown in Table VII, and the objective function value is 578.125.

VI. CONCLUSIONS AND FUTURE WORK

Considering the more demand number for emergency supplies and time-sensitive situation, scheduling model of relief supplies with the goal of minimum disaster losses was established and solved under different conditions. In order to achieve the goal, the requirements for relief supplies of each emergency demand points should be met in supplies the aspect of time and quantity which means the required supplies must be delivered to the demand points in limited time. The scheduling strategy proposed

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**TABLE VI. INITIAL CONDITIONS OF EXPERIMENTAL OF MODEL III**

<table>
<thead>
<tr>
<th>( t_j )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( S_3 )</th>
<th>( S_4 )</th>
<th>( S_5 )</th>
<th>( S_6 )</th>
<th>( S_7 )</th>
<th>( S_8 )</th>
<th>( S_9 )</th>
<th>( S_{10} )</th>
<th>( z_i )</th>
<th>( r_j )</th>
<th>( a_{ij} )</th>
<th>( \lambda_{ij} )</th>
<th>( \beta )</th>
<th>( \text{Demand} )</th>
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<td>[10,12,14]</td>
<td>[11,13,15]</td>
<td>[12,14,16]</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_2 )</td>
<td>[14,16,18]</td>
<td>[10,11,12]</td>
<td>[18,21,24]</td>
<td>[7,8,9]</td>
<td>[11,12,13]</td>
<td>[13,15,17]</td>
<td>[16,18,20]</td>
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<tr>
<td>( F_3 )</td>
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<td>[15,17,19]</td>
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<tr>
<td>( F_4 )</td>
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<td>[6,8,10]</td>
<td>[15,17,19]</td>
<td>[15,16,17]</td>
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<tr>
<td>( F_5 )</td>
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</table>

Reserve 40 50 55 45 60 60 40 45 65 40 500
in this paper take into account the needs of all emergency demand points which can effectively prevent further expansion of the disaster loss. We are interested in how to determine the value of penalty factor \( \beta \) for further research by the scientific and rational way.

APPENDIX LINGO CODE OF EXPERIMENTAL EXAMPLE OF SOLVING MODEL III

model:
sets:
warehouses/wh1..wh10/: capacity;
customers/v1..v5/: demand;
links(warehouses,customers): trans_time1, trans_time2, volume, beta, ft;
endsets
data:
capacity=40 50 55 45 60 60 40 45 65 40;
demand=100 120 90 110 80;
trans_time1=14 14 9 8 10 8 10 13 6 19;
9 18 15 15 13 10 7 11 15 12;
13 11 6 9 7 10 13 9 15 14;
12 16 20 11 10; 7 8 12 10 17;
8 9 9 21 12;
19 11 10 5 14;
trans_time0= 17 16 11 9 12
9 11 15 8 22
11 21 17 17 15
12 8 12 16 14
16 12 7 11 8
13 15 11 18 15
14 18 23 13 12
8 10 13 12 18
9 11 10 25 13
22 12 12 7 16;
trans_time2=20 18 13 10 14
10 12 17 10 25
13 24 19 19 17
14 9 13 17 16
19 13 8 13 9
15 17 13 21 16
16 20 26 15 14
9 12 14 14 19
10 13 11 29 14; 25 13 14 9 18;
limited_time=10;
enddata

min=@sum(links: beta*(1-ft)*(trans_time2-limited_time)*volume);
@for(links(I,J):
beta=@IF(trans_time2-limited_time#le# 0, 0, @IF((trans_time2-limited_time #gt# 0) #and# (trans_time2-limited_time #lt# 5), 1, @IF((trans_time2-limited_time #gt# 5) #and# (trans_time2-limited_time #le# 10), 2, @IF((trans_time2-limited_time #gt# 10) #and# (trans_time2-limited_time #le# 20), 10,100))));
@for(links(I,J):
ft=@IF(limited_time #lt# trans_time1, 0, @IF((limited_time #ge# trans_time1) #and# (limited_time #lt# trans_time0), 2*(limited_time-trans_time1)/(trans_time2-trans_time1))^2, @IF((limited_time #ge# trans_time0) #and# (limited_time #lt# trans_time2), 1-2*((trans_time2-limited_time)/(trans_time2-trans_time1))^2,1)));
@for(customers(J):
@sum(warehouses(I): volume(I,J))=demand(J));
@for(warehouses(I):
@sum(customers(J): volume(I,J))<=capacity(I));
end

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REFERENCES