The Near-Optimal Preventive Maintenance Policies for a Repairable System with a Finite Life Time by Using Simulation Methods

Chun-Yuan Cheng
Department of Industrial Engineering and Management, Chaoyang University of Technology, Taichung County, Taiwan
Email: cycheng@cyut.edu.tw

***Abstract*—For a repairable and deteriorating system, the theoretical preventive maintenance (PM) models are usually complicated and need numerical methods to obtain the optimal PM policies since the system's failure rate function is changed after each PM. It makes the application of the theoretical model not quite suitable for real cases. Moreover, the theoretical optimal PM solution is obtained by evaluating the expected cost rate of the system over an infinite time span. Yet, in reality, a deteriorating system always has a finite life time. Hence, searching for an optimal PM solution for a deteriorating system over an infinite time span might not be rational. Therefore, we consider using Monte Carlo simulation method to mimic the complicated failure process of a system with PM activity and then to obtain a range of the near-optimal PM policies. In this paper, the inverse transformation method and the rejection method are applied to generate the time-between-failures (TBF) random variates. The algorithms for generating the RVs are developed. The procedure of finding the near-optimal PM policies are also provided. Then, examples of using the proposed simulation method to obtain the near-optimal policies for the age-reduction PM model in a finite time span are presented and discussed.**

***Index Terms*—Preventive maintenance, Simulation method, Time-between-failure random variates, Age reduction, Inverse Transformation Method, Rejection Method.**

I. INTRODUCTION

For a deteriorating and repairable system, the system's failure rate function is changed after performing a preventive maintenance (PM) activity. Thus, the theoretical PM models are usually complicated and need numerical methods to obtain the optimal PM policies. Some theoretical PM models can be found in [1-4]. This limits the application of the theoretical model in real world. Furthermore, the theoretical optimal PM policies are obtained based on the long-term expectation of failure occurrences over the infinite time span [4-8]. Yet, in reality, the life time of a system is always finite. For systems with a finite life time, although there do exist some PM models in literature, however, the models are complicated and need numerical methods to find the optimal solution [9-12]. Hence, the optimal theoretical solution may not suitable for the real case of a single system with finite life time. In practical, a near-optimal PM policy is good enough for applications which can be obtained by using Monte Carlo simulation method [13]. However, the research of using simulation method in solving the PM problems can not be commonly found in literature.

When using Monte Carlo simulation method to obtain a near-optimal PM policy, the critical step is to generate the time-between-failures (TBF) random variates (RV) to mimic the system's failure process which is affacted by the PM activities. Percy and Kobbacy [14] investigate the scheduling of PM for repairable systems with renewals and minimal repairs models by using the simulation method. However, they did not present the RV generation method. Leemis and Schmeiser [15] describe algorithms based on the probability density function and hazard rate for generating a continuous non-negative random variates. Cheng and Liaw [16] applies the inverse transformation method to generate the RVs of the TBF for a PM model with age reduction effect. The algorithm developed by Cheng and Liaw [16] requires the derivation of the cumulative distribution function (CDF) of the TBF for each PM which is usually complicated. Cheng, Guo, and Liu [13] presented three RV generation methods for the TBF of a PM model and compared the accuracy among the three methods. They found that all three RV generation methods have high accuracy while the rejection method (or called acceptance-rejection method) is the simplest and easy-to-use method. Since the inverse transformation method and the rejection method are commonly applied in generating RVs [17, 18], in this paper, we apply these two methods to develop the algorithms of RV generation for the PM model with age reduction effect. Examples of finding the near-optimal PM policies are also provided and discussed.

II. THE THEORITICAL FEATURE OF THE PM MODEL WITH AGE REDUCTION

A. Nomenclature

- L the finite life time span for the system or equipment
- N the number of PM performed in the finite life time span (L)

doi:10.4304/jcp.6.3.548-555

T the time interval of each periodic PM where $T = L/(N+1)$

$x_{i,j}$ the generated time between the j-1st and the j^{th} failures in the i^{th} PM cycle, $j = 1, 2, \ldots, k_i$, $i = 1, 2, \ldots, N+1$.

$t_{i,j}$ the generated occurrence time of the j^{th} failure in the i^{th} PM cycle where $t_{i,j} = t_{i,j-1} + x_{ij}$

$u_{i,j}$

x_m the generated time between the m-1st and the m^{th} failures, $m = 0, 1, \ldots$

t_m the generated occurrence time of the m^{th} failures

γ the age reduced (restored) after each PM

$\lambda(t)$ The original failure rate function (before the 1st PM action)

$\lambda_i(t)$ The failure rate function at time t where t is in the i^{th} PM cycle and $\lambda_0(t)=\lambda(t)$

$F(t)$ the cumulated distribution function (CDF) of the TBF at age t

$R(t)$ the reliability at age t

u_1 the random number for generating the RV with majorizing failure rage function, $\lambda(t)$, where $u_1 \sim Uniform(0,1)$

u_2 the random number for making the decision of rejecting the generated RV, where $u_2 \sim Uniform\ (0,1)$

C_{pm} Cost of each PM

C_{mr} Minimal repair cost of each failure

TC The total maintenance cost function in the finite life time span

B. Assumptions

- The system has a finite useful life time L.
- The system is deteriorating and repairable over time where the failure process follows the non-homogenous Poisson Process (NHPP) with increasing failure rate (IFR). Weibull distribution with failure rate function:

$$\lambda(t) = \left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1}. \qquad (1)$$

is used to illustrate the examples in this paper, where β is the shape parameter and θ is the scale parameter.

- The periodic PM actions with constant interval (T) are performed over the finite time span L.
- The system's age can have a younger age (called the effective age) by reducing γ units of time after each PM. Hence, the failure rate function at time t in the i^{th} PM cycle can be written as

$$\lambda_i(t) = \lambda(t - i\gamma)\,. \qquad (2)$$

- Minimal repair is performed when failure occurs between each PM.
- The time required for performing PM, minimal repair, or replacement is negligible.

C. The PM Model and the Theoretical Optimal Policy

The PM model with age reduction is applied in this paper. Figure 1 illustrates the failure rate function of this PM model with 3 PM actions. In order to study the accuracy of near-optimal policies obtained from the proposed simulation method, we have to find the theoretical optimal policies for the age-reduction PM model over a finite time span. The theoretical optimal policies are obtained based on Cheng, Liaw, and Wang [19] and Yeh and Chen [11] where the decision variables are the number of PM (N) and the restored age (γ) for each PM. We summarize the procedure for finding the theoretical optimal PM policy over a finite time span as follows.

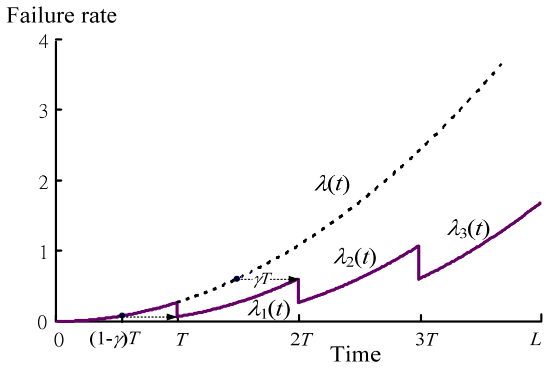

Figure 1. The failure rate function of the age-reduction PM Model with N=3.

The first step is to find the expected cost function for the PM model as shown below.

$$C(N,\gamma) = (N-1)C_{pm} + C_{pr} + C_{mr}\Lambda(N,\gamma), \qquad (3)$$

where $\Lambda(N,\gamma)$ is the expected number of failures occurred in the finite time span L and is defined as

$$\Lambda(N,\gamma) = \sum_{i=0}^{N-1}\int_{iT}^{(i+1)T}\lambda_i(t)dt = \sum_{i=0}^{N-1}\int_{iT}^{(i+1)T}\lambda(t-i\gamma)dt \qquad (4)$$

where $T = L/(N+1)$. Second step is to obtain the restored age of the PM effect (γ) as a function of N by taking the partial derivative of γ of the expected cost function shown in (3) and letting it equal to zero, i.e.,

$$\frac{\partial C(N,\gamma)}{\partial\gamma} = 0\,.$$

Third, since the cost function is a convex function, the optimal value γ^* and N^* of the theoretical PM policy can be obtained by numerically searching

$$\min_N C(N,\gamma),\quad N = 1,2,\ldots$$

III. The Near-Opimal PM Policies by the Simulation Mehtod

A. The Concept of Random Variate Generation

There exist some useful methods for generating random variate with specific distribution, such as the inverse transformation method, the composition (linear combination) method, and the rejection method. The inverse transformation method is generally applicable and can be computationally efficient if the CDF can be analytically inverted, but may be complicated in computation for some probability distributions. The basic algorithm of the inverse transformation method [15, 17, 18] is listed as follows.

(1) Invert the CDF $F(t)$.

(2) Generate $u \sim Uniform(0,1)$.

(3) Obtain t where $t = F^{-1}(1-u)$.

The composition method is typically used when the probability density function (pdf) can be written as a convex combination of n other pdf's. The rejection method is usually applied in cases where the form of probability density function (pdf), $f(x)$, makes the inverse transformation method difficult to use.

The rejection technique requires finding a majorizing function $f^*(t)$ which bounds the pdf $f(t)$, i.e., $f^*(t) > f(t)\ \forall t$. The majorizing function must integrate to a finite value so that it can be scaled to be a pdf, $g(t)$, i.e.,

$$g(t) = \frac{f^*(t)}{\int_0^\infty f^*(\tau)d\tau}.$$

The rejection method of using the majorizing function is illustrated in Figure 2. Values are generated from $g(t)$, then accepted or rejected so that the accepted random variates will have pdf $f(t)$.

The basic algorithm of the rejection method is shown below.

1. Generate t from $g(t)$ and u from $Uniform(0,1)$.
2. IF $u < f(t)/f^*(t)$,
   accept t as a realization of $f(t)$;
   else
   reject the value of t and repeat Step 1.

Instead of using a majorizing function, Leemis and Schmeiser [15] present a thinning algorithm by using a majorizing failure rate function for the rejection method which is applied in this paper and is shown as follows.

1. Find a majorizing hazard function $\lambda^*(t)$ such that $\lambda^*(t) \geq \lambda(t)$ for all $t > 0$.
2. Let $t = 0$.
3. Generate y from $\lambda^*(y)$; $t \leftarrow t + y$.
4. Generate u from $Uniform(0,1)$.
5. IF $u < \lambda(t)/\lambda^*(t)$,
   accept t as a realization of $\lambda(t)$;
   else
   reject the value of t and repeat Step 3.

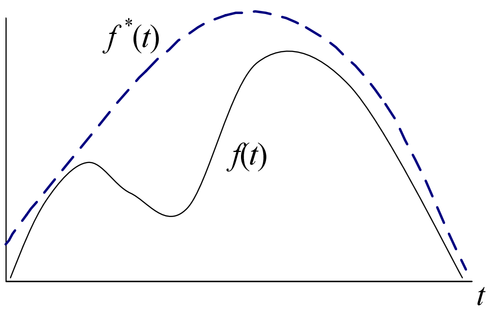

Figure 2. The illustration of the rejection method by using the majorizing function

B. The Generation of Time-Between-Failure RVs

(1) The Inverse Transformation Method

In this paper, we apply the modified inverse transformation method presented by Cheng and Liaw [16] to develop an algorithm for generating the TBF random variates from a PM model with age reduction effect. Since the minimal repair is assumed for each failure in this age-reduction PM model, for the i^{th} PM cycle, the CDF of the j^{th} TBF ($x_{i,j}$) given that the j-1st failure occurred at $t_{i,j-1}$ can be obtained as

$$F(x_{i,j}|t_{i,j-1}) = 1 - R(x_{i,j}|t_{i,j-1}) = 1 - \exp\left\{-\int_{t_{i,j-1}}^{t_{i,j-1}+x_{i,j}} \lambda_i(t')dt'\right\} \quad (5)$$
$$\text{for } i = 0,1,\ldots,N;\ j = 1,2,\ldots,k_i.$$

Cheng and Liaw [16] originally assume that the last failure in the i^{th} PM cycle is irrelative to the first failure in the i+1st PM cycle. However, this assumption may not be reasonable because the failure occurrence of the system follows the non-homogenous Poisson process (NHPP) and the PM is imperfect (i.e., the PM will not renew the system to zero failure rate). Therefore, The algorithm developed for the modified inverse transformation method assumes that the first failure in the i+1st PM cycle is affected by the last failure in the i^{th} PM cycle. It turns out that the CDF of the first TBF in the i+1st PM cycle given that the k_i^{th} failure occurred at t_{i,k_i} can be obtained as

$$\begin{aligned} F(x_{i+1,1}|t_{i,k_i}) &= 1 - R(x_{i+1,1}|t_{i,k_i}) \\ &= 1 - \exp\left\{-\left[\int_{t_{i,k_i}}^{(i+1)T} \lambda_i(t')dt' + \int_{(i+1)T}^{t_{i+1,1}} \lambda_{i+1}(t')dt'\right]\right\}. \end{aligned} \quad (6)$$

For a Weibull failure distribution, providing that $u_{i,j} \sim Uniform(0,1)$, based on (5) and (6), the TBF $x_{i,j}$ can be generated as follows.

(1) For $i = 0$; $j = 1, 2, \ldots, k_1$ and $i = 1, 2, \ldots, N$; $j = 2, 3, \ldots, k_i$:

$$x_{i,j} = \left\{-\theta^\beta \ln(1-u_{ij}) + \left[t_{i,j-1} - i\gamma\right]^\beta\right\}^{1/\beta} - t_{i,j-1} + i\gamma, \quad (7)$$

where $t_{0,0} = 0$, $t_{i,j} = t_{i,j-1} + x_{i,j}$;

(2) for $i = 1, 2, \ldots, N$; $j = 1$:

$$x_{i,1} = \left\{\begin{matrix} \left[i(T-\gamma)\right]^\beta + \left[t_{i-1,k_{i-1}} - (i-1)\gamma\right]^\beta \\ -\left[iT - (i-1)\gamma\right]^\beta - \theta^\beta \ln(1-u_{i-1,1}) \end{matrix}\right\}^{1/\beta} - t_{i-1,k_{i-1}} + i\gamma, \quad (8)$$

where $t_{i,1} = t_{i-1,k_{i-1}} + x_{i,1}$. The inverse transformation method of generating the TBF random variates for the age-reduction PM model is illustrated in Figure 3. The algorithm for the modified inverse transformation method is developed as follows.

1. Specify the values of the following parameters: β, θ, γ, N, L.
2. Let $i = 0$, $t_{0,0}$=0.
3. Let $j = 1$.
4. Generate $u_{i,j}$ from $Uniform(0,1)$.

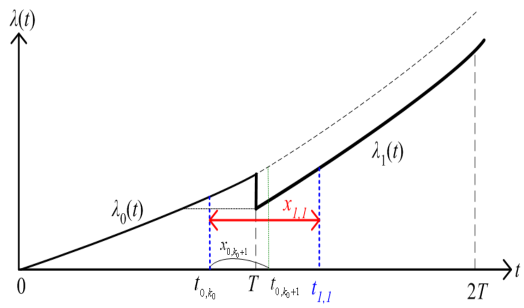

Figure 3. The illustration of the inverse transformation method of generating TBF random variates for the age-reduction PM model

5. Obtain the value of $x_{i,j}$ using (7); let $t_{i,j} = t_{i,j-1}+x_{i,j}$.
6. If $t_{i,j} < iT$, let $j = j + 1$ and go back to Step 4, else go to Step 7.
7. If $t_{i,j} < L$,
   obtain the value of $x_{i+1,1}$ using (8);
   let $t_{i+1,1} = t_{i,k_i} + x_{i+1,1}$;
   let $i = i + 1$ and $j = 2$;
   go back to Step 4,
   else stop.

(2) The Rejection Method

In this paper, a thinning algorithm of the rejection method provided by Leemis and Schmeiser [18] is applied for generating the RVs from the non-homogeneous Poisson processes. A majorizing function $\lambda^*(t)$ must be found which bounds the failure rate function $\lambda(t)$. It can be seen from Figure 1 that the original failure rate function $\lambda(t)$ satisfies the condition $\lambda(t) \geq \lambda_i(t)$ for any $t \geq 0$ where $\lambda_i(t)$ is the failure rate function of the i^{th} PM cycle as defined in (2). Thus, $\lambda(t)$ can be defined as the majorizing function, i.e., $\lambda^*(t) = \lambda(t)$.

When using the rejection method, two random numbers, u_1 and u_2, from *Uniform*(0,1) are required for generating each RV from the PM model with age reduction effect. We use u_1 to generate random variates from the majorizing function $\lambda(t)$ by using the inverse transformation method. Basic on the concept of the inverse transformation method stated in previous and the equation of the CDF for the TBF which is shown in (5), we have

$$u_1 = F(x_m|t_{m-1}) = 1 - \exp\left\{-\int_{t_{m-1}}^{t_{m-1}+x_m} \lambda(\tau)d\tau\right\} \quad \text{for } m = 1,2,\ldots \tag{9}$$

where m is the number of failures generated, $F(x_m \mid t_{m-1})$ represents the conditional CDF at time $t_{m-1} + x_m$ given that the system is surviving after the minimal repair at time t_{m-1}.

When a system has Weibull failure distribution, based on (1) and (9), we can generate RVs from the majorizing function $\lambda(t)$ by the following equation.

$$x_m = \left[-\theta^\beta \ln(1-u_1) + (t_{m-1})^\beta\right]^{1/\beta} - t_{m-1}. \tag{10}$$

Then, x_m is accepted if $u_2 < \lambda_i(t)/\lambda(t)$. The algorithm for generating the time-to-failure random variates from the age-reduction PM model is shown as follows.

1. Specify the values of the following parameters: β, θ, γ, N, L.
2. Let $t_{0,0}=0$, $t_0=0$.
3. Let $m = 0$, $i = 0$, $j = 1$.
4. Generate random number u_1.
5. Obtain the value of x_m according to (10); let $t_m = t_{m-1}+x_m$.
6. If $t_m < iT$, go to Step 7 else go to Step 10.
7. Generate random number u_2.
8. Calculate $\lambda(t_m)$ and $\lambda_i(t_m)=\lambda(t_m-i\gamma)$.
9. If $u_2 \leq \lambda_i(t_m)/\lambda(t_m)$,
   let $t_{ij} = t_m$; $j = j+1$; $m = m+1$;
   go back to Step 4
   else
   $m = m+1$; go back to Step 4.
10. If $t_m < L$,
   let $i = i + 1$ and $j = 1$; go back to Step 7
   else stop.

C. The Comparision of theProposed RV Generation Methods Using Experiment Examples

To compare the accuracy of the two RV generation methods proposed in this paper, we assume the finite life time period (L) be 6 time units, the number of PM (N) be 5, PM interval (T) be 1. The PM restoration effect is set as γ = 0.8. The Weibull failure distribution with scale parameter θ = 0.4 is assumed. Then, two experiments with β = 2.2 and 3.2 are constructed for each RV generating method. There are 30 runs for each experiment.

The accuracy of each proposed RV generation method

TABLE I.
THE COMPARISON OF THE PROPOSED **RV** GENERATION METHODS

(a) For $\beta = 2.2$

i	$E(k_i)$	$\bar{k}_{Ii}$	$\bar{k}_{Ri}$	p-value ($\bar{k}_{Ii} = \bar{k}_{Ri}$)	MAD_{Ii}	MAD_{Ri}	p-value ($\bar{k}_{Ii} = E(k_i)$)	p-value ($\bar{k}_{Ri} = E(k_i)$)
0	7.5070	7.0333	7.0333	1	1.9676	1.9676	0.8457	0.8457
1	10.9939	11.6333	11.4333	0.8341	2.6350	3.2341	0.8483	0.9115
2	14.7380	15.000	16.2667	0.2580	3.3159	3.4731	0.9526	0.7235
3	18.6720	18.333	18.8333	0.6728	4.0656	3.5437	0.9448	0.9683
4	22.7621	22.8333	22.4667	0.7897	4.4175	4.1016	0.9892	0.9542
5	26.9862	29.2667	27.2333	0.1163	4.8740	3.7676	0.6931	0.9548
all	101.659	104.1000	103.2667	0.7490	7.3000	8.7561	0.8005	0.8782

(b) For $\beta = 3.2$

i	$E(k_i)$	$\bar{k}_{Ii}$	$\bar{k}_{Ri}$	p-value ($\bar{k}_{Ii} = \bar{k}_{Ri}$)	MAD_{Ii}	MAD_{Ri}	p-value ($\bar{k}_{Ii} = E(k_i)$)	p-value ($\bar{k}_{Ri} = E(k_i)$)
0	18.7676	18.8333	18.8333	1	3.66432	3.6643	0.9883	0.9883
1	33.5259	33.7333	34.9333	0.4956	5.2632	5.6948	0.9744	0.8405
2	54.0830	56.000	54.13333	0.3105	5.4000	5.2111	0.7982	0.9939
3	80.7884	81.06667	78.43333	0.2623	7.2526	7.7910	0.9747	0.7993
4	113.917	113.5667	112.2333	0.6351	10.0389	7.4889	0.9767	0.8557
5	153.699	152.1333	154.3667	0.5484	10.8932	11.8068	0.9058	0.9645
all	454.7809	455.3333	452.9333	0.6708	17.9041	15.2375	0.9804	0.9277

is evaluated. In Tables I, we present the following values in the i^{th} PM cycle for β = 2.2 and 3.2: the expected number of failures ($E(k_i)$), the average number of failures generated by the inverse transformation method and the rejection method ($\bar{k}_{Ii}$ and $\bar{k}_{Ri}$, respectively), the p-value obtained from the hypothesis test of H_o: $E(\bar{k}_{Ii}) = E(\bar{k}_{Ri})$, the mean absolute deviation of the generated number of failures from the expected number of failures for the inverse transformation method and the rejection method (MAD_{Ii} and MAD_{Ri}, respectively), and the p-values obtained from the hypothesis tests of H_o: $\bar{k}_{Ii} = E(k_i)$ and H_o: $\bar{k}_{Ri} = E(k_i)$.

From Table I, we can see that the two RV generation methods have the same results in the initial PM cycle (i = 0) because they produce the same failure observations before any PM action is performed. It can be found from the p-values of H_o: $E(\bar{k}_{Ii}) = E(\bar{k}_{Ri})$ for all the PM cycles that there are no significances between the results of the two RV generation methods for both β = 2.2 and 3.2. It can also be noted from the values of MAD and the p-values of H_o: $\bar{k}_{Ii} = E(k_i)$ and H_o: $\bar{k}_{Ri} = E(k_i)$ that the inverse transformation method and the rejection method have similarly high accuracy in generating the TBF random variates for the age-reduction PM model. Therefore, we apply the rejection method to find the near-optimal PM policies since the rejection method uses a majorizing failure rate function which is easier and simpler for generating the TBF random variates from the failure distributions with complicated formula.

D. Procedure of Finding the Near-Optimal PM Policies

Once the algorithm for generating the TBF random variates is developed, we can find the near-optimal PM policies by the following procedure.

1. Specify the failure distribution, the PM model, and the type of restoration.
2. Set the values for the required parameters, such as the parameters of the failure distribution ($f(t)$), repair cost (C_{mr}), PM cost (C_{pm}), life time (L), number of simulation runs, maximum number of PM to be simulated (N), .and the restoration effect (γ) if it has to be predetermined.
3. Develop the failure rate function, the CDF of the TBF, and the cost function (TC).
4. Apply the inverse transformation method or the rejection method to generate the TBF random variates for each simulation run.
5. Calculate the total cost for each simulation run.
6. Establish the tables of simulation results (similar to Tables II, III or IV, V).
7. Find the near-optimal policies from the tables established in Step 6.

IV. Examples and Discussion

In the examples, we assume the finite life time period (L) be 6 time units. The Weibull failure distribution with shape parameter β = 3.2 and scale parameter θ = 0.4. The minimal repair cost is set as 3.1036 per failure. The cost of each PM is assumed as function of the PM restoration effect, which is $C_{pm} = a+b\gamma = 5+100\gamma$. The near-optimal solutions are obtained by Monte Carlo simulation method with 30 runs. The theoretical optimal solution is also calculated using the algorithm provided by Yeh and Chen [11].

A. Example 1

In Example 1, for simplicity, the restoration effect γ for each N is predetermined using the theoretical method as shown in Table II. The 30-run simulation results for N = 1 to 6 using the rejection method are presented in Table III. The smallest (best) total maintenance cost (TC) of each run is highlighted with shadow background. It can be seen from Table III that, for each N, the average value of TC from the 30-run simulation is very close to the value obtained by using the theoretical method. Both methods (simulation and theoretical model) provides the same optimal policy of having the optimal number of PM N^*=3 and the optimal restoration effect γ^*=0.4781. It has

TABLE II.
THE VALUE OF γ CORRESPONDING TO EACH N FOR EXAMPLE 1

N	1	2	3	4	5	6
γ	1	0.6667	0.4781	0.3655	0.2957	0.2483

TABLE III.
THE RESULTS OF THE 30 SIMULATION RUNS FOR EXAMPLE 1

Run#	N	1	2	**3**	4	5	6
	γ	1	0.6667	**0.4781**	0.3655	0.2957	0.2483
1		216.730	189.894	**180.155**	194.132	194.575	210.016
2		229.144	196.101	192.570	**187.925**	210.093	194.498
3		250.869	196.101	**189.466**	197.236	213.197	206.912
4		250.869	186.790	**180.155**	200.340	188.368	213.120
5		204.315	199.205	204.984	**187.925**	219.404	197.602
6		263.284	**189.894**	201.880	194.132	197.679	197.602
7		213.626	**165.065**	183.259	197.236	185.264	203.809
8		232.248	214.723	**192.570**	194.132	206.990	194.498
9		241.558	**177.480**	183.259	200.340	191.472	206.912
10		216.730	189.894	**180.155**	194.132	188.368	206.912
11		219.833	211.619	**204.984**	209.650	213.197	216.223
12		222.937	189.894	189.466	**181.718**	197.679	197.602
13		247.766	**177.480**	180.155	209.650	197.679	216.223
14		247.766	196.101	**180.155**	206.547	197.679	203.809
15		198.108	196.101	189.466	**181.718**	200.782	194.498
16		216.730	192.998	**183.259**	200.340	185.264	203.809
17		226.040	205.412	**189.466**	191.029	194.575	206.912
18		195.004	202.308	211.191	**191.029**	210.093	191.394
19		216.730	**183.687**	186.362	191.029	200.782	213.120
20		226.040	199.205	**183.259**	203.443	200.782	206.912
21		232.248	186.790	189.466	**184.822**	194.575	203.809
22		204.315	**196.101**	198.777	197.236	197.679	200.705
23		207.419	186.790	**180.155**	206.547	188.368	206.912
24		210.522	208.516	195.673	**187.925**	206.990	197.602
25		219.833	205.412	**195.673**	203.443	197.679	213.120
26		257.076	192.998	**173.948**	215.858	191.472	219.327
27		216.730	211.619	195.673	**172.407**	210.093	188.291
28		216.730	**189.894**	201.880	200.340	197.679	216.223
29		210.522	**168.169**	180.155	206.547	191.472	216.223
30		247.766	**186.790**	189.466	187.925	197.679	200.705
Average		225.316	193.101	**189.569**	195.891	198.92	204.843
Theoretical		221.495	191.076	**189.728**	192.850	197.222	202.051

demonstrated that the experimental results obtained by simulation method are consistent with those obtained by the theoretical model when large sample runs are generated.

It should be noted that the best solution of N, γ, and TC (marked with shadow) resulted from each simulation run are different from those obtained by the theoretical model. It is because the optimal solution of the theoretical model is obtained by taking the expectation result over the infinite time interval or over a large number of identical systems in a finite time interval. However, the simulation method can provide different policies for a single system having a finite life time. This is more close to the real situation where an organization will not possess a large number of identical systems nor will purchase an identical system in every replacement cycle for suiting the assumption of infinite time span.

It can be seen from Table III that the best solutions of each simulation run (marked with shadow) can be categorized into three near-optimal policies: (N=2, γ=0.6667), (N=3, γ=0.4781), and (N=4, γ=0.3655). Table IV lists the simulation runs in each category and presents the average, the smallest, and the largest values of the minimal TC for each category of the near-optimal policy.

Among these best solutions, the average of the minimal TC (184.1143) is smaller than the theoretical minimal TC (189.7280). The results demonstrate that the theoretical PM model over a finite time span might not be suitable for the problem of considering only a single system in a finite time span.

Therefore, in practical, when considering a single system to be preventively maintained in a finite time period, especially for short time period, more than one single near-optimal policy is suggested. In this example, either (N=2, γ=0.6667) or (N=3, γ=0.4781) or (N=4, γ=0.3655) may be chosen as the best (near-optimal) PM policy. Further examining the simulation results shown in Table IV, we can see that the minimal TCs marked with shadow are smaller than the theoretical optimal TC (189.7280) in which 6 out of 9 (67%) for Policy 1, 10 out of 13 (77%) for Policy 2, 7 out of 8 (88%) for Policy 3, and 23 out of 30 (77%) for overall policies are better than the theoretical optimal policy. Therefore, for a finite-time-span PM problem, these results show strong evidence that the simulation method not only flexibly provides near-optimal policies but also gives better-than-the theoretical optimal policies with high confidence.

TABLE IV.
THE NEAR-OPTIMAL POLICIES FOR EXAMPLE 1

Policy 1 (N^*=2, γ^*=0.6667)		Policy 2 (N^*=3, γ^*=0.4781)		Policy 3 (N^*=4, γ^*=0.3655)	
Run#	Min. TC	Run#	Min. TC	Run#	Min. TC
6	189.894	1	180.155	2	187.925
7	165.065	3	189.466	5	187.925
9	177.480	4	180.155	12	181.718
13	177.480	8	192.570	15	181.718
19	183.687	10	180.155	18	191.029
22	196.101	11	204.984	21	184.822
28	189.894	14	180.155	24	187.925
29	168.169	16	183.259	27	172.407
30	186.790	17	189.466		
		20	183.259		
		23	180.155		
		25	195.673		
		26	173.948		
No of Runs	9		13		8
Runs Better than Theoretical	6 (67%)		10 (77%)		7 (88%)
Avg.	181.6177		185.6462		184.4337
Min.	165.0652		173.9480		172.4072
Max.	196.1012		204.9840		191.0288
Overall average of min. TC: 184.1143 The Rate of better-than-theoretical solutions: 77%					

B. Example 2

In the second example, we assume that the PM restoration value (γ) for each N is a decision variable and has to be determined by the simulation method. Therefore, different values of γ over the range of (0, 1) are used for each N where N is ranging from 1 to 6 in the simulation experiments. Only the experiments of γ = 0.2T, 0.5T, 0.8T, 0.985T, and γ^* (the theoretical optimal value) are shown in this example. All other parameters are given with the same values as in Example 1. A 30-run simulation is performed for each combination of (N, γ) to find the smallest TC with the corresponding best value of N and γ. The results are presented in Table V. Again, among these best solutions, the average of the minimal TC (182.5808) is smaller than the theoretical minimal TC (189.7280). This result shows that the best solution obtained from Monte Carlo simulation is better than the optimal theoretical solution.

Table VI lists the simulation runs in each category of the near-optimal policy and presents the average, the smallest, and the largest minimal TC of each near-optimal policy for Example 2. Likewise, the simulation method provides three near-optimal policies (with N = 2, 3, and 4). Since γ is a decision variable, each near-optimal policy has more than one value for γ. The values of γ are in the range of (0.6375, 0.6667) in Policy 1 (N = 2); it falls in the range of (0.400, 0.4925) in Policy 2 (N = 3); in Policy 3 (N =), γ has values in the range of (0.3655, 0.3940). From Table VI, Similarly, we can see that the minimal TCs marked with shadow are smaller than the theoretical optimal TC (189.7280) where 6 out of 8 (75%) for Policy 1, 10 out of 11 (91%) for Policy 2, 9 out of 11 (82%) for Policy 3, and 25 out of 30 (83%) for overall policies are better than the theoretical optimal policy.

Note that the value of γ specified for each policy of Example 1 (as shown in Table III) is covered in the range of the corresponding policy of Example 2. Moreover, it can also be found that although Examples 1 and 2 have similar results, the near-optimal policy of Example 2 has better solution (smaller average TC, narrower range of TC, and higher rate of better-than-theoretical solutions) than the corresponding policy in Example 1. it implies that the simulation method can obtain better solutions if γ is a decision variable than it is predetermined a value.

TABLE V.
THE RESULTS OF THE 30 SIMULATION RUNS FOR EXAMPLE 2

Run #	N	γ	TC	Run #	N	γ	TC
1	3	0.4781	180.1552	16	3	0.4925	181.3716
2	4	0.3655	187.9252	17	3	0.4925	187.5788
3	3	0.4925	187.5788	18	4	0.3655	191.0288
4	4	0.3825	176.1036	19	2	0.6567	181.6868
5	4	0.3655	187.9252	20	3	0.4781	183.2588
6	2	0.6667	189.8940	21	4	0.3655	184.8216
7	2	0.6667	165.0652	22	4	0.3940	190.0144
8	3	0.4000	187.7612	23	3	0.4000	175.3468
9	2	0.6567	175.4796	24	4	0.3655	187.9252
10	3	0.4000	178.4504	25	4	0.3825	191.6216
11	3	0.4925	196.8896	26	3	0.4781	173.9480
12	4	0.3655	181.7180	27	4	0.3655	172.4072
13	2	0.6667	177.4796	28	2	0.6667	189.8940
14	3	0.4781	180.1552	29	2	0.6667	168.1688
15	4	0.3655	181.7180	30	2	0.6375	184.0540
Average					3.1	0.4816	182.5808
Theoretical					3	0.4781	189.7280

TABLE VI.
THE NEAR-OPTIMAL POLICIES FOR EXAMPLE 2

Policy 1: N=2			Policy 2: N=3			Policy 3: N=4		
Run #	γ	TC	Run #	γ	TC	Run #	γ	TC
6	0.6667	189.8940	1	0.4781	180.1552	2	0.3655	187.9252
7	0.6667	165.0652	3	0.4925	187.5788	4	0.3825	176.1036
9	0.6567	175.4796	8	0.4000	187.7612	5	0.3655	187.9252
13	0.6667	177.4796	10	0.4000	178.4504	12	0.3655	181.718
19	0.6567	181.6868	11	0.4925	196.8896	15	0.3655	181.718
28	0.6667	189.8940	14	0.4781	180.1552	18	0.3655	191.0288
29	0.6667	168.1688	16	0.4925	181.3716	21	0.3655	184.8216
30	0.6375	184.0540	17	0.4925	187.5788	22	0.394	190.0144
			20	0.4781	183.2588	24	0.3655	187.9252
			23	0.4000	175.3468	25	0.3825	191.6216
			26	0.4781	173.9480	27	0.3655	172.4072
No of Runs		8			11			11
Runs Better than Theoretical		6 (75%)			10 (91%)			9 (82%)
Avg	0.6606	178.9653		0.4620	182.9540		0.3712	184.8372
Min	0.6375	165.0652		0.4000	173.9480		0.3655	172.4072
Max	0.6667	189.8940		0.4925	196.8896		0.3940	191.6216
Overall average of min. TC: 182.581 The Rate of better-than-theoretical solutions: 83%								

V. CONCLUSIONS

In this paper, we apply the inverse transformation method and the rejection method to develop the algorithms of generating the time-between-failure random variates for the age-reduction PM model to find the near-optimal PM policies.

We conclude from the results of this paper that, for the age-reduction PM model in a finite time span, more than one near-optimal policy can be obtained by using Monte Carlo simulation method. Each of the near-optimal solution can be the best PM policy for any single system having a finite life time. The simulation results have demonstrated that the theoretical PM model might not be suitable for a single system in a finite time span. It can be further concluded from this research that (1) the PM policies obtained from the proposed simulation methods can provide even better solution for an organization such as a car rental company who possesses many identical vehicles; (2) When considering the PM problem of a single system, the decision is more flexible because any of the near-optimal policies can work well.

The proposed simulation methods can be extended to solve more complicated real world situation, such as considering the random shocks in a PM model which is very difficult to be solved by the theoretical model.

ACKNOWLEDGMENT

This research has been supported by the National Science Council of Taiwan under the project number NSC96-2221-E-324-010.

REFERENCES

[1] H. Pham and H. Wang, “Invited review: imperfect maintenance”, *European Journal of Operational Research*, Vol.94, pp425-438,1996.

[2] S. Martorell, A. Sanchez, and V. Serradell, “Age-dependent reliability model considering effects of maintenance and working conditions”, *Reliability Engineering and System Safety*, Vol.64, pp.19-31, 1999.

[3] S.-H. Sheu and C.-C. Chang, “An Extended Periodic Imperfect Preventive Maintenance Model With Age-Dependent Failure Type”, *IEEE Transactions on Reliability*, Vol.58, No. 2, pp397 – 405, 2009.

[4] M. Bartholomew-Biggs, M. J. Zuo, and X. Li, “Modeling and optimizing sequential imperfect preventive maintenance”, *Reliability Engineering and System Safety*, Vol.94, pp53– 62, 2009.

[5] D. N. P. Murthy and D. G. Nguyen, “Optimal Age-Policy with Imperfect Preventive Maintenance,” *IEEE Transactions on Reliability*, Vol.R-30, No.1, pp.80-81, 1981.

[6] T. Nakagawa, “Periodic and Sequential Preventive Maintenance Policies”, *Journal of Applied Probability* R-23/2, pp536-542, 1986.

[7] D.H. Park, G.M. Jung, and J.K. Yum, “Cost Minimization for Periodic Maintenance Policy of a System Subject to Slow Degradation”, *Reliability Engineering and System Safety*, Vol.68, pp105-112, 2000.

[8] C.-Y. Cheng and M. Chen, “The periodic maintenance policy for a Weibull life-time system with degradation rate reduction under reliability limit,” *Asia-Pacific Journal of Operational Research*, Vol.25, Issue: 6, pp793-805, 2008.

[9] T. Nakagawa and S. Mizutani, “A summary of maintenance policies for a finite interval”, *Reliability Engineering and System Safety*, Vol.94: pp89-96, 2009.

[10] J. Pongpech and D. N. P. Murthy, “Optimal Periodic Preventive Maintenance Policy for Leased Equipment,” *Reliability Engineering & System Safety*, Vol.91, pp.772-777, 2006.

[11] R. H. Yeh and C. K. Chen, “Periodical Preventive-Maintenance Contract for a Leased Facility with Weibull Life-Time,” *Quality & Quantity*, Vol.40, pp.303-313, 2006.

[12] C.-Y. Cheng and H.-H. Liu, “The Finite-Time-Period Preventive Maintenance Policies with Failure Rate Reduction under A Warranty Consideration,” *Journal of the Chinese Institute of Industrial Engineers*, Vol.27, No.2, pp. 81-89, 2010.

[13] C.-Y. Cheng, R. Guo, and M.-L. Liu, 2009, "Random Variates Generating Methods of Time-Between-Failures for the Repairable Systems under Age-Reduction Preventive Maintenance," *Proceedings of Sixth International Conference on Informatics in Control, Automation and Robotics (ICINCO2009)*, July 2-5, 2009, Milan, Italy, pp325-330.

[14] D. F. Percy, K. A. H. Kobbacy, "Determining economical maintenance intervals," International Journal of Production Economics, Vol.67, pp87-94, 2000.

[15] L. Leemis and B. Schmeiser, "Random Variate Generation for Monte Carlo Experiments," *IEEE Transactions on Reliability*, Vol.R-34, No. 1, pp81-85, 1985.

[16] C.-Y. Cheng and C.-F. Liaw, "Statistical estimation on imperfectly maintained system," *European Safety & Reliability Conference 2005 (ESREL 2005)*, Jun. 27-30, 2005, Tri-City, Poland, pp 351-356, 2005.

[17] S. M. Ross, *Simulation*, Academic Press, San Diego, 1997.

[18] C. P. Robert and G. Casella, *Monte Carlo Statistical Methods*, (second edition). New York: Springer-Verlag, 2004.

[19] C.-Y. Cheng, C.-F.Liaw, and M.Wang,. "Periodic preventive maintenance models for deteriorating systems with considering failure limit," *4th International Conference on Mathematical Methods in Reliability—Methodology and Practices*, Santa Fe, New Mexico, June, 2004.

Chun-Yuan Cheng received her M.S. and Ph.D. degrees in industrial engineering from Auburn University, Alabama, USA. She is currently an associate professor at Chaoyang University of Technology, Taiwan. Her research interests include modeling of reliability and maintainability, simulation, and quality engineering.