A Self-Adaptive Differential Evolution Algorithm with Dimension Perturb Strategy

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Abstract—**Differential Evolution (DE) has been proven to be an efficient and robust algorithm for many real optimization problems. However, it still may converge toward local optimum solutions, need to manually adjust the parameters, and finding the best values for the control parameters is a consuming task. In this paper that proposed a dimension perturb strategy and self-adaptive F value in original DE to increase the exploration ability and exploitation ability. Self-adaptive has been found to be highly beneficial for adjusting control parameters. The performance of self-adaptive differential evolution algorithm with dimension perturb strategy (PSADE) is showed on the following performance measures by benchmark functions: the solution quality and solution stability. This paper has found that PSADE can efficiently find the global value of these functions.**

Index Terms—**Differential Evolution, Dimension Perturb Strategy, Self-adaptive**

I. INTRODUCTION

The differential evolution algorithm (DE) was proposed by Storn and Price in 1996 [12] [14] [15], the algorithm is a popular optimization algorithm in recent years. Previous studies can be found that DE shows better results than other algorithms such as genetic algorithm (GA) and particle swarm optimization (PSO), in addition, DE is used in various fields widely.

Differential evolution is based on random population based global algorithm, like other evolutionary algorithm framework. It is not only well-known, robust, easy to use, fast convergence, but also requiring relatively little control parameters in solving optimization problems. Although DE has the advantages mentioned above, it encounters the crucial flaws of evolutionary computation, like trapped in the local solution easily, and adjust the parameters manually [12] [17].

In the past studies, the differential algorithm had much in common the improved methods that has improve control parameters [1] [4] [7] [16], modify the algorithm framework for process [3] [11] and with other algorithms [9] [17], the main purpose of these methods are trying to improve exploration ability and exploitation ability. Exploration means that the search to the global optimum capacity in the solution space, when the exploration capability is poor, the algorithm is easily trapped in the local optimum. And the exploitation ability means in the vicinity of global optimum solution can dig out the better solutions, if without adequate exploitation capacity that will cause instability in convergence, stretching computation time and lowering the quality of the solution.

Furthermore, the user must find the best parameter values for different problems. Finding the best values for the control parameters is a time consuming task [17].

Therefore, this paper presented a modified the control parameter and dimension perturb strategy. Through the above mentioned ways to improve

DE has to manually adjust the parameters and the optimal solution trapped into the local optimum issues. Followed by high-dimensional experiment to prove the modified algorithm can achieve a balance between the exploration ability and the exploitation ability.

The following of this paper is organized as follows: this paper briefly introduce the original DE algorithm in section II. Section III, A self-adaptive differential evolution algorithm with dimension perturb strategy (PSADE) is presented in detail. In section IV, the experimental settings and the experiment results are reported the PSADE performed on the benchmark functions. And this paper elaborates the findings in final section.

II. LITERATURE REVIEW

A. Evolution Computation

Evolutionary Computation (EC) is not only popular innovative search technology in the last decade, but also significantly alter the structure of the existing engineering computing. Evolutionary computation using computer models of evolution and selection process, to simulate Darwin "survival of the fittest" as the basis of natural biological evolution. The computer model, so-called

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evolutionary algorithm, is for solving many NP-hard optimization problems in engineering and science related fields [8].

Evolutionary algorithms (EAs) are population-based search algorithms to simulate the evolution of individual select, mutate and recombine process. According to the concept of evolutionary algorithm, the developed specifications include the following steps [8]:

- The initial phase: the initial population randomly generated.
- Assessment phase: assessment fitness for each individual of population. If meet the termination conditions will be terminated. Otherwise, continue the following steps.
- The selection phase: first, the individual selected from the group as a parent. Second, the parent through the various genetic operators to produce new offspring. Finally, assess the fitness value of offspring.
- Production phase: the decision to replace some or entire individual of the current group to become the next generation and then back to assessment phase.

B. Differential Evolution Algorithm

Differential evolution is a floating-point encoding of evolutionary algorithms for continuous global optimization solution space, it also can be discrete coding manner [2] [7] [10]. Differential evolution algorithm is similar to genetic algorithm used by the individuals composed of population to search for the optimal solution. But the biggest difference between genetic algorithm and differential algorithm is the mutation operators. Genetic algorithm only use small probability to implement the mutation operator.

On the other hand, mutation in the DE is the use of arithmetic formula to combine the individual in each generation. The mutation plays the role of explores at the beginning and becomes a developer at later evolution process, its individuals within their group will become increasingly similar [6] [12].

The original DE is introduced in more detail with reference to three main operators: mutation, recombination and selection [13] [14] [15].

- Mutation

For each individual vector, $x_i(t)$, of generation t, a mutant vector is created by

$$v_i(t) = x_{r1}(t) + F(x_{r2}(t) - x_{r3}(t)) \tag{1}$$

where $x_{r1}(t)$, $x_{r2}(t)$ and $x_{r3}(t)$ are randomly selected with $i \neq r_1 \neq r_2 \neq r_3$, and the F is scale factor used to effect the amplification of the difference vector, $x_{r2} - x_{r3}$, $F \in [0,2]$.

- Recombination

DE adds the diversity of population that uses a discrete recombination way where elements from the target vector, $x_i(t)$, are combined with elements from the mutant vector, $v_i(t)$, to produce the trial vector, $u_i(t)$.

$$u_{ij}(t) = \begin{cases} v_{ij}(t) & if\ rand < CR\ or\ j = r \\ x_{ij}(t) & otherwise \end{cases} \tag{2}$$

where *j=1,…,Nd* refers to a specific dimension, *Nd* is the number of parameters of single individual, and *rand~U(0,1)*, in the above, *CR* is the probability of crossover, $CR \in [0,1]$, and r is the randomly selected index, *r~U(1,…,Nd)*. In other words, the trial vector inherits directly some of elements of the mutant vector, Thus, even *CR*=0, at least one of the elements of the trail vector randomly selected.

- Selection

To select the trial vector $u_i(t)$ or target vector $x_i(t)$, DE employs a very easy selection process. Only when the fitness of the trial vector is better than the target vector that the generated trial vector $u_i(t)$ replaces the target vector $x_i(t)$ be a member of the next generation. For example, if this paper has the same functions in this paper for a minimization problem, the following selections rule such as:

$$x_i(t+1) = \begin{cases} u_i(t), & f(u_i(t)) < f(x_i(t)) \\ x_i(t), & else \end{cases} \tag{3}$$

where if the trial vector has smaller or equal fitness value than the corresponding target vector, the trial vector will replace the vector, otherwise, the target vector still remain in the population.

Recently, self-adaptive has been proven to be very useful in the automatic and dynamic adjustment of the evolution algorithm of control parameters such as mutation rate and crossover rate. Self-adaptive evolution strategy allows re-configuration to adapt to any general problems [2].

In DE, self-adaptive often used in mutation and recombination of the control parameters, the differential evolution algorithm to set the control parameters mutation and recombination than the third set control parameters on number of population is more sensitive [2].

Sum up, we can be divided into two main types of setting parameter values; parameter tuning and parameter control, parameter tuning is a more common approach is to find out the true meaning of the parameter values before the implementation of the algorithm, and then adjustment algorithm using these values. The parameter control is the parameter value during generation will change and this change can be divided into three [2][5].

- Deterministic parameter control:

It's changed with a number of decisive rules for parameter value.

- Adaptive parameter control:

Parameter value assignment strategy may involve some form of feedback mechanism used to determine the direction of the search with (or) significant changes in policy parameters.

- Self-adaptive parameter control:

The evolutionary algorithm can be applied to the evolution of the concept of self-adaptive parameters. Here the parameters are coded to the individual (genes), the better values of these encoded parameters lead to

better individuals. In other words, it will be easier to survive and produce offspring.

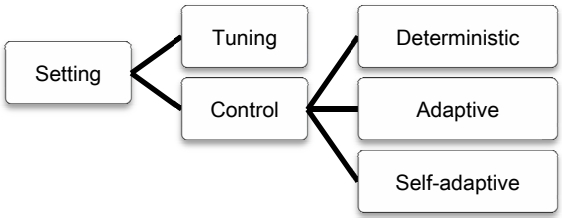


Figure 1. Types of setting parameters [5]

III. A Self-Adaptive Differential Evolution Algorithm with Dimension Perturb Strategy(PSADE)

In this section, this research will improve the structure of differential algorithm and import dimension perturb strategy. This research anticipates that when the individuals trap into the local optimum, it can give individuals the opportunity to escape the local optimum. Hence, the DE can increase the diversity of population, and to continue the evolution in the same time.

In addition, this study take the most widely used DE/rand/1 strategy of differential algorithm with self-adaptive in F, F values are about with the convergence rate in solving different optimization problems. If scholars want to manually adjust to the best parameter in the implementation process, they will have to by very many experiments can be found. In other words, finding the best setting parameter value from different experiment is a very time-consuming work. Therefore, the concept is to retain good F values and let the individual of the next stage as a reference.

A. Dimension Perturb Strategy

First, the current iteration of the best individual vector carry out the action dimension perturb, that is, randomly selected to perform the exchange of two elements of the action from the best individual vector. Its main objective is that at the beginning of evolution, due to the differences of each individual vector is very large, such a mechanism can be achieved through the jumping ability to global exploration. Then in the latter part of evolution, the differences between individual vectors will slowly shrink, and then through the perturb mechanism can enhance the population diversity and the ability to escape from the local optimum.

B. F value of Self-adaptive

The F value is related to the solution convergence rate, and different test functions may have the best value itself. The concept is to retain good F values and let the individual of the next stage as a reference.

This study assumes F normally distributed in a range with mean Fa and standard deviation 0.15. In the beginning, Fa is set at 0.5 and different F values conform this normal distribution are for each individual in the current population. The method is then calculated as

$$v_i(t) = x_{r1}(t) + F_i(t) * (x_{r2}(t) - x_{r3}(t)) \quad (4)$$

where

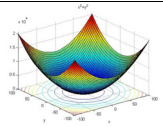
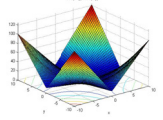
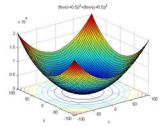
$$F_i(t) = N(Fa, 0.15) \quad (5)$$

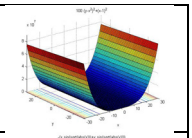
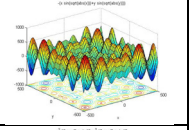
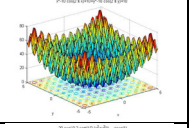
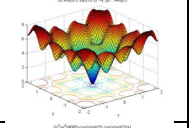
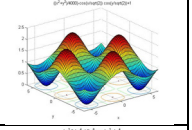
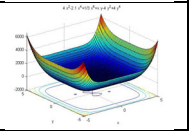
These F values for all individuals remain for several generations and then a new set of F values is generated under the same normal distribution. During every generation, the F values associated with trial vectors successfully entering the next generation are saved. After a specified number of generations, F has been changed for several times under the same normal distribution with mean Fa and standard deviation 0.15, and after every trail vector to be successful reservations via selection mechanism to the next generation, F value will be recorded along. With this new normal distribution's mean and the standard deviation 0.15, we repeat the above process. In addition, if there are no successful individuals during this period, the Fa will not change. As a result, the proper F value range for the current problem can be learned to suit this problem.

IV. Experiment Result

In this study, the test function can be divided into the unimodal as well as the multimodal functions, which contains small and large local optimum of two optimal types, the assessment criteria will be used to solve the solution to the mean and standard deviation for precision and stability of performance assessment.

Table I. Benchmark functions

Benchmark functions		f_{min}	S	Graphic
Sphere	$f_1(x) = \sum_{i=1}^{n} x_i^2$	0	$[-100,100]^D$	
Schwefel's problem 2.22	$f_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	0	$[-10,10]^D$	
Step	$f_3(x) = \sum_{i=1}^{n} (\lfloor x_i + 0.5 \rfloor)^2$	0	$[-100,100]^D$	

Rosenbrock	$f_4(x)=\sum_{i=1}^{n-1}\left(100\left(x_i-x_{i-1}^2\right)^2+\left(x_{i-1}-1\right)^2\right)$	0	$[-30,30]^D$	
Schwefel 's Prblem 2.26	$f_5(x)=-\sum_{i=1}^{n}\left(x_i \sin\left(\sqrt{\lvert x_i\rvert}\right)\right)$	-12569.5	$[-500,500]^D$	
Rastrigin	$f_6(x)=\sum_{i=1}^{n}\left(x_i^2-10\cos\left(2\pi x_i\right)+10\right)$	0	$[-5.12,5.12]^D$	
Ackley's	$f_7(x)=-20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D}x_i^2}\right)-\exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\right)+20+e$	0	$[-32,32]^D$	
Griewank	$f_8(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	0	$[-600,600]^D$	
Six-hump Camel-back	$f_9(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	-1.0316285	$[-5,5]^D$	

A. *Experiment with Original Differential Evolution*

Table II. Experiment parameters setting with original differential evolution

NP	50
Dimension	30
F	0.5
CR	0.9; PSADE1~N(0.5,0.1)
Initial Fa	0.5
Update Fa Generation	50
Generation	1000

This section will be conducted comparison experiment with DE/rand/1, DE/best/2 and DE / current to best / 1. The CR 0.9 of PSADE refer to Storn with Price, and the PSADE1 refer to Qin and Huang, who made use of the normal distribution of mean 0.5 and standard deviation 0.1 for differential individuals in the current population [11]. Other parameters set as shown in table II.

Table V summarizes the results obtained by applying the different methods to the unimodal problems. The result shows that PSADE and PSADE1 performed better than (or at least equal to) the other strategies in all the benchmark functions except the Rosenbrock function where DE/best/2 outperformed the other strategies. However, PSADE1 performed significantly better than PSADE.

Table V summarizes the results obtained by applying the different methods to the multimodal problems. The result shows that PSADE1 performed consistently better than the other strategies in all the test functions. The improvement is even more significant for the Rastrigin function. Furthermore, examining the standard deviations of all the algorithms, PSADE1 achieved the smallest standard deviation, illustrating that PSADE and PSADE1 are more stable and thus more robust than the other versions of DE.

Fig. 2 shows average best fitness curves for the different strategies with 30 independent runs for selected benchmark functions. Fig. 2 shows that PSADE1 has a faster convergence rate than DE/rand/1, DE/best/2, DE/Current to best/1 and PSADE. On the other hand, the figure shows that PSADE1 converges insignificantly faster than DE/best/2. Hence, from table V and Fig. 2 it can be concluded that, PSADE1 reached the global optimum solution faster than the other strategies in all functions except for the Rosenbrock function.

B. *Solution of the effectiveness of Fa update frequency*

This section will examine the updated *Fa* effectiveness. To find out the self-adaptive update frequency will affect or not affect the performance of the solution.

Set of experimental parameters such as table III.

Table III. Experiment parameters setting with update *Fa* frequency

NP	50
Dimension	30
F	0.5
CR	0.9; PSADE1~N(0.5,0.1)
Initial Fa	0.5
Update Fa Generation	25;50;100
Generation	1000

PSADE1 can offer the best output by updating the *Fa* value strategy with every 25 iterations, on the other hand, PSADE1 advantage with every 100 iterations to update the *Fa* value strategies offer more prominent effect output in Rosenbrock function.

Solving multi-modal functions, can still be observed PSADE1 with updates every 25 iteration strategy *Fa* values are output to provide better accuracy and stability.

Fig. 3 shows average best fitness curves for the different strategies with 30 independent runs for selected benchmark multimodal functions. Fig. 3 shows that PSADE1 (25) and PASDE1 (50) achieved faster convergence rates than others. Besides, PSADE1 (25) converged insignificantly faster than PSADE1 (50).

Table VI shows the dynamic *CR* values strategy is better than fix *CR* values strategy. And different updating strategy of *Fa* frequency will also affect the accuracy and stability of the solution. Whether in the unimodal or multimodal of the benchmark functions, the updating the *Fa* value strategy with every 25 iterations provided the most outstanding results.

C. With BBDE in high-dimension experiment

Table IV. Experiment parameters Setting with BBDE[10]

NP	50
Dimension	100
F	0.5
CR	0.9; PSADE1~N(0.5,0.1)
Initial Fa	0.5
Update Fa Generation	25
NFEs	50000

In this section, the function dimension raised to 100. This paper will choose the Omran et al, who published BBDE [10] algorithm to compare the object in 2009. The article has put forward an improved mechanism and control parameters. The BBDE is based on self-adaptive differential algorithm with almost free parameter fine-tuning of the hybrid differential improvement algorithm.

The main motivation is that when encountering new problems, PSO and DE are required to optimize the control parameters set, it can be regarded as the best set parameters dependence. In addition, self-adaptive differential algorithm have been studied to show good solution results, and it can reduce the time that to find the best parameter value of each problems. Therefore, this study will compare this research, to verify the method of this study and conduct precision on the performance comparison.

Set of experimental parameters in table IV. The four of the unimodal functions, the BBDE and PSADE1 presented results superior to the traditional differential algorithm. Hence, comparison results can be seen that BBDE and PSADE1 has the advantage to solve Sphere and Schwefel's problem 2.22 through the data.

The Step function is not a continuous problem type of the unimodal function, PSADE and PSADE1 still can maintain the effectiveness of the stability of its solution. But for the Rosenbrock function, BBDE are more prominent than other algorithms. According to unimodal functions of experimental data, PSADE1 and BBDE have faster convergence, more stable and more accurate than the traditional differential algorithm.

For the multimodal functions, the Rastrigin function and Griewank function has a lot of local optimum solutions. Table VII can be found that the PSADE1 has better accuracy than BBDE and traditional differential algorithm. On the other hand, BBDE and the traditional differential algorithm are not particularly outstanding to solve these functions.

For the Ackley function, BBDE and PSADE1 are able to meet the better stability and accuracy. Therefore, in multimodal test functions, PSADE1 algorithm presented the excellent results in the same condition. In other words, when PSADE1 encounter more complex and more local optimum solution multimodal functions, the application of perturb strategy have the opportunity to escape from the region and self-adaptive of *F* values can fast convergence in every function.

V. CONCLUSION

The differential algorithm through the self-adaptive parameter *F* and the dimension perturb strategy is to make up for the shortcomings. In order to enhance the exploration ability, exploitation ability and increase the population diversity, so that algorithms can be more appropriate speed convergence towards the optimal solution and effectively promote to escape from the local optimum.

Expectations effectively enhance the effectiveness of the algorithm for solving accuracy and stability. Meanwhile, the conclusions are sorted out from the experiment.

First, the parameters tuning is a time-consuming work, therefore, this study use self-adaptive mechanism for *F* value, it can reduce the need for different functions to do manually adjust the parameters to find the best setting time. Second, when trapped into the local optimum, the algorithm with dimension perturb strategy can enhance the capacity of the escaping from local optimum area. Third, if the problem complexity increases, the PSADE1 is able to maintain a stable and accurate status for solving effectiveness. Finally, the convergence evolution diagram can understand that the proposed algorithm has fast convergence speed. In other words, PSADE1 can be reached with less iteration to evolution.

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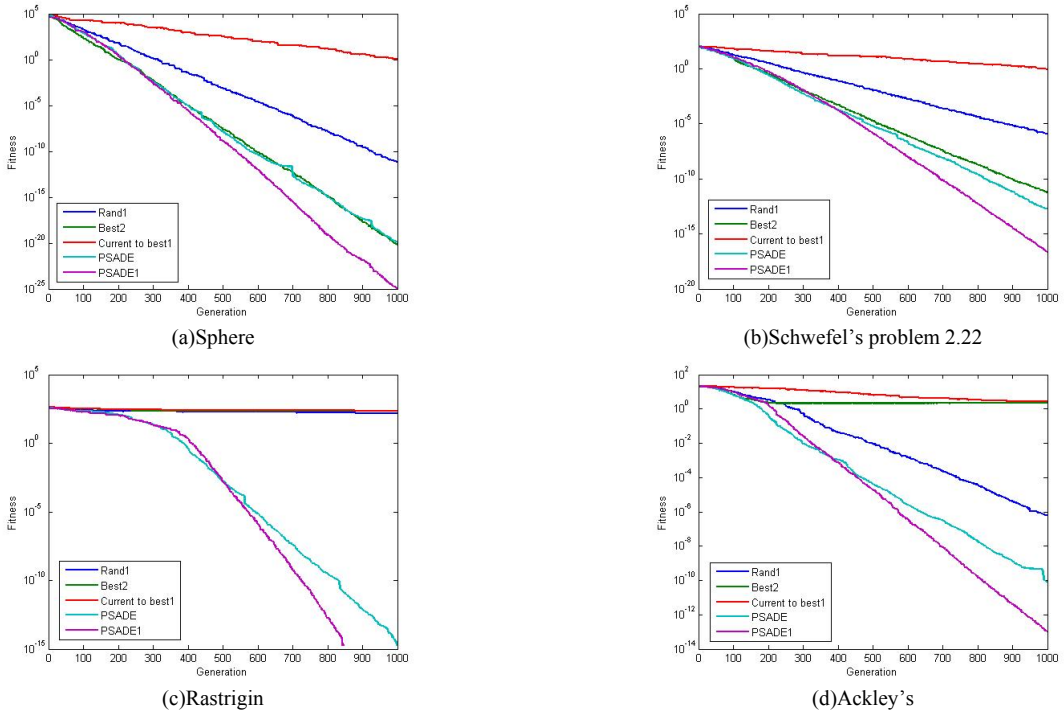
Table V. Mean and standard deviation (SD) of the optimization results for 30 runs

	rand1	best2	currenttobest1	PSADE	PSADE1
Sphere	3.123e-011 (4.097e-011)	6.352e-021 (9.549e-021)	1.867e+000 (8.064e-001)	1.818e-019 (2.848e-019)	**1.712e-025 (3.139e-025)**
Schwefel's problem 2.22	1.154e-006 (4.293e-007)	9.996e-012 (1.202e-011)	1.053e+000 (2.267e-001)	6.534673e-014 (6.144e-014)	**4.392549e-017 (2.794e-017)**
Step	**0 (0)**	3.633e+000 (4.319e+000)	6.100e+000 (2.482e+000)	**0 (0)**	**0 (0)**
Rosenbrock	1.299e+002 (2.729e+002)	**1.685e+001 (1.661e+001)**	3.207e+002 (3.162e+002)	2.569e+002 (4.459e+002)	3.767e+001 (2.721e+001)
Schwefel 's Prblem 2.26	-7.402e+003 (9.969e+002)	-9.888e+003 (6.148e+002)	-4.958e+003 (4.100e+002)	**-1.256e+004** (6.702e-008)	**-1.256e+004 (1.098e-009)**
Rastrigin	1.631e+002 (2.888e+001)	1.951e+002 (1.743e+001)	2.315e+002 (1.582e+001)	7.153e-013 (2.334e-012)	**0 (0)**
Ackley's	1.426e-006 (7.270e-007)	1.199e+000 (7.978e-001)	2.289e+000 (5.257e-001)	8.465e-011 (9.985e-011)	**7.170e-014 (3.272e-014)**
Griewank	2.053e-003 (4.344e-003)	1.206e-002 (1.088e-002)	9.675e-001 (5.725e-002)	6.583e-004 (2.586e-003)	**0 (0)**
Six-hump Camel-back	**-1.0316285 (4.5168e-016)**	**-1.0316285 (4.5168e-016)**	**-1.0316285 (4.5168e-016)**	**-1.0316285 (4.5168e-016)**	**-1.0316285 (4.5168e-016)**



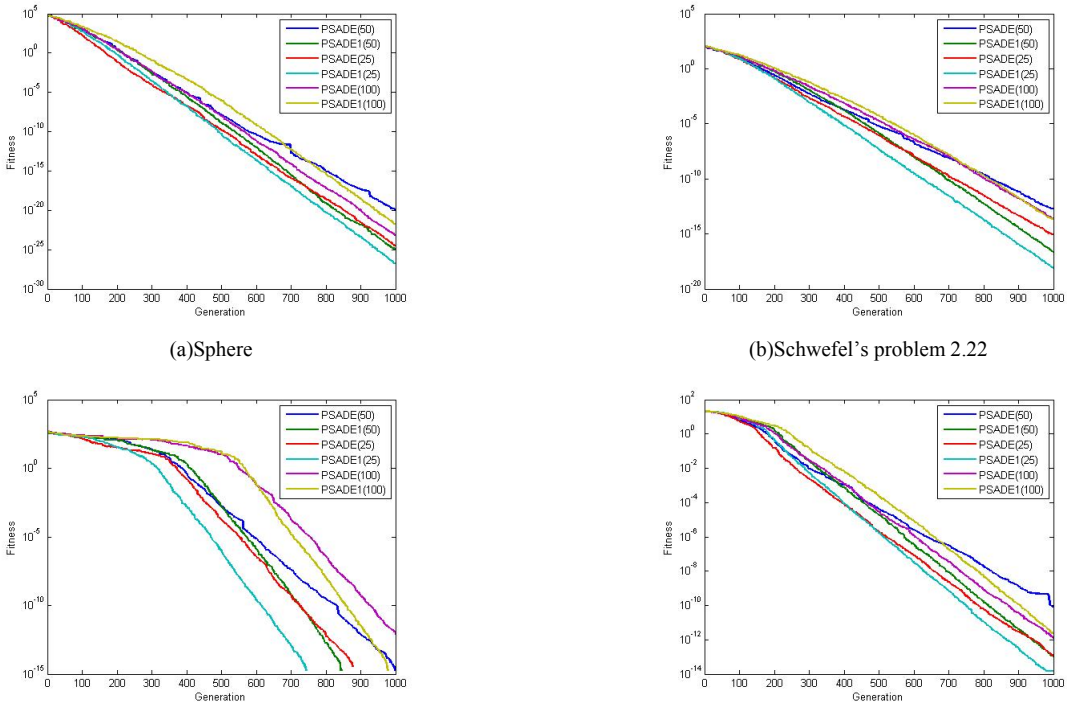
(a)Sphere (b)Schwefel's problem 2.22

(c)Rastrigin (d)Ackley's

Figure 2. Convergence Graph for the optimum solution

Table VI. Mean and standard deviation (SD) of the optimization results for 30 runs

	PSADE (25)	PSADE1 (25)	PSADE (50)	PSADE1 (50)	PSADE (100)	PSADE1 (100)
Sphere	2.470e-024 (4.586e-024)	**7.899e-028 (8.669e-028)**	1.818e-019 (2.8488e-019)	1.712e-025 (3.1394e-025)	2.858e-023 (8.199e-023)	1.572e-022 (1.349e-022)
Schwefel's problem 2.22	4.121e-016 (4.555e-016)	**6.771e-019 (2.867e-019)**	6.534e-014 (6.144e-014)	4.392e-017 (2.794e-017)	1.585e-014 (8.835e-015)	2.490e-014 (1.273e-014)
Step	**0 (0)**	**0 (0)**	**0 (0)**	**0 (0)**	**0 (0)**	**0 (0)**
Rosenbrock	1.073e+002 (1.596e+002)	5.379e+001 (4.030e+001)	2.569e+002 (4.459e+002)	3.767e+001 (2.721e+001)	**3.522e+001 (2.009e+001)**	**2.700e+001** (4.043e-001)
Schwefel 's Prblem 2.26	-1.233e+004 (1.288e+003)	**-1.256e+004 (2.026e-012)**	**-1.256e+004** (6.702e-008)	**-1.256e+004** (1.098e-009)	**-1.256e+004** (4.262e-003)	**-1.256e+004** (5.210e-004)
Rastrigin	**0 (0)**	**0 (0)**	7.153e-013 (2.334e-012)	**0 (0)**	1.275e-011 (6.833e-011)	1.776e-016 (7.151e-016)
Ackley's	3.457e-013 (3.114e-013)	**1.048e-014 (2.822e-015)**	8.465e-011 (9.985e-011)	7.170e-014 (3.272e-014)	9.140e-013 (9.041e-013)	2.677e-012 (1.012e-012)
Griewank	4.351e-003 (7.216e-003)	**0 (0)**	6.583e-004 (2.586e-003)	**0 (0)**	2.465e-004 (1.350e-003)	**0 (0)**



(a)Sphere (b)Schwefel's problem 2.22

(c)Rastrigin (d)Ackley's

Figure 3. Convergence Graph for the optimum solution

Table VII. Mean and standard deviation (SD) of the optimization results for 30 runs

	rand1	best2	BBDE[10]	PSADE	PSADE1
Sphere	32.091642 (40.145155)	1.959712 (2.750338)	**0.000000 (0.000000)**	0.000046 (0.000045)	**0.000000 (0.000000)**
Schwefel's problem 2.22	1.544770 (1.189442)	0.760741 (0.671501)	**0.000000 (0.000000)**	0.000151 (0.000071)	**0.000000 (0.000000)**
Step	158.8333 (220.6941)	932.7 (522.7707)	2.70000 (5.01824)	**0.000000 (0.000000)**	**0.000000 (0.000000)**
Rosenbrock	85618.17 (168799.9)	72253.2 (93453.1)	**312.63207 (195.54631)**	835.0324 (525.9118)	562.1907 (475.7651)
Rastrigin	774.7837 (71.39962)	806.7959 (232.3332)	616.194754 (38.11584)	1.292809 (3.239625)	**0.001120 (0.001623)**
Ackley's	2.435670 (0.472041)	6.881239 (1.129922)	**0.000000 (0.000001)**	0.000528 (0.000187)	0.000004 **(0.000001)**
Griewank	1.168208 (0.225261)	0.465042 (0.252081)	0.001640 (0.005296)	0.009988 (0.016258)	**0.000000 (0.000000)**